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Time-Varying Correlations between Selected Exchange Rates: A Robust DCC-GARCH Model Approach

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Abstract

Effective diversification of global portfolios and risk management depends to a large extent on the degree of correlation between the returns of financial assets. The Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroscediency (DCC-GARCH) model addresses the time-varying correlation and volatility among financial assets. The model has a two-step process whereby the volatility of each individual asset is first estimated using the univariate GARCH and the time-varying correlation between those assets is then captured using the DCC framework. Although the DCC-GARCH setup allows for the use of any univariate GARCH model for the variance of each financial asset, when constructing a DCC-GARCH model, most researchers typically use only one univariate GARCH model for all the returns of the financial asset. This study proposes a robust DCC-GARCH model in analyzing the dynamics of time-varying correlations of financial data in which the optimal univariate GARCH models (with the lowest information criteria) for each financial asset are used to build up the DCC-GARCH model. An empirical application of this model in analyzing the time-varying conditional correlation between four exchange rates shows an improvement in results (relative to the information criterion values) compared to the case where only one univariate GARCH model is used to construct the DCC-GARCH model. Standardized residuals in the quasi maximum likelihood estimation (QMLE) procedure for DCC-GARCH parameters, are typically assumed to follow a multivariate Gaussian distribution. One stylized fact of financial data returns is that the residuals are heavy-tailed. We consider the case where the standardized residuals follow a multivariate Gaussian, and the case where they follow either a multivariate Student's t-distribution or a multivariate Laplace distribution. Results show that a robust DCC-GARCH model performs better when the standardized residuals follow a multivariate Student's t-distribution.

Keywords: multivariate GARCH, DCC-GARCH, time-varying correlation, exchange rates, portfolio diversification.

1. Introduction

The understanding and prediction of the temporal dependence in the second-order moments of asset returns is of great importance in financial time series (Black 1976). This feature is better recognized through a multivariate modeling framework, as it leads to more useful empirical models than working with separate univariate models.

One of the applications of the Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) model is the study of correlations between assets. Extending from making use of univariate models to multivariate models leads to better decision tools in areas such as portfolio selection, asset pricing, hedging, and the forecasting of value-at-risk (Orskaug 2009). A well-known multivariate volatility model within the class of multivariate generalizations of GARCH models suitable for investigating the volatility of financial data and the dynamic correlations of market interdependence is the Dynamic Conditional Correlation (DCC) model of Engle (2002). Other models within this class include the Constant Conditional Correlation (CCC) model of Bollerslev (1990), the Baba, Engle, Kraft, and Kroner (BEKK) model of Engle and Kroner (1995), and the Exponentially Weighted Moving Average (EWMA) model of Morgan (Morgan *et al.* 1997).

In our study, we choose the DCC model over the BEKK and EWMA models, given the positive definiteness of the time-dependent conditional correlation matrix it guarantees at every point in time. This model is also relatively parsimonious as the number of parameters grows linearly with the number of assets, making the model suitable for a large set of financial assets. The DCC-GARCH model, which parameterizes the conditional correlations directly, is estimated in two steps. First, a series of univariate GARCH models are estimated for each asset. Second, the model parameters of the correlation between the assets are estimated using the standardized residuals obtained from the first step. In this way, the number of parameters to be estimated in the correlation process becomes independent of the number of series to be correlated, giving a clear computational advantage over other MGARCH models (Lestano and Kuper 2016).

Even though the setup of the DCC-GARCH model allows the usage of any univariate GARCH model for the variance of each financial asset, most researchers usually make use of just one univariate GARCH model for all the financial asset returns in constructing the DCC-GARCH model. Wu, Md Yusof, and Misiran (2024) used the DCC-GARCH model to study the correlation between three trading daily stocks in the agricultural sector of the Chinese stock market. They assumed, according to Bollerslev, Chou, and Kroner (1992), that the GARCH(1,1) was the best model that defined the variance equation of the univariate GARCH models and used it to build up the DCC-GARCH model. Similarly, Maharana, Panigrahi, Chaudhury, Uprety, Barik, and Kulkarni (2025), in their paper on economic resilience in post-pandemic India, used the GARCH(1,1) for each time series (index) as an estimate of the univariate GARCH model that was used to build up a VAR-DCC-GARCH model. They, however, acknowledged the need to further adjust the lag structure if the results of the GARCH(1,1) model diagnostics for autocorrelation are not satisfied. Ampountolas (2022) used the DCC-GARCH model to evaluate the price volatilities of a range of four cryptocurrencies. The sGARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), and TGARCH(1,1) models were used arbitrarily to build up the DCC-GARCH model going by the conclusion of Hansen and Lunde (2005). Burckhardt (2012) in his research on the correlation of foreign exchange rates also admitted the need of not excluding the possibility of using several lags in the univariate GARCH models that are used to build up the DCC-GARCH model. Irani, Al. Al. Hadood, Gökmenoğlu, and Athari (2025) equally used an autoregressive integrated moving average, the ARFIMA (2,d,1) - GARCH (1,1) process, to build the DCC-GARCH model in their paper on the impact of financial market uncertainty and financial crises on dynamic stock - foreign exchange market correlations. In their article on the dynamic conditional correlation analysis of financial market interdependence, Lestano and Kuper (2016) first applied the GARCH(1,1) with an AR(2) filter to remove serial correlation and used the filtered series in estimating the DCC model.

The use of a single univariate GARCH model for each time series, mostly the GARCH(1,1), to build up the DCC-GARCH model excludes the possibility that each of the time series could be properly described using different GARCH models.

We propose a more robust DCC-GARCH model in analyzing the dynamics of time-varying correlations of financial data. Optimal univariate GARCH models are those that provide the best fit and possibly the best forecasting performance for a given dataset. In this study, an optimal univariate GARCH model will refer to one of the models; (GARCH(p, q), EGARCH(p, q), and GJR-GARCH(p, q), with $p, q \in \{1, 2\}$) that has the lowest information criterion. All the optimal univariate GARCH models for each financial asset are used to build the DCC-GARCH model. This is in contrast to the situation where a single univariate GARCH model is applied to all the financial assets.

The standardized residuals of the DCC-GARCH model are usually assumed to follow a multivariate Gaussian distribution. One of the empirical findings of the returns of financial data is that the residuals are heavy-tailed and thus, non-Gaussian. When the residuals have a heavy-tailed distribution, such as the Student's t-distribution and Laplace distribution, QMLE can be used to estimate the model parameters. This approach ensures that the volatility estimates remain reliable and robust, despite the presence of extreme values in the data. We analyze and compare results of the DCC-GARCH model with the assumption that the standardized residuals follow: (i) multivariate Gaussian distribution, (ii) multivariate Student's t-distribution, and (iii) the Laplace distribution.

The main contribution of this study is the analysis of the dynamics of the time-varying correlation relationship between some selected exchange rates using the proposed robust DCC-GARCH model. In terms of trading volume, the foreign exchange market is one of the largest capital markets (Butt, Ramzan, Wong, Chohan, and Ramakrishnan 2023). Exchange rates are a key macroeconomic variable that reflects the interdependence between national economies and the international financial market (Brice and Jules 2024). Companies and investors pay particular attention to exchange rates so as to manage exchange risk effectively and diversify their portfolios, respectively. Speculators also look to the co-movement of exchange rates in order to take advantage of market opportunities. The following exchange rates will be considered in this study: USD/XAF, GBP/XAF, JPY/XAF, and CNY/XAF (where USD=United States Dollar, GBP=Great British Pound, JPY=Japanese Yen, CNY=Chinese Yuan, XAF=ISO symbol for the Franc of the Financial Community of Africa (FCFA)). The euro has been left out since XAF is pegged to the euro. Even though some research has been carried out on modeling the above exchange rates using univariate GARCH models (see Oben (2018), Dinga, Claver, Kum, and Che (2023), Dinga, Claver, Kum, and Che (2025)), to the best of our knowledge, the DCC-GARCH model, which is used to analyze the dynamics of the correlation relationship between these exchange rates, has not been considered by previous studies. This study also emphasizes the importance of making use of a heavy-tailed multivariate standardized residual distribution in the implementation of the DCC-GARCH model. It is hoped that the results from this study will lead to the understanding of the foreign exchange market dynamics for effective diversification.

The rest of the paper is organized as follows: Section 2 introduces univariate and multivariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. Section 3 presents the data analysis and results, while the discussion of results and conclusion is found in Section 4.

2. GARCH models

2.1. Preliminaries

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are widely used in time series analysis to model and forecast time-varying volatility. In studying GARCH models, the important preliminary concepts we require include an understanding of basic time series concepts like stationarity, autocorrelations, heteroskedasticity, log returns, conditional mean, and conditional variance. We equally have to be familiar with statistical tests that check data suitability for GARCH modeling.

Most financial returns usually show similar statistical patterns, commonly referred to as stylized facts (Francq and Zakoian 2019). Some stylized facts for daily returns include uncorrelated returns but auto-correlated squared returns, (heavy-tailed) residual distributions that are non-normally distributed with non-zero skewness and a kurtosis that is greater than 3, and volatility clustering, which suggests the presence of time-dependent variance (heteroscedasticity) (Nelson 1991). Traditional models used in time series analysis, such as the ARMA models, assume that the variance is time invariant (homoscedastic). GARCH models, on the other hand, have the property of time-varying conditional variance and can therefore capture volatility clusters and other stylized facts common to financial time series. GARCH models may be either univariate or multivariate.

When applying the GARCH model to the price history of a financial asset, we make sure that the data is in a consistent format and covers a significant period of time to capture patterns of volatility. The price series are converted into return series as GARCH models are typically applied to returns rather than prices. Supoose r_t is the log return for a financial asset with price P_t at time t. Also, let P_{t-1} be the price at the previous time step. Then the square of the log return is

$$r_t^2 = (\log P_t - \log P_{t-1})^2 . \tag{1}$$

Equation (1) is often used as a proxy in the modeling of volatility (σ_t) given that volatility is a latent variable (Poon 2005).

The conditional mean of log returns is the expected value of the returns at time t given all past information up to time $t-1$ and is given by $\mu_t = \mathbb{E}(r_t|\mathcal{F}_{t-1})$, where $\mathcal{F}_{t-1}$ is the information available at time $t-1$. On the other hand, the conditional variance of log returns is given by $\sigma_t^2 = \mathrm{Var}(r_t|\mathcal{F}_{t-1}) = \mathbb{E}[(r_t - \mu_t)^2|\mathcal{F}_{t-1}]$.

We note that μ_t captures the predictable component of the returns (expected future return); meanwhile, σ_t^2 captures the time-varying volatility of the returns. Squaring the log returns helps capture the volatility (or variance) of the returns, as it emphasizes larger deviations.

2.2. Some univariate GARCH models

Engle (1982) noticed that the volatility of time series could be modeled using an ARMA-type process which he referred to as the Autoregressive Conditional Heteroscedasticity (ARCH) model. This model was subsequently generalized to the Generalized ARCH (GARCH) model by Bollerslev (1986).

GARCH(p,q) model

Let $z_t \sim D(0,1)$ be a sequence of independent and identically distributed (i.i.d) random variables which is assumed to be independent of the volatility σ_t and ε_{t-k} for $k > 0$. Then ε_t is called a GARCH(p,q) process if it has mean equation given by Equation (2) and variance equation given by Equation (3) respectively where $\omega > 0, \alpha_i \geq 0$ and $\beta_j \geq 0$.

$$r_t = \mu_t + \varepsilon_t, \ \varepsilon_t = \sigma_t z_t, \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^2. \tag{3}$$

The GARCH (p,q) model assumes that the response of volatility to a shock depends on the strength of the shock only ignoring information on the sign of ε_{t-i}. This suggests that ε_{t-i} has the same effect on volatility irrespective of whether $\varepsilon_{t-i} > 0$ or $\varepsilon_{t-i} < 0$ as seen in Equation (3). This is however not true as empirical evidence suggests that positive and negative values of ε_{t-i} of the same magnitude may sometimes lead to different responses in the volatility, known as leverage effect (Lim and Sek 2013). Various extensions of the GARCH model have therefore been developed in order to take into consideration this asymmetric characteristic of volatility, some of which include;

EGARCH(p, q) model

This model was developed by Nelson (1991), and captures the asymmetric response of volatility by including an asymmetric parameter in the model. The mean equation is similar to Equation (2) and the variance equation is given by,

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^{q} \beta_j \log\left(\sigma_{t-j}^2\right), \tag{4}$$

where α_i measures the magnitude of the shock, β_j measures the persistence in conditional volatility of the shocks to the market while γ_i is the asymmetric parameter that measures the leverage effect. The model is asymmetric when $\gamma_i \neq 0$ and has leverage effect when $\gamma_i < 0$.

GJR-GARCH (p, q) model

This model that was developed by Glosten, Jagannathan, and Runkle (1993) has variance equation defined as,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I_{t-i})\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^2, \tag{5}$$

where the indicator function $\mathbb{I}_t = 0$ if $\varepsilon_{t-i} \geq 0$ and 1 otherwise. σ_t is positive when $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\alpha_i + \gamma_i \geq 0$. The model is asymmetric when $\gamma_i \neq 0$ and has leverage effect when $\gamma_i > 0$ for $\varepsilon_{t-i} < 0$.

The parameters of univariate GARCH models are estimated using the QMLE procedure wherein the Maximum Likelihood Estimation (MLE) procedure is used to estimate the parameters even when the assumption that the residuals are normally distributed is false (Christoffersen 2011).

2.3. Multivariate GARCH models

The multivariate GARCH models broaden the GARCH framework to encompass several time series, enabling the modeling of time-varying volatilities and correlations between multiple assets.

Portfolio allocation decisions are influenced by the degree of covariation of prices or volatility following a shock. A negative covariance between two companies indicates the presence of diversification opportunities since there is a trade off of the profit and loss between the two companies. Investors and investment managers take particular interest in the co-movements of financial assets as it provides insights into the potential for diversification. Diversification opportunities between assets drop when the correlation between them is high (Solnik 1974). These issues can be directly studied using a multivariate model. This raises the question of the specification of the dynamics of the covariance and correlations between financial time series.

DCC-GARCH model

The dynamic conditional correlation (DCC) model combines univariate GARCH models for each time series with a dynamic correlation structure. The multivariate equivalent of univariate GARCH models is used to model the volatility of a vector of assets where the important variables are modeled together allowing for a dynamic and more realistic relationship between these variables. The DCC-GARCH model of Engle (2002) addresses the time-varying volatilities and correlations among financial assets (Özdemir 2022). This model is an extension of the CCC-GARCH model of Bollerslev (1990). This extension was necessary because empirical studies showed that the assumption of constant conditional heteroscedasticity in the CCC-GARCH model was too restrictive. The DCC-GARCH model is a simple model that can be used to identify the relationship between more than one univariate models by parameterizing the conditional correlation between these univariate estimators. Given that one of the statistical regularities of daily (log) returns is the fact that the distribution of the residuals is heavy-tailed, a version of the DCC-GARCH model with a multivariate Student's t-distribution and multivariate Laplace residual distribution will be used in this study, in comparison with that of the multivariate normal distribution, as suggested by Pesaran and Pesaran (2007) and Shiferaw (2019). In this way, the robustness of the estimated values to possible deviations from normality will be assessed.

According to Afuecheta, Okorie, Nadarajah, and Nzeribe (2024), let $\boldsymbol{\varepsilon_t} = (\varepsilon_{1,t} \cdots \varepsilon_{n,t})'$, be an $n \times 1$ vector of residuals from a portfolio with n financial assets and an $n \times 1$ vector of log returns $\mathbf{r_t}$. Then $\mathbf{r_t} = \boldsymbol{\mu_t} + \boldsymbol{\varepsilon_t}$, where $\boldsymbol{\mu_t} = \mathbb{E}(\boldsymbol{r_t}|\mathcal{F}_{t-1})$ and $\mathbb{E}(\boldsymbol{\varepsilon_t}|\mathcal{F}_{t-1}) = 0$. In this paper, $\boldsymbol{\mu_t}$ is modeled as an ARMA series (see Francq and Zakoian (2019)). The vector of residuals $\boldsymbol{\varepsilon_t}$ can be expressed as:

$$\boldsymbol{\varepsilon_t} = \boldsymbol{H_t}^{\frac{1}{2}} \boldsymbol{v_t}, \tag{6}$$

where $\boldsymbol{v_t} \sim N(\mathbf{0}, \boldsymbol{I_n})$ is the standardized residual, $\boldsymbol{H_t}$ is an $n \times n$ matrix of conditional variances of $\boldsymbol{\varepsilon_t}$ at time t and $\boldsymbol{H_t}^{\frac{1}{2}}$ is any $n \times n$ matrix at time t that can be obtained from the Cholesky factorization of $\boldsymbol{H_t}$. In a DCC-GARCH model, $\boldsymbol{H_t}$ is decomposed into a diagonal matrix of time-varying standard deviations $\boldsymbol{A_t} = \text{diag}(\sigma_{11,t}, \cdots, \sigma_{nn,t})$, and a time-varying conditional correlation matrix $\boldsymbol{R_t}$. This decomposition is expressed as,

$$\boldsymbol{H_t} = \boldsymbol{A_t R_t A_t}. \tag{7}$$

The matrix $\boldsymbol{A_t}$ is obtained from a univariate GARCH model since the conditional variance evolves according to some $\sigma^2_{ii,t},\ i = 1, 2, \cdots, n$ univariate GARCH process.

Many researchers usually use the GARCH $(1, 1)$ process according to Hansen and Lunde (2005) and thus assume that,

$$\sigma^2_{ii,t} = \omega_i + \alpha_{i1}\varepsilon^2_{i,t-1} + \beta_{i1}\sigma^2_{ii,t-1}. \tag{8}$$

This assumption may not always be correct. It is possible that the optimal univariate model is not the GARCH model or has more than one lag. Thus in this study, the optimal GARCH model of Equation (9) as per AIC with $P_i \in \{1, 2\}$ and $Q_i \in \{1, 2\}$ will be considered.

$$\sigma^2_{ii,t} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip}\varepsilon^2_{i,t-p} + \sum_{q=1}^{Q_i} \beta_{iq}\sigma^2_{ii,t-q},\ i = 1, 2, \cdots, n. \tag{9}$$

Here, $\omega_i > o,\ \alpha_{ip} > 0,\ \beta_{tq} \geq 0$, and $\sum_{p=1}^{P_i} \alpha_{ip} + \sum_{q=1}^{Q_i} \beta_{tq} < 1$ (Engle Robert and Kevin 2001).

EGARCH and GJR-GARCH will also be considered. From Equation (6), the standardized residuals,

$$\boldsymbol{v_t} = \boldsymbol{A_t}^{-1}\boldsymbol{\varepsilon_t}.$$

$\boldsymbol{R_t}$ is the time-varying conditional correlation matrix of the standardized residuals $\boldsymbol{v_t}$ and is given as;

$$\boldsymbol{R_t} = \text{diag}(\boldsymbol{B_t})^{-\frac{1}{2}}\ \boldsymbol{B_t}\ \text{diag}(\boldsymbol{B_t})^{-\frac{1}{2}},$$

where

$$\boldsymbol{B_t} = (1 - a - b)\overline{\boldsymbol{B}} + a\boldsymbol{v_{t-1}}\boldsymbol{v'_{t-1}} + b\boldsymbol{B_{t-1}}, \tag{10}$$

and a and b are non-negative real numbers such that,

$$a + b < 1. \tag{11}$$

The elements of $\boldsymbol{R_t}$ are of the form,

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}\, q_{jj,t}}}.$$

$\boldsymbol{B_t}$ and $\overline{\boldsymbol{B}}$ are $n \times n$ positive-definite matrices representing the conditional and unconditional variance-covariance matrices of $\boldsymbol{v_t}$ resulting from the estimation of the univariate GARCH models respectively.

$$\overline{\boldsymbol{B}} = \mathrm{Cov}(\boldsymbol{v_t v_t'}),$$

and a and b control the reaction to shocks and persistence respectively. If Equation (11) is true, then $\boldsymbol{R_t}$ varies over time because $\boldsymbol{B_t}$ will change over time. $\boldsymbol{R_t}$ is stationary and positive definite. If Equation (11) does not hold, the DCC-GARCH model would converge to the CCC-GARCH model with a constant conditional correlation matrix $\boldsymbol{R}$ (Gabauer 2020).

2.4. Estimating the parameters of the DCC - GARCH model

The parameters of the DCC-GARCH model are estimated using the QMLE procedure. We consider cases where the standardized residuals are assumed to be: (i) normally distributed, (ii) Student's t-distributed, and (iii) Laplace distributed.

Multivariate normally distributed residuals

For $\boldsymbol{v_t} \sim N(\boldsymbol{0}, \boldsymbol{I_n})$, the joint distribution of $v_1, \cdots, v_T$ is given by,

$$f(\boldsymbol{v_t}) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2}} \exp(-\frac{1}{2}\boldsymbol{v_t' v_t}).$$

We recall that $\boldsymbol{\varepsilon_t} = \boldsymbol{H_t}^{\frac{1}{2}} \boldsymbol{v_t}$. Through linear transformation of variables technique, the likelihood function for the paramters of the model $\boldsymbol{\theta}$ is given by,

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{t=1}^{T} (2\pi)^{-\frac{n}{2}} \left|\boldsymbol{H_t}\right|^{-\frac{1}{2}} \exp(-\frac{1}{2}\boldsymbol{\varepsilon_t' H_t^{-1} \varepsilon_t}). \tag{12}$$

According to Engle Robert and Kevin (2001), the model parameters $\boldsymbol{\theta}$, may be divided into two groups; $\boldsymbol{\theta} = (\boldsymbol{\phi}, \boldsymbol{\psi})$, where $\boldsymbol{\phi_i} = (\omega_i, \alpha_{1i}, \cdots, \alpha_{pi}, \beta_{1i} \cdots, \beta_{qi})$; $i = 1, 2, \cdots, n$, are the univariate GARCH parameters for the ith time series. The parameters of $\boldsymbol{B}_t$ in Equation (10) are $\boldsymbol{\psi}$=(a, b). Substituting $\boldsymbol{H_t} = \boldsymbol{A_t R_t A_t}$ in Equation (12) and taking the logarithm, we have,

$$\begin{aligned}\log(\mathcal{L}(\boldsymbol{\theta})) &= -\frac{1}{2}\sum_{t=1}^{T}\Big[n\log(2\pi) + \log(|\boldsymbol{H_t}|) + \boldsymbol{\varepsilon_t' H_t}^{-1}\boldsymbol{\varepsilon_t}\Big] \\ &= -\frac{1}{2}\sum_{t=1}^{T}\Big[n\log(2\pi) + 2\log(|\boldsymbol{A_t}|) + \log(|\boldsymbol{R_t}|) + \boldsymbol{\varepsilon_t' A_t}^{-1}\boldsymbol{R_t}^{-1}\boldsymbol{A_t}^{-1}\boldsymbol{\varepsilon_t}\Big]. \end{aligned} \tag{13}$$

The DCC model has a two stage estimation process: the estimation of the parameters $\boldsymbol{\phi}$ of the univariate GARCH models for each time series is done in the first stage and in the second stage, the correctly specified log-likelihood function in Equation (13) given the parameter $\boldsymbol{\phi}$ is used to estimate the parameters of $\boldsymbol{\psi}$. In stage one, the matrix $\boldsymbol{R_t}$ in Equation (13) is replaced with the identity matrix $\boldsymbol{I_n}$ to give the quasi-likelihood function of Equation (14) written as the sum of individual GARCH specification likelihoods as,

$$\log(\mathcal{L}_1(\boldsymbol{\phi})) = -\frac{1}{2}\sum_{i=1}^{n}\left[T\log(2\pi) + \sum_{t=1}^{T}\left(\log(\sigma_{it}^2) + \frac{\varepsilon_{it}^2}{\sigma_{it}^2}\right)\right]. \tag{14}$$

The $\log(\mathcal{L}_1(\boldsymbol{\phi}))$ function is maximized by separately maximizing each of the GARCH log likelihood functions. From Equation (14), the parameters of $\boldsymbol{\phi}$ and the conditional variance $\sigma_{ii,t}^2$, $i = 1, 2, \cdots, n$ for each time series are estimated. The conditional variances are then used to standardize the residuals obtained, from which the unconditional variance-covariance matrix $\overline{\boldsymbol{Q}}_{\boldsymbol{t}}$ can be estimated.

Given the estimated parameters $\hat{\boldsymbol{\phi}}$ in stage one, the parameters $\boldsymbol{\psi}$ are estimated in stage two using the correctly specified log-likelihood function in Equation (13). Ignoring the constant terms (including $\boldsymbol{A_t}$) results in the second stage quasi-likelihood function is given by,

$$\log(\mathcal{L}_2(\boldsymbol{\psi})) = -\frac{1}{2}\sum_{t=1}^{T}[\log(|\boldsymbol{R_t}|) + \boldsymbol{v_t'}\boldsymbol{R_t}^{-1}\boldsymbol{v_t}]. \tag{15}$$

Multivariate Student's t-distributed residuals

The joint distribution of the standardized residuals $v_1, \cdots, v_t$, when $\boldsymbol{v_t}$ follows a multivariate Student's t distribution is given by:

$$f(\boldsymbol{v_t}|w) = \prod_{t=1}^{T}\frac{\Gamma(\frac{w+n}{2})}{\Gamma(\frac{w}{2})(\sqrt{\pi(w-2)})^n(1+\frac{\boldsymbol{v_t'}\boldsymbol{v_t}}{w-2})^{\frac{n+w}{2}}}, \tag{16}$$

where Γ is the gamma function, and w is the shaping parameter (degrees of freedom). The likelihood function for $\boldsymbol{\varepsilon_t} = \boldsymbol{H_t^{\frac{1}{2}}}\boldsymbol{v_t}$ is given by,

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{t=1}^{T}\frac{\Gamma(\frac{w+n}{2})}{\Gamma(\frac{w}{2})(\sqrt{\pi(w-2)})^n\left|\boldsymbol{H_t^{\frac{1}{2}}}\right|(1+\frac{\boldsymbol{\varepsilon_t'}\boldsymbol{H_t}^{-1}\boldsymbol{\varepsilon_t}}{w-2})^{\frac{n+w}{2}}}. \tag{17}$$

Similar to the case for multivariate normal distribution, the parameter is divided into two groups: $\boldsymbol{\theta} = (\boldsymbol{\phi}, \boldsymbol{\psi}) = (\phi_1, \cdots, \phi_n; \boldsymbol{\psi})$, where $\boldsymbol{\phi} = (\omega_i, \alpha_{1i}, \cdots, \alpha_{pi}, \beta_{1i}, \cdots, \beta_{qi}), i = 1, 2, \cdots, n$ are the univariate GARCH parameters for the ith time series and $\boldsymbol{\psi} = (a, b, w)$. By taking the logarithm of Equation (17) and substituting Equation (10), the log-likelihood is given by,

$$\log(\mathcal{L}(\boldsymbol{\theta})) = \sum_{t=1}^{T}[\log\Gamma(\frac{w+n}{2}) - \log\Gamma(\frac{w}{2}) - \frac{n}{2}\log\pi(w-2) - \frac{1}{2}\log(|\boldsymbol{A_t}\boldsymbol{R_t}\boldsymbol{A_t}|)],$$

which results to,

$$\log(\mathcal{L}(\boldsymbol{\theta})) = -\frac{w+n}{2}\log(1+\frac{\boldsymbol{v_t'}\boldsymbol{A_t}^{-1}\boldsymbol{R_t}^{-1}\boldsymbol{A_t}^{-1}\boldsymbol{v_t}}{w-2}). \tag{18}$$

A two stage estimation procedure will equally be used to estimate the parameters just as in the case where the distribution is Gaussian. Stage one is identical to that in which the distribution is Gaussian, and the result is the quasi-likelihood function of Equation (14) as per the principle of QMLE (Engle Robert and Kevin 2001). In stage two, $\boldsymbol{A_t}$ is treated as a constant term since conditioning is done on the parameters from Stage one. $\boldsymbol{A_t}$ is thus excluded from the log-likelihood estimator in Equation (18) to give the second stage estimation which assumes that the residuals follow a multivariate Student's t-distribution as

$$\log(\mathcal{L}_2(\boldsymbol{\psi})) = -\frac{w+n}{2}\log(1+\frac{\boldsymbol{v_t'}\boldsymbol{R_t}^{-1}\boldsymbol{v_t}}{w-2}). \tag{19}$$

Multivariate Laplace distributed residuals

The joint distribution of the residual vector $\boldsymbol{\varepsilon_t} = \boldsymbol{H_t}^{\frac{1}{2}}\boldsymbol{v_t}$, when $\boldsymbol{\varepsilon_t}$ follows a multivariate Laplace distribution (see Rossi and Spazzini (2010)) is given by,

$$f\left(\boldsymbol{\varepsilon_t};\boldsymbol{m},\boldsymbol{H_t}\right)=\frac{2\exp\left(\boldsymbol{\varepsilon_t'H_t^{-1}m}\right)}{(2\pi)^{n/2}\left|\boldsymbol{H_t}\right|^{1/2}}\left(\frac{\boldsymbol{\varepsilon_t'H_t^{-1}\varepsilon_t}}{2+\boldsymbol{m'H_t^{-1}m}}\right)^{w/2}K_w\left(\sqrt{\left(2+\boldsymbol{m/H_t^{-1}m}\right)\left(\boldsymbol{\varepsilon_t'H_t^{-1}\varepsilon_t}\right)}\right),$$

where the vector $\boldsymbol{m}$ is the location parameter and the matrix $\boldsymbol{H_t}$ is the scale parameter of this distribution.

This distribution has mean $\boldsymbol{m}$ and conditional covariance matrix $\boldsymbol{H_t}+\boldsymbol{mm'}$. $w=(2-n)/2$ and $K_w(u)=\frac{(u/2)^w\Gamma(1/2)}{\Gamma(w+1/2)}\int_1^\infty e^{-ut}\left(t^2-1\right)^{w-1/2}dt$, $w\geq -1/2$ is the modified Bessel function of the second kind as shown in Abramowitz (1964). The log-likelihood function is given by,

$$\log\mathcal{L}\left(\boldsymbol{\theta}\right)=\sum_{t=1}^{T}\left[\boldsymbol{\varepsilon}_t'\boldsymbol{H}_t^{-1}\boldsymbol{m}-\frac{1}{2}\log\left|\boldsymbol{H}_t\right|+\frac{w}{2}\left(\log\left(\boldsymbol{\varepsilon}_t'\boldsymbol{H}_t^{-1}\varepsilon_t\right)-2\log\left(2+\boldsymbol{m}'\boldsymbol{H}_t^{-1}\boldsymbol{m}\right)\right)\right]$$
$$+\sum_{t=1}^{T}\left[\log K_w\left(\sqrt{\left(2+\boldsymbol{m}/\boldsymbol{H}_t^{-1}m\right)\left(\boldsymbol{\varepsilon}_t'\boldsymbol{H}_t^{-1}\varepsilon_t\right)}\right)\right]. \quad (20)$$

Then, following Equation (7), Equation (20) is written as,

$$\log\mathcal{L}(\boldsymbol{\theta})=\sum_{t=1}^{T}\left[\boldsymbol{\varepsilon}_t'(\boldsymbol{A}_t\boldsymbol{R}_t\boldsymbol{A}_t)^{-1}\boldsymbol{m}-\frac{1}{2}\log\left|\boldsymbol{A}_t\boldsymbol{R}_t\boldsymbol{A}_t\right|+\frac{w}{2}\left(\log\left(\boldsymbol{\varepsilon}_t'(\boldsymbol{A}_t\boldsymbol{R}_t\boldsymbol{A}_t)^{-1}\boldsymbol{\varepsilon}_t\right)\right)\right]$$
$$+\sum_{t=1}^{T}\left[\frac{w}{2}\left(-2\log(2+\boldsymbol{m}'(\boldsymbol{A}_t\boldsymbol{R}_t\boldsymbol{A}_t)^{-1}\boldsymbol{m})\right)\right]$$
$$+\sum_{t=1}^{T}\left[\log K_w\left(\sqrt{(2+\boldsymbol{m}'(\boldsymbol{A}_tR_t\boldsymbol{A}_t)^{-1}\boldsymbol{m})\,(\boldsymbol{\varepsilon}_t'(\boldsymbol{A}_t\boldsymbol{R}_t\boldsymbol{A}_t)^{-1}\boldsymbol{\varepsilon}_t)}\right)\right]. \quad (21)$$

We can now move on to maximize Equation (21) w.r.t. $\boldsymbol{\theta}$.

3. Data analysis and results

3.1. Data description and preliminary results

The datasets used in this study are the daily closing USD/XAF, GBP/XAF, JPY/XAF, and CNY/XAF exchange rates. They were downloaded from Stock Market Quotes & Financial News (2017). The time span under consideration was January 2, 2017, through September 30, 2022, and there were 1499 data points in each data set. The R software (Team 2020) and RStudio 2022.12.0 (Team 2022) were used for data analysis in this work.

The number of units of one currency that exchanges for a unit of another currency is referred to as the exchange rate (Dinga *et al.* 2023). A plot of these exchange rates is shown in Figure 1. The output is seen to be non-stationary with a non-constant mean and variance throughout the sample period. A drop in all the exchange rates can be noticed around early 2020 as a result of the COVID-19 pandemic.

The graphs for the daily returns for the selected currencies are shown in Figure 2. Clusters of both large and small log return values appear on the graphs, demonstrating volatility clustering. The year 2020 experienced significant volatility in the CNY/XAF exchange rate. The significant fluctuations noted may likely be linked to the disruption brought on by the COVID-19 pandemic. Furthermore, the log returns exhibit stationarity, as indicated by the graphs.

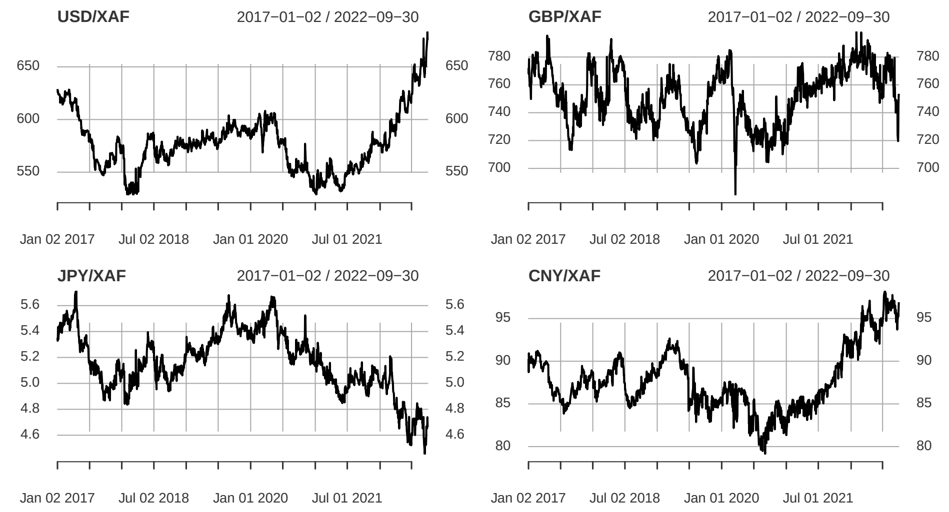

Figure 1: Daily exchange rates

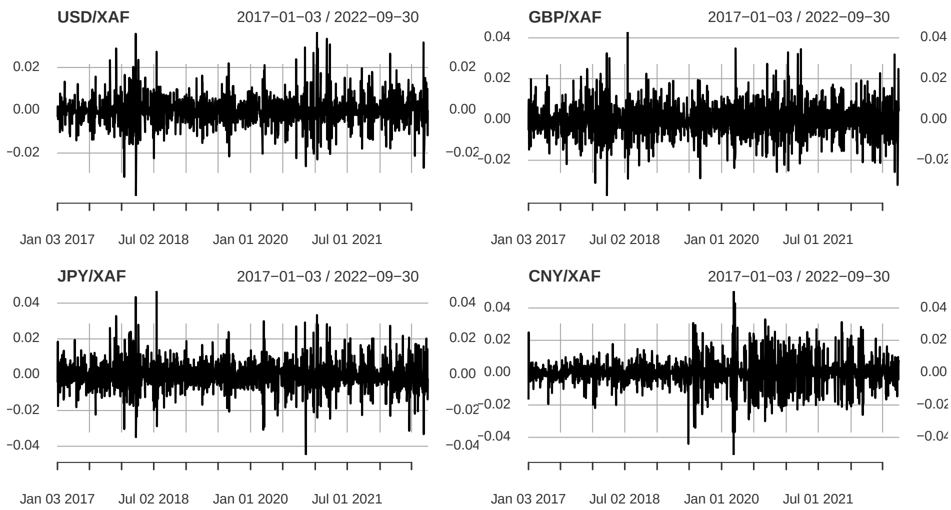

Figure 2: Daily exchange rate returns

Descriptive statistics

Summary information for the daily exchange rate returns is shown in Panel A of Table 1. The minimum value for the daily log returns for each of the four exchange rates is negative. The smallest log return value is −0.0508, and it is that for the CNY/XAF exchange rate. The maximum values are all positive. The CNY/XAF value is 0.0503, and it is the largest value. With the exception of the USD/XAF exchange rate, which has a positive mean return of 0.0007, all others exhibit a mean return of zero. Regarding variability, USD/XAF has the lowest standard deviation (0.0067), while CNY/XAF has the highest (0.0084). The skewness of the returns is positive and significantly different from zero (with the exception of the CNY/XAF returns), indicating that these series have asymmetric distributions. Apart from GBP/XAF, the kurtosis values of the rest are greater than 3. Thus, the distributions are leptokurtic, indicating non-Gaussian and heavy-tailed distributions of residuals, which is a typical characteristic or stylized fact of financial returns (Cont 2001).

Jarque-Bera (JB) test for normality

The Jarque-Bera (JB) test for normality of Bera and Jarque (1981) with p-values less than

Table 1: Descriptive statistics and Pearson correlation values

	Panel A: Descriptive statistics			
Returns	USD/XAF	GBP/XAF	JPY/XAF	CNY/XAF
Minimum	-0.0399	-0.0370	-0.0448	-0.0508
Maximum	0.0365	0.0428	0.0466	0.0503
Mean	0.0007	0.0000	0.0000	0.0000
St. Dev.	0.0067	0.0080	0.0080	0.0084
Skewness	0.3374	0.2049	0.1932	-0.0367
Kurtosis	4.8688	2.4654	3.7036	5.0348
JB test	1514.5	392.12	869.63	1589.3
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16
	Panel B: Pearson (Unconditional) Correlation Coefficients			
	USD/XAF	GBP/XAF	JPY/XAF	CNY/XAF
USD/XAF	1.000			
GBP/XAF	0.7056	1.0000		
JPY/XAF	0.7884	0.7204	1.0000	
CNY/XAF	0.0338	-0.0164	-0.0621	1.0000

0.05 leads to the rejection of the null hypothesis that the data is normally distributed at the 5% level of significance. From Table 1, all the p-values are less than 0.05. Thus, all four exchange rates are non-Gaussian, and therefore a non-Gaussian multivariate distribution, such as the multivariate Student's t-distribution and multivariate Laplace distribution, may be more suitable in modeling the exchange rates.

Pearson's correlation

Panel B of Table 1 presents Pearson's unconditional correlation values between the exchange rate returns. The values are both positive and negative, suggesting that the exchange rates do not all move in the same direction. High positive Pearson's correlation above 0.7 exists between the following pairs: USD/XAF and GBP/XAF, USD/XAF and JPY/XAF, GBP/XAF and JPY/XAF. A low positive Pearson's correlation of 0.03 exists between USD/XAF and CNY/XAF. A low negative correlation of -0.02 and -0.06 exists between GBP/XAF and CNY/XAF, and between JPY/XAF and CNY/XAF, respectively.

Pearson product-moment-based unweighted ordinary results of correlation values, however, provide an average correlation without handling the variations in the correlations (Özdemir 2022). Time-varying conditional correlation between pairs of exchange rates will be investigated using the DCC-GARCH model.

Preliminary tests

Various preliminary stationary and diagnostic tests are required before the GARCH family of models is used in modeling financial time series. In this study, the tests considered include tests for stationarity, the Ljung-Box (LB) Q test of Ljung and Box (1978) that tests for serial dependence on the first m lags of the time series returns, and Engle's Lagrange Multiplier (LM) test of Engle (1984) that tests for the presence of ARCH effects in the returns of the time series. The tests for stationarity include the Augmented Dickey Fuller (ADF) test of Dickey and Fuller (1979), the Phillips-Perron (PP) test of Phillips (1988), and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test of Kwiatkowski, Phillips, Schmidt, and Shin (1992).

The results of the different tests are presented in Table 2. The ADF and PP tests test the null hypothesis that the time series has a unit root (non-stationary), while the KPSS test tests the null hypothesis that the time series is stationary. From Table 2, the null hypothesis is rejected at the 5% level of significance, following the ADF and PP test results, and it is not rejected by the KPSS test. Thus, the returns are stationary. LB and LM tests at 5, 10 and 20 lags are statistically significant, leading to the rejection of the null hypothesis of no serial correlation and constant variance, respectively. It can therefore be concluded that serial correlation and ARCH effects are present in the time series returns, necessitating the use of GARCH models.

Multivariate portmanteau tests, notably the rank-based and robust tests, which motivate the use of multivariate heteroscedasticity models, are equally applied to the returns (Tsay 2013). The results, reported in Table 2, show that all the multivariate Portmanteau tests reject the null hypothesis of no conditional heteroscedasticity, implying the presence of ARCH effects. Hence, the multivariate DCC-GARCH model can be used to model the time series.

Table 2: Preliminary test results

	Unit root tests			
Returns	USD/XAF	GBP/XAF	JPY/XAF	CNY/XAF
ADF	-12.375	-12.71	-12.387	-12.98
p-value	0.01	0.01	0.01	0.01
PP test	-1800.5	-1642	-1718.2	-1749
p-value	0.01	0.01	0.01	0.01
KPSS test	0.4533	0.0231	0.0628	0.1814
p-value	0.0542	0.100	0.100	0.100
	Ljung-Box (LB) Test			
	USD/XAF	GBP/XAF	JPY/XAF	CNY/XAF
LB(lag=5)	105.48	94.1	107.83	204.07
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16
LB(lag=10)	107.47	97.848	112.43	215.78
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16
LB(lag=20)	111.09	105	116.71	230.3
p-value	1.243e-14	1.587e-13	1.11e-15	<2.2e-16
	Lagrange Multiplier (LM) Test			
	USD/XAF	GBP/XAF	JPY/XAF	CNY/XAF
LM(lag=5)	85.065	42.905	60.698	272.97
p-value	<2.2e-16	3.863e-08	8.719e-12	<2.2e-16
LM(lag=10)	93.083	47.106	63.136	277.32
p-value	1.308e-15	9.034e-07	9.199e-10	<2.2e-16
LM(lag=20)	106.67	53.314	70	289.83
p-value	7.934e-14	2.607e-05	1.822e-07	<2.2e-16
	Multivariate tests			
	Test statistic		p-value	
Rank-based test	194.8498		< 0.05	
Robust test	351.4052		2.2204e-16	

3.2. Parameter estimation

Optimal univariate GARCH model for the variance of each exchange rate time series

The Akaike Information Criteria (AIC) is used to determine the best univariate model that describes the variance of each of the exchange rates. The optimal model is the one with the lowest AIC. Our focus is on the GARCH, EGARCH, and GJR-GARCH models.

In financial time series analysis, choosing the right error distribution is crucial for accurately modeling and forecasting data. The Normal, Skewed Normal, Student's t-Distribution (STD), Skewed Student's t-Distribution (SSTD), Generalized Error Distribution (GED), and the Skewed Generalized Error Distribution (SGED) are assumed as error distributions. We consider ARMA($\hat{p}, \hat{q}$), where $\hat{p}, \hat{q} = 0, 1, 2$, and GARCH($p, q$), where $p, q = 1, 2$.

Using the arfima function in the R software, optimal values for ARMA($\hat{p}, \hat{q}$) that describe the mean equation for the exchange rate returns are shown in Table 3. By the AIC, the optimal values for the univariate ARMA($\hat{p}, \hat{q}$)$-$GARCH(p, q) models that describe the mean and volatility equations for each of the exchange rates together with the corresponding error distribution are also given.

We used the Multivariate normal (MVN), Multivariate Student's t (MVT), and Multivariate Laplace (MVL) distributions in constructing the DCC-GARCH model. Their results were compared, as per the values of the information criteria. Each of the optimal univariate models M_1, M_2, M_3 and M_4 was used in constructing the DCC-GARCH model. All the optimal univariate GARCH models were also used to construct the DCC model. The results with the corresponding Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Shibata (SH), Hannan-Quinn (HQ), and log-likelihood (LL) values are shown in Table 4.

Table 3: Optimal ARMA($\hat{p}, \hat{q}$)$-$GARCH(p, q) models

USD/XAF	ARMA(0,1)	$M_1 = ARMA(0,1)$- GJR-GARCH(1,1)-SGED
GBP/XAF	ARMA(0,2)	$M_2 = ARMA(0,2)$- EGARCH(1,1)-SSTD
JPY/XAF	ARMA(1,1)	$M_3 = ARMA(1,1)$- GARCH(1,1)-SSTD
CNY/XAF	ARMA(1,1)	$M_4 = ARMA(1,1)$- GJR-GARCH(2,2)-SGED

Innovation

From Table 4, the lowest values for AIC, BIC, SH, and HQ occur when models M_1 to M_4 are used to build the DCC-GARCH model with the multivariate Student's t-distribution. In this case, all the optimal univariate GARCH models, M_1 to M_4, are first applied to the data to remove serial correlation, and the standardized residuals of the filtered series are used in the estimation of the DCC model. This is an innovation given that the use of just one univariate GARCH model is common practice. The most commonly used is the standard GARCH(1,1), for all the time series returns in constructing the DCC-GARCH model (Mohammed, Mwambi, and Omolo 2024).

It is apparent that for all models, M_1 to M_4, and a combination of all, the information criteria with the multivariate Student's t and Laplace distributions is lower than that for the multivariate normal distribution. In Table 4, models with the lowest information criteria are marked with an asterisk (*). Thus, models with heavy-tailed residual distributions perform better when they are used to model exchange rates. The suitability of the multivariate Student's t-distribution in modeling the residuals of the DCC-GARCH model is consistent with the results obtained by Boudt, Galanos, Payseur, and Zivot (2019).

The estimated results for the univariate GARCH and DCC models with multivariate Student's t-distribution for the residual terms are shown in Table 5. The estimates of the DCC correlation parameters a and b are all statistically significant, and $a + b < 1$. This suggests that $\boldsymbol{B_t}$, and hence $\boldsymbol{R_t}$, vary over time, and they are stationary and positive definite.

Time-varying correlation between pairs of exchange rates

The graphs of the daily conditional volatility are shown in Figure 3, and those of the dynamic correlation coefficient of the pairwise combinations of the four exchange rates are shown in

Table 4: Information Criteria and Log-likelihood Values of DCC Models

Univariate Model	AIC-MVN	BIC-MVN	SH-MVN	HQ-MVN	LL-MVN
M_1	−29.746	−29.619	−29.747	−29.699	22315.99
M_2	−29.619	−29.477	−29.621	−29.566	22224.80
M_3	−29.711	−29.583	−29.712	−29.663	22289.30
M_4	−29.776	−29.592	−29.778	−29.707	22354.37
M_1,M_2,M_3,M_4	−29.864*	−29.744*	−29.865*	−29.819*	22402.23
	AIC-MVT	BIC-MVT	SH-MVT	HQ-MVT	LL-MVT
M_1	−30.161	−30.029	−30.162	−30.112	22627.30
M_2	−30.091	−29.946	−30.093	−30.037	22579.37
M_3	−30.148	−30.016	−30.149	−30.099	22617.49
M_4	−30.185	−29.997	−30.187	−30.115	22661.46
M_1,M_2,M_3,M_4	−30.209*	−30.084*	−30.210*	−30.162*	22661.20
	AIC-MVL	BIC-MVL	SH-MVL	HQ-MVL	LL-MVL
M_1	−30.058	−29.930	−30.059	−30.010	22549.10
M_2	−29.996	−29.854	−29.997	−29.943	22507.09
M_3	−30.046	−29.918	−30.047	−29.998	22540.17
M_4	−30.087*	−29.902	−30.089*	−30.018	22586.91
M_1,M_2,M_3,M_4	−30.081	−29.960*	−30.082	−30.036*	22564.39

(*) Model with the lowest AIC value among competing models

Figure 4. The heteroscedastic nature of the exchange rate conditional volatility is shown in Figure 3. Before mid-2019, the CNY/XAF exchange rate was the least volatile compared to the other exchange rates. The onset of the COVID-19 pandemic made the CNY/XAF exchange rate very volatile. From Figure 4, it can be seen that currency correlations are non-stationary, dynamic, and time-varying with pronounced apparent structural changes in the correlation process as equally reported in Engle (1990). An analysis of the daily volatility and correlation relationships confirms the findings of most scholars that high volatility of markets is directly linked with strong correlations between them such that markets tend to behave as one during times of crisis (Junior and Franca 2012). This can be seen somewhere between 2017-2018 and 2020-2021. In 2017, the rise of Donald Trump as US president and his declaration of Jerusalem as Israel's capital, Theresa May of the UK negotiating Brexit, and extreme weather events such as Hurricane Harvey and Hurricane Irma in the US, contributed to the high volatility and strong positive correlations between the exchange rates. The global spread of the COVID-19 pandemic around March 2020 also had a similar effect. The recovery from the pandemic, coupled with the 2020 oil price war between Russia and Saudi Arabia, caused an energy crisis and high global inflation in 2021, as seen in the graphs.

For ease of explanation, let us denote U=USD/XAF, G=GBP/XAF, J=JPY/XAF, and C=CNY/XAF. There is empirical evidence of time-varying correlation between U and G and between U and J within the range $[0.2, 0.8]$. Another piece of evidence is between U and C and between G and C, within the range $[-0.2, 0.4]$. The next is between G and J, in the range $[0.3, 0.8]$, and the last is between J and C, in the range $[-0.2, 0.2]$. High and positive co-movements can be identified between the following three pairs of exchange rates: U and G, U and J, and G and J. This means that there are fewer opportunities for diversification between these currency pairs. A very low correlation exists between U and C, G and C, and

Table 5: Parameter estimates for DCC-GARCH model, using M_1 to M_4 as univariate GARCH

	USD/XAF		GBP/XAF		JPY/XAF		CNY/XAF	
Parameter	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
μ	0.0001	0.2424	–0.0000	0.3685	0.0000	0.6566	–0.0000	0.6621
ar_1					0.1154	0.2947	0.1255	0.0601
ma_1	0.2290	< 0.05	–0.3061	< 0.05	–0.4095	0.0001	–0.4938	< 0.05
ma_2			–0.0749	< 0.05				
ω	0.0000	< 0.05	–0.8895	0.1541	0.0000	< 0.05	0.0000	0.3307
α_1	0.1145	< 0.05	–0.0127	0.6640	0.1135	< 0.05	0.0441	0.0376
β_1	0.8266	< 0.05	0.9078	< 0.05	0.7794	< 0.05	0.8844	< 0.05
γ_1	–0.0476	0.2593	0.2338	< 0.05			0.0779	0.1117
	Correlation Parameters							
	Estimate		p-value					
a	0.0526		< 0.05					
b	0.8598		< 0.05					

J and C. A low correlation is a good indication for portfolio diversification. Thus, the 6 pairs of exchange rate returns fall into two groups: 3 pairs with high co-movements and correlation between them and 3 pairs with very little co-movements and correlation between them, as shown in Figure 5.

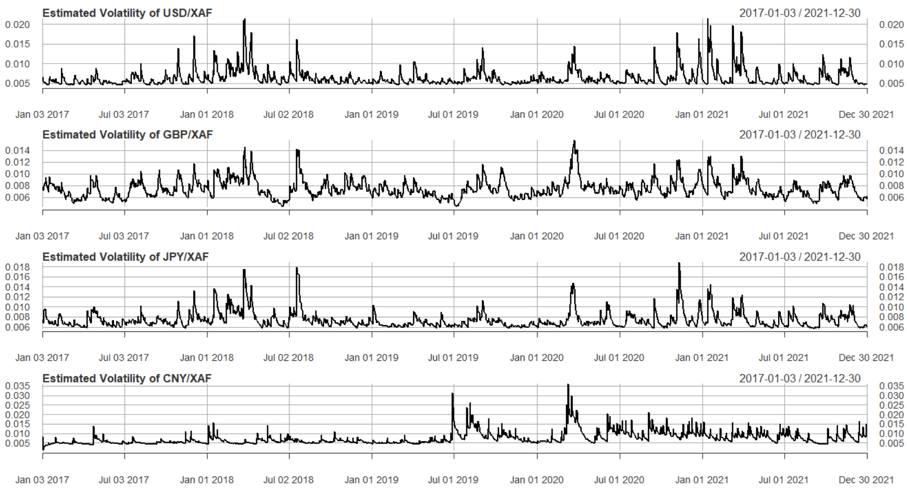

Figure 3: Daily USD/XAF, GBP/XAF, JPY/XAF and CNY/XAF conditional volatility

Prediction of time-varying correlation between pairs of exchange rates

Figure 6 shows the fitted and predicted conditional correlation between the pairs of exchange rates using the DCC-GARCH model. All of the data is plotted, and a one day ahead rolling prediction is carried out for each pair of exchange rates with a prediction window of 100 days. In this figure, the black line is the fitted conditional correlation, and the orange line is the predicted conditional correlation. It can be observed that the fitted correlation fluctuates around the unconditional correlation, and in the long run, the predicted values of the conditional correlation appear to converge to the value of the unconditional correlation (as shown in Panel B of Table 1).

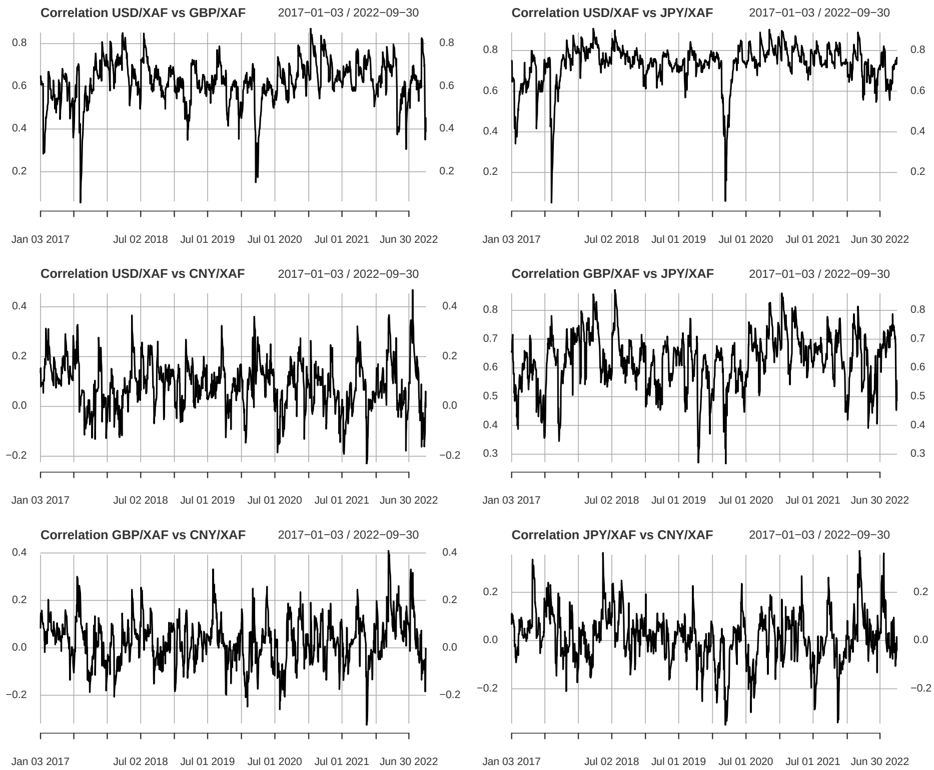

Figure 4: DCC time-varying correlations between pairs of exchange rates

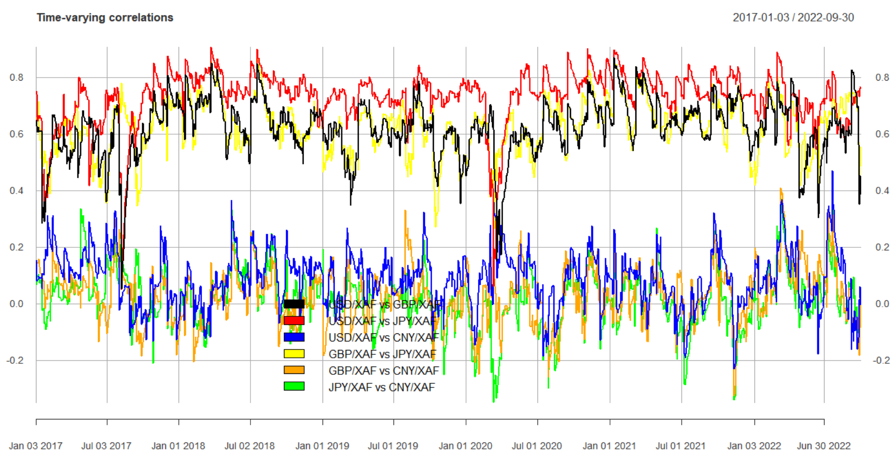

Figure 5: Combined DCC time-varying correlations between pairs of exchange rate returns

4. Discussion and conclusion

The DCC-GARCH is a powerful multivariate volatility model within the GARCH multivariate class of models, which is suitable for investigating the volatility of financial data and dynamic correlations in financial markets.

The objective of this study was to propose a robust DCC-GARCH model that is constructed using standardized residuals from all optimal univariate GARCH models for each time series. The robust DCC-GARCH model is then used to analyze the dynamics of the time-varying correlation relationship between selected exchange rates, in particular USD/XAF, GBP/XAF, JPY/XAF, and CNY/XAF. It was found that if all the optimal univariate GARCH models (with lowest AIC values) for each exchange rate return are used to construct a DCC-GARCH

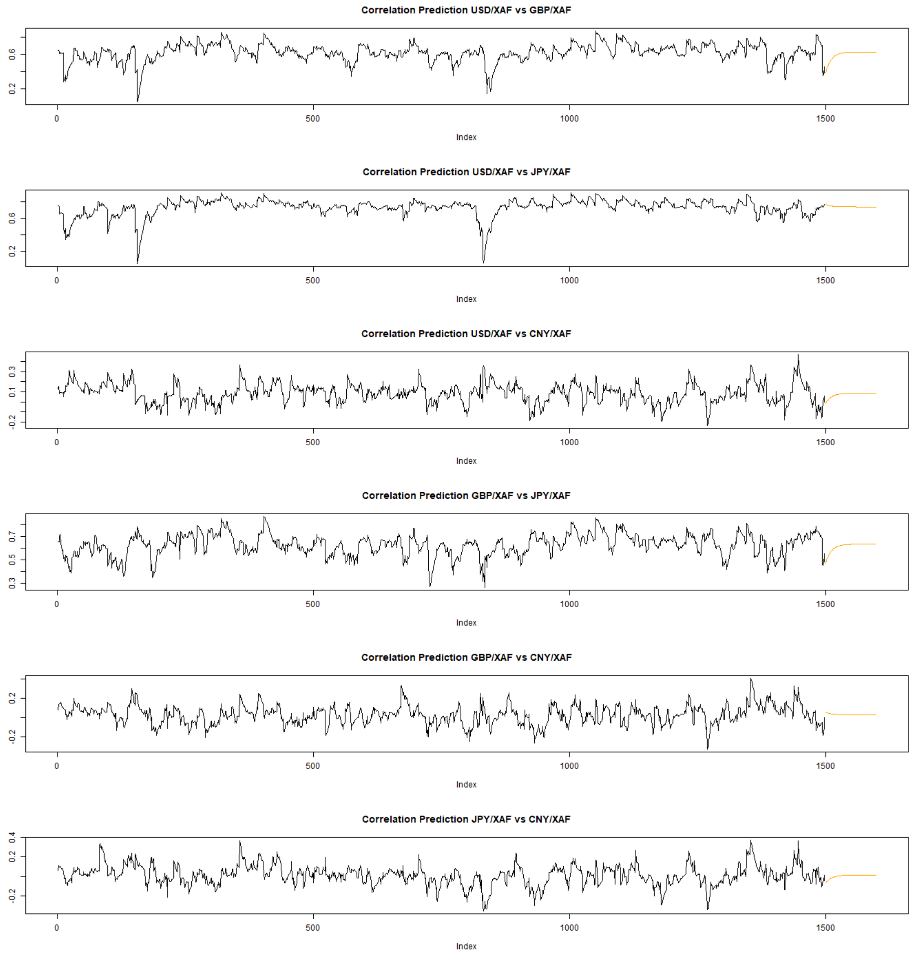

Figure 6: Fitted (in black) and Predicted (in orange) DCC time-varying correlations between pairs of exchange rates

model, the resulting multivariate model has the lowest information criterion and is therefore more efficient than if individual GARCH models are used. Second, the resulting model is better if the standardized residuals are assumed to follow a heavy-tailed distribution, that is, the multivariate Student's t-distribution in this case, compared to if we assumed that they follow the Gaussian distribution. Although there are a few studies that use heavy-tailed residual distributions to construct a DCC-GARCH model, to the best of our knowledge, no study has used the approach presented in this paper.

Empirical evidence of the time-varying correlation relationship between the selected exchange rates is also of the utmost importance for foreign-exchange investors. Understanding these time-related correlations will help investors to optimize their portfolio allocation by investing in pairs of exchange rates with low correlation and co-movement between them, and consider alternative strategies such as sector rotation in periods of high correlation.

For further research, other heavy-tailed residual distributions, such as the multivariate generalized error distribution, may also be used as standardized residual distributions. The results obtained should be compared with those where the standardized residuals are assumed to follow the multivariate Student's t-distribution, Laplace, or Gaussian distributions. Hybrid models such as Copula-DCC-GARCH can also be used to analyze the time-varying correlations between exchange rates and compare the results with the DCC-GARCH model.

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