Austrian Journal of Statistics

AUSTRIAN STATISTICAL SOCIETY

Volume 44, Number 1, 2015

Regular Issue

Österreichische Zeitschrift für Statistik

ÖSTERREICHISCHE STATISTISCHE GESELLSCHAFT

Austrian Journal of Statistics; Information and Instructions

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Austrian Journal of Statistics

Volume 44, Number 1, 2015

Editor: Matthias TEMPL

http://www.ajs.or.at

Published by the AUSTRIAN STATISTICAL SOCIETY

http://www.osg.or.at

Österreichische Zeitschrift für Statistik

Jahrgang 44, Heft 1, 2015

ÖSTERREICHISCHE STATISTISCHE GESELLSCHAFT

Impressum

Editor: Matthias Templ, Statistics Austria & Vienna University of Technology

Book Reviews: Ernst Stadlober, Graz University of Technology
Printed by Statistics Austria, A-1110 Vienna

Published approximately quarterly by the Austrian Statistical Society, C/o Statistik Austria
Guglgasse 13, A–1110 Wien

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Editorial

The new web-site and editorial system (http://www.ajs.or.at) has been launched one year ago. Four issues (in Volume 43) were published in 2014, one special issue on official statistics, one regular issue and a conference proceedings in form of a double issue. With the relauch of the website the style of the manuscripts have been changed, as well as the cover pages. Various people asked about the paintings at the cover page. However, this will be kept as a secret until Volume 44, Issue 3. Without giving too much away, it can be said that the initial intention was not to draw a “A” (for **A**ustrian), but if you feel better you can think on a “A” until the secret will be give away.

It is great to look forward to Volume 44 (2015). Beside this issue, a special issue on the Q2014 conference is in progress as well as a special issue on R and one additional regular issue.

According to submissions to AJS, the year 2014 was not bad. More than 20 papers are currently in review. However, the share of submissions from authors living in Austria and papers covering topics related to Austrian society, economy, environment, and generally to science is still low. It would be wonderful if this can be changed whereas all scientists, PhD students and practioneer working in Austria are asked to contribute and to submit a manuscript.

The first paper in this issue by Alexandra Grand and Regina Dittrich is related to psychometrics and formulates a beta Bradley-Terry regression model for modelling metric paired comparison data. A self-rated survey is conducted with the aim to investigate in emotions. Interesting results are obtained: don’t get anger when learning maths!

The second paper by Wolfgang Voit and Arne C. Bathke investigate in the subjects statistics and probability at secondary schools in the federal state of Salzburg and research on the question about the relevance of these subjects. Good news! Statistics and probability are currently gaining in importance. Get motivated by this paper.

One can discuss if all these measurements are fuzzy, at least for continuous quantities. Fundamental basics are formulated in the third paper by Reinhard Viertl. We are looking forward to receive new papers considering multivariate aspects as well.

The fourth paper by Jehad Al-Jararha and Als’ Bataineh concentrates on the estimation of a finite population ratio using two auxiliary variables. The paper presents some interest, because the proposed estimator does indeed seem to be less variable than the classical ratio estimator.

The fifth contribution contains an interview with Ewald Kutzenberger who was one of the important persons in official statistics in Austria. It also highlights the foundation of the Austrian law on statistics, and the whole interview is a highlight itself.

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Austrian Journal of Statistics
January 2015, Volume 44, 3–15.
http://www.ajs.or.at/

Modelling Assumed Metric Paired Comparison Data – Application to Learning Related Emotions

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Abstract

In this article we suggest a beta regression model that accounts for the degree of preference in paired comparisons measured on a bounded metric paired comparison scale. The beta distribution for bounded continuous random variables assumes values in the open unit interval $(0, 1)$. However, in practice we will observe paired comparison responses that lie within a fixed or arbitrary fixed interval $[-a, a]$ with known value of a. We therefore transform the observed responses into the interval $(0, 1)$ and assume that these transformed responses are each a realization of a random variable which follows a beta distribution. We propose a simple paired comparison regression model for beta distributed variables which allows us to model the mean of the transformed response using a linear predictor and a logit link function – where the linear predictor is defined by the parameters of the logit-linear Bradley-Terry model. For illustration we applied the presented model to a data set obtained from a student survey of learning related emotions in mathematics.

Keywords: beta regression, logistic Bradley-Terry model, metric paired comparisons.

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1. Introduction

The method of paired comparisons is a well established method for analysing preferences in many sciences. In a paired comparison study individuals are asked to repeatedly judge which of the pairwise presented objects (of a set of J objects) they prefer according to an attribute. The aim is to obtain an ordering of the objects on a preference continuum. In each of the $\binom{J}{2}$ paired comparisons (denoted by (jk)) there are two possible decisions: preference for object j, $jk(j)$, or preference for object k, $jk(k)$. For the analysis of such paired comparison data we refer to the well known Bradley-Terry (BT) model (Bradley and Terry 1952), see Equation (4).

Paired comparisons with only two response categories are easy to perform but in a way they forces the judge to decide for one of the two presented objects, which could be questionable if the judge has no preference. Several authors (e.g. Rao and Kupper 1967; Kousgaard 1976; Dittrich, Hatzinger, and Katzenbeisser 1998) have extended the BT model to incorporate a third category (ties) for indifference, i.e. no preference. To obtain more information the BT model has further been extended by allowing the judge to specify the degree of preference on an ordered response scale – for example on a seven point scale with labelled ordered response categories: *strong preference for object j*, *moderate preference for j*, *mild preference for j*, *no preference*, *mild preference for k*, *moderate preference for k*, *strong preference for object k* (see e.g. Agresti 1992; Böckenholt and Dillon 1997; Dittrich, Hatzinger, and Katzenbeisser 2004; Dittrich, Francis, Hatzinger, and Katzenbeisser 2007). However, judges may differ in their interpretation of labelled response categories or in their response scale usage. Little literature was found about metric paired comparisons where the judge is asked to report the degree of preference for a particular object in a given comparison (jk) on a bounded metric scale. Although there still might exist the problem of possible response style bias, more (detailed) information may be obtained and individuals often felt to be more consistent when using continuous response formats (McKelvie, 1978 cited in Ferrando 2002).

De Ruiz (1990), for example, proposed a logistic model for a paired comparison experiment on a continuum of response where the parameters are estimated with an integrated minimum mean squared error approach. Stern (2011) suggested a model that accounts for continuous paired comparison data. In this study cricket teams (the objects) were pairwise compared in the matches according to a predefined rule to receive the *margin* (i.e. magnitude) of victory. In each cricket match (paired comparison) Stern (2011) obtained the margin of victory by calculating the proportion of available resources which were unused by the winning team. The sign of the margin of victory indicates which team won the match. Stern (2011) treated this paired comparison outcome which was transformed to take values in the unit interval (0, 1), as the response variable in a beta regression model. The parameters of interest (the relative strengths of the cricket teams) were estimated using a penalized log-likelihood method. This penalty function allows specification of the trade-off between the relative importance of the information that a team wins or loses (i.e. the dichotomous outcome) and the continuous margin of victory (the degree).

Following the approach of Stern (2011), the purpose of this article is to suggest a simple beta regression model for (assumed) metric paired comparison data without using a penalty function, where only the degree of preference should be indicative of the preference parameters. In each paired comparison the decision has to be communicated respectively specified on a given response scale with (arbitrary) fixed bounds. We assume that the more one object dominates the other according to a particular attribute the more likely a judge states a high degree of preference for this object in a given paired comparison. The responses made are bounded and (assumed to be) interval scaled so that we found the assumption of a beta distributed response variable appropriate. Note that the response variable still has to be transformed to lie within the open unit interval (0, 1), see Section 2.1. The proposed paired comparison approach for (assumed) metric responses is suitable in situations where a *relative* preference ordering is of interest.

A practical example for the usage of a bounded (assumed) metric response scale in a paired comparison study would be the measurement of relative preferences for J different services (the objects) in tourism where the judge is asked to state on an assumed metric response scale how much more she/he would prefer one of the pairwise presented services. We assume that in the process of coming up with a mark on a bounded response scale in each paired comparison, a person first imagines the worth of each of the two services and then compare these. Finally the judge has to communicate respectively specify her/his degree of preference for one of the objects being compared on a given (arbitrary) bounded response scale. Let us assume that person A would appreciate the possibility of a rent-a-car service and do not require wireless LAN. Confronting person A in a paired comparison with these two services, person A might quantify her/his preference for the rent-a-car service somewhere at the end of the response scale indicating a very high preference for this service compared to the other one. As a result, over all paired comparisons and persons, we obtain estimates of worth parameters (see Section 2.1) for each of the J services which can be located on a continuum. A high value of the estimated worth parameter indicates high preference for a certain service in relation to the other services. In many fields in tourism it is unrealistic to serve all possible services tourists would wish to receive, so a specialization in a few of them is required (due to economical and other reasons).

Therefore it might be interesting which of the services of a set of J services to pursue and specialize in. In such a case an ordering of the services on a continuum would be a guidance, where also distances between the J services can be reasonably interpreted. Using another approach like a rating scale of preference, we might observe the possible phenomenon that when asking judges they rate lots of services equally high which is less informative. Choosing a paired comparison approach the persons are forced to make several relative, pairwise judgements about the objects and therefore to think more about their responses.

Another example could be a *derived* paired comparison study where the time persons need for solving J mathematical tasks (the objects) is collected. And where in a second step the tasks are pairwise compared by computing for each individual the time difference, i.e. the degree of solving one maths task more rapid than the other. This relative time degree (the derived response) can then be located on a bounded metric paired comparison scale. As these paired comparisons will not originate from real paired comparisons, we term this kind of data *derived* paired comparison data. On basis of these derived responses worth parameters for each of the J maths tasks can be estimated. They can be located on a time continuum where high values indicate that a long time is needed to solve a certain task (implying that this is a relative difficult task) relative to the other tasks and vice versa. As a result, we could obtain information about the rank order (e.g.: the most and least time intense maths task of a set of J maths tasks, respectively) and interpret the distances between the parameters of the maths tasks (e.g.: the distance between the first and second ranked maths task is larger than between the second and third ranked maths task). Moreover we could check if the ordering of the maths task parameters changes for various groups of students (e.g. female students might solve a certain task significantly quicker than male students). The maths tasks can further be classified according to one or more attribute(s) maths tasks can have and incorporate object-specific covariates into the model. Such information might also be useful for psychological assessments or for longitudinal studies, for example.

In this article we applied the beta regression model to a data set of a study concerning learning related emotions in maths. The application (see Section 3) covers paired comparisons where students were asked to mark how much more often one emotion is typically experienced while learning compared to another on a visual scale with arbitrary fixed bounds. Here the emotions are the objects of interest. We still have to bear in mind that we do not know exactly if the same marks of different judges on a visual horizontal line imply that they have experienced the same degree of frequency (see Aitken 1969). However, we assume that the responses are ordered for all judges and on an interval scale level. We term such a scale an *assumed* metric paired comparison response scale.

Emotions are very subjective, complex interrelated constructs for which it is very difficult to come up with suitable measures. Using a traditional IRT approach we would require item sets each measuring one (latent) emotion of interest and the quality of the items has to be checked according to the assumptions made by the IRT models. The responses in these models are sort of *absolute* in the sense that a subject might agree (or not agree) or an item can be solved (or not solved). One model assumption in the IRT approach is that the probability for a response depends on item as well as subject parameters. If the model holds we would get *unidimensional* interval scaled measures for each of the latent constructs of interest separately. The assumption of unidimensionality requires that the item parameters are the same (up to a constant) for all subgroups of respondents. Differences between subgroups would indicate IRT model violations in this context.

Furthermore, in the framework of Item Response Theory (IRT) only a few authors have analysed (bounded) continuous responses (see e.g. Noel and Dauvier 2007 or Müller 1987). In these approaches the respondents were asked for *absolute continuous* judgements for each item.

However, we are not interested in scaling emotions but in directly modelling *relative* responses by comparing emotions to obtain an ordering of a set of J emotions on an interval scaled continuum where we could also interpret the distances between the estimated parameters representing the emotions (i.e. the objects) of interest. In the paired comparison approach the probability for a response only depends on the strength (or worth) of the involved emotions and the effect of the subject characteristics can be modelled explicitly by interaction effects between emotions and subject characteristics. Therefore the proposed metric paired comparison model allows the incorporation of subject covariates, where possible effects of the subject covariates on the ordering of the emotion parameters can be assessed. This can be seen as an advantage of the paired comparison approach compared to the IRT models where item differences between subgroups are interpreted as model violations. As in the IRT models it is also possible to incorporate object-specific covariates. Model selection can easily be done through a likelihood ratio test of nested models.

2. Beta regression model

The beta distribution is restricted to the standard unit interval $(0, 1)$. Let Y be a beta distributed random variable with realization y; the density of Y is then given by:

$$f(y; p, q) = \frac{1}{B(p, q)} y^{p-1}(1-y)^{q-1} = \frac{\Gamma(p+q)}{\Gamma(p) \cdot \Gamma(q)} y^{p-1}(1-y)^{q-1} \ , \ 0 < y < 1 \ . \tag{1}$$

Here, the shape parameters of the beta density are symbolized with p and q, where $p > 0, q > 0$. The beta function $B(\cdot, \cdot)$ is related to the gamma function $\Gamma(\cdot)$ by $B(p, q) = \frac{\Gamma(p) \cdot \Gamma(q)}{\Gamma(p+q)}$ and the term $\frac{1}{B(p,q)}$ is a normalizing constant so that (1) integrates to unity. Several authors substituted one (e.g. Kieschnick and McCullough 2003) or both (e.g. Paolino 2001; Ferrari and Cribari-Neto 2004; Smithson and Verkuilen 2006) shape parameters of the beta distribution (1) and imposed a regression structure for the mean response or for both the mean and the precision parameter to simplify interpretation.

The beta regression model, as introduced by Ferrari and Cribari-Neto (2004), is an alternative parameterization of the beta density for modelling continuous response variables restricted to $(0, 1)$. To be able to model the mean of the response variable as a function of explanatory variables (via a linear predictor) along with a precision parameter, Ferrari and Cribari-Neto (2004) proposed the different parameterization of the beta density:

$$f(y; \mu, \phi) = \frac{\Gamma(\phi)}{\Gamma(\mu\phi) \cdot \Gamma((1-\mu)\phi)} y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1} \ , \ 0 < y < 1 \ ,$$

where $\mu = \frac{p}{p+q}$ and $\phi = p+q$ (i.e. $p = \mu\phi$ and $q = (1-\mu)\phi$), $0 < \mu < 1$, $\phi > 0$. The parameter μ is a location parameter (the mean of the response variable), ϕ is a precision parameter and ϕ^{-1} a dispersion parameter; the expectation of Y, $\mathrm{E}(Y) = \mu$ and $\mathrm{VAR}(Y) = \frac{\mu(1-\mu)}{1+\phi}$.

In this article we refer to a beta regression model that includes a linear predictor and a link function for both the location parameter μ and the precision parameter ϕ. This model is similar to a generalized linear model (GLM). For a random sample we write $Y_i \sim B(\mu_i, \phi_i)$, $i = 1, \ldots, N$, where N is the sample size. The beta regression model is defined as (cf. Smithson and Verkuilen 2006; Cribari-Neto and Zeileis 2010; Simas, Barreto-Souza, and Rocha 2010):

$$\begin{aligned} g_1(\mu_i) &= \boldsymbol{x}_i^T \boldsymbol{\lambda} = \eta_{1i} \ , \\ g_2(\phi_i) &= \boldsymbol{z}_i^T \boldsymbol{\gamma} = \eta_{2i} \ , \end{aligned} \tag{2}$$

where $g(\cdot)$ is a link function. $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_k)^T$ and $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_h)^T$, $k + h < N$, are $k \times 1$ and $h \times 1$ column vectors of unknown regression parameters, respectively. $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$ are covariate vectors: $\boldsymbol{x}_i = (x_{i1}, \ldots, x_{ik})^T$, $\boldsymbol{z}_i = (z_{i1}, \ldots, z_{ih})^T$ and η_{1i} and η_{2i} are the linear predictors.

The notation of a beta regression model as defined in (2) is suitable for modelling paired comparison data, where we want to model the mean of the response made on a bounded metric scale (i.e. the dependent variable) as a function of a set of covariates (independent variables) and parameters of the objects (λ) via a linear predictor. In the following section we present a beta regression model for paired comparisons where we assume that the precision parameter ϕ is constant for all observations, i.e. $g_2(\phi) = \gamma$.

2.1. Beta regression model for paired comparison data

In most of the paired comparison studies the (assumed) metric responses, the $y_{jk,i}$'s, of the ith individual, $i = 1, \ldots, N$ in the $\binom{J}{2}$ comparisons, will lie in an interval $[-a, a]$. The bounds $-a$ and a are predefined with known value of a which refer to theoretical or arbitrary values. Following the proposed transformation method of Smithson and Verkuilen (2006) we can transform each random variable $Y_{jk,i}$ of a given paired comparison (jk) of person i into the standard unit interval $(0, 1)$ by two steps:
First we squeeze $Y_{jk,i}$ with realization $y_{jk,i}$ in $[-a, a]$ into the interval $[0, 1]$ and obtain the transformed random variable $Y^*_{jk,i}$ by means of the transformation:

$$Y^*_{jk,i} = \frac{Y_{jk,i} + a}{2a}, \text{ with } \quad 0 \leq Y^*_{jk,i} \leq 1 \ .$$

In a second step we make sure that the two times transformed random variable $Y^{**}_{jk,i}$ cannot take on the values zero and one, i.e. takes on values in the interval $(0, 1)$:

$$Y^{**}_{jk,i} = \frac{Y^*_{jk,i} \cdot (N - 1) + 0.5}{N}, \text{ with } \quad 0 < Y^{**}_{jk,i} < 1 \ ,$$

where N is the sample size. In our study the $Y_{jk,i}$'s are always (i.e. in each paired comparison (jk)) associated with the first object (j) being compared. For example: In the interval $[-50, 50]$, an observed response $y_{jk,i} = -40$ denotes that object j is 40 units preferred compared to object k by judge i. And $y_{jk,i} = 40$ indicates that object k is 40 units preferred compared to object j. For further discussion about the transformation see e.g. the supplementary material of Smithson and Verkuilen (2006). As a result of the transformation process we obtain values in the interval $(0, 1)$, where values < 0.5 indicate preference for the first object (j) in a paired comparison (jk) and values > 0.5 preference for the second object (k). The closer the value to zero the greater the degree of preference for object j (and vice versa) where the most and the least favourable response for object j is about 0 and about 1. A tied response (i.e. no preference; $y_{jk,i} = -a + \frac{a+a}{2} = 0$, the middle of the response scale) is indicated by the two times transformed value of $y^{**}_{jk,i} = 0.5$.

In the case of a response variable $Y_{jk,i}$ with observation $y_{jk,i}$ on a bounded $[-a, a]$ metric paired comparison scale, let the transformed variable $Y^{**}_{jk,i}$ be a beta distributed random variable, $Y^{**}_{jk,i} \sim B(\mu_{jk,i}, \phi)$. We further assume independence between the decisions of the N judges and between the paired comparisons.

The log-likelihood function over all paired comparisons (jk) (where $j < k; j = 1, 2, \ldots, J - 1; k = 2, 3, \ldots, J$) and over all N judges is defined by:

$$\ell(\mu, \phi) = \sum_{i=1}^{N} \sum_{j=1}^{J-1} \sum_{k=j+1}^{J} \ell_{jk,i}(\mu_{jk,i}, \phi) \ , \tag{3}$$

where the log-likelihood, $\ell_{jk,i}(\mu_{jk,i}, \phi)$ associated with the transformed response $Y^{**}_{jk,i}$ of judge i in a given paired comparison (jk), is given by:

$$\ell_{jk,i}(\mu_{jk,i}, \phi) = \ln \Gamma(\phi) - \ln \Gamma(\mu_{jk,i}\phi) - \ln \Gamma((1 - \mu_{jk,i})\phi) + (\mu_{jk,i}\phi - 1) \ln y^{**}_{jk,i} + \\ \{(1 - \mu_{jk,i})\phi - 1\} \ln(1 - y^{**}_{jk,i}), \quad y^{**}_{jk,i} \in (0, 1) \ .$$

As link function $g(\cdot)$ for $\mu_{jk,i}$, we choose the logit link and for ϕ the identity link:

$$\begin{aligned} g_1(\mu_{jk,i}) &= \text{logit}\,(\mu_{jk,i}) = \ln\left(\frac{\mu_{jk,i}}{1-\mu_{jk,i}}\right) = \eta_{1jk,i} \ , \\ g_2(\phi) &= \gamma \ . \end{aligned}$$

In general, the expectation $\mu_{jk,i}$ of $Y^{**}_{jk,i}$ can be obtained by inverting the logit link function:

$$\mu_{jk,i} = g_1^{-1}(\eta_{1jk,i}) = \frac{\exp(\eta_{1jk,i})}{1+\exp(\eta_{1jk,i})} \ .$$

We apply the logit-linear Bradley-Terry model to the logistic mean structure. The Bradley-Terry model (Bradley and Terry 1952) defines the probability that object j is preferred in the comparison (jk), $P_{jk(j)}$, as follows:

$$P_{jk(j)} = \frac{\pi_j}{\pi_j + \pi_k} \ , \tag{4}$$

where the π_j's are positive worth parameters specified with the requirement that $\sum_{j=1}^{J} \pi_j = 1$. The worth parameters can be interpreted as locations of the objects on a preference continuum that ranges from zero to one. The probability of preferring object k over object j, $P_{jk(k)}$ is: $1 - P_{jk(j)}$ or $P_{jk(k)} = \frac{\pi_k}{\pi_j+\pi_k}$. Hence, the log-odds of preferring object j compared to object k is:

$$\ln\left(\frac{P_{jk(j)}}{P_{jk(k)}}\right) = \ln\left(\frac{\pi_j}{\pi_k}\right) = \lambda_j - \lambda_k = \eta_{1jk,i} \ ,$$

where $\lambda_j = \ln \pi_j$ or $\pi_j = \exp(\lambda_j)$. The parameter λ_j characterizes object j. For identifiability we set the object parameter λ_J to be zero. The λs are related to the worth parameter π by $\pi_j = \frac{\exp(\lambda_j)}{\sum_{j=1}^{J}\exp(\lambda_j)}$.

We parameterized the logistic mean structure of the beta regression model for a given paired comparison (jk) for judge i so that logit $(\mu_{jk,i}) = \lambda_j - \lambda_k$, where $\mu_{jk,i} = g_1^{-1}(\lambda_j - \lambda_k)$ is the expected degree of preference for object j compared to object k (cf. Stern 2011). The beta Bradley-Terry regression (BBTR) model for judge i for the paired comparison (jk) is defined as:

$$\begin{aligned} \text{logit}\,(\mu_{jk,i}) &= \lambda_j - \lambda_k \ , \\ \phi &= \gamma \ . \end{aligned}$$

Note that a tied response ($y^{**}_{jk,i} = 0.5$) means that the judge has no preference so that logit $(\mu_{jk,i}) = 0$.

In general, for all comparisons (jk) and for all judges i the BBTR model is given by:

$$\begin{aligned} \text{logit}\,(\mu_{jk,i}) &= \boldsymbol{x}_{jki}^T \boldsymbol{\lambda} = \eta_{1jk,i} \ , \\ \phi &= \gamma \ . \end{aligned} \tag{5}$$

The vector $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_J)^T$ is a $J \times 1$ column vector of unknown object parameters, and $\boldsymbol{x}_{jki}$ is the corresponding vector of covariates for a particular paired comparison (jk) for judge i, $\boldsymbol{x}_{jki} = (x_{jki,1}, \ldots, x_{jki,J})^T$ with $x_{jki,j} \in (-1, 0, 1)$ (see Table 1). The expectation of $Y^{**}_{jk,i}$ is: $\text{E}(Y^{**}_{jk,i}) = \mu_{jk,i}$.

The parameters of the BBTR model (5) for paired comparisons can be estimated by maximizing the log-likelihood (3) using a quasi Newton method. The underlying design structure of the beta regression model (5) was constructed in R (R Development Core Team 2013) by using elements of the package `prefmod` (Hatzinger 2012). For model estimation we used the R-package `betareg` (Zeileis, Cribari-Neto, Grün, Kosmidis, Simas, and Rocha 2013; see also Cribari-Neto and Zeileis 2010).

Table 1: Design structure of a BBTR model for $J = 5$.

paired comparisons $(jk)i$	**response** $y^{**}_{jk,i}$	$\boldsymbol{\lambda_1}$ $x_{jki,1}$	$\boldsymbol{\lambda_2}$ $x_{jki,2}$	$\boldsymbol{\lambda_3}$ $x_{jki,3}$	$\boldsymbol{\lambda_4}$ $x_{jki,4}$	$\boldsymbol{\lambda_5}$ $x_{jki,5}$
(12)1	$y^{**}_{12,1}$	1	−1	0	0	0
(13)1	$y^{**}_{13,1}$	1	0	−1	0	0
(23)1	$y^{**}_{23,1}$	0	1	−1	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(35)1	$y^{**}_{35,1}$	0	0	1	0	−1
(45)1	0.5	0	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(35)111	$y^{**}_{35,111}$	0	0	1	0	−1
(45)111	$y^{**}_{45,111}$	0	0	0	1	−1

The design structure consists of a column with entries of the values of the transformed responses $y^{**}_{jk,i}$ and a $\binom{J}{2}\cdot N \times J$ design matrix with J columns for the objects of the paired comparison study (here labelled with $1, 2, \ldots, J$). Each row represents one of the $\binom{J}{2}$ paired comparisons (ordered according to a predefined rule: $(12), (13), (23), (14), (24), (34), (15), \ldots, (J-1, J)$) which were stacked over all judges N. Note that we set λ_J equal to be zero for parameter estimation. Example: For $J = 5$ we obtain 10 paired comparisons and for N=111 the dimension of the design matrix is 1110 × 5.

2.2. Extensions of the BBTR model for paired comparisons

The BBTR model (5) can be extended for modelling possible effects of categorical and numerical subject covariates on the preference ordering of objects or incorporating object-specific covariates into the model. In the following we give some examples for BBTR models, each for the paired comparison (jk) for judge i.

Examples:
One categorical subject covariate S with s levels, $s = 1, 2$ (cf. Dittrich *et al.* 1998):

$$\operatorname{logit}(\mu_{jk,i|s}) = \lambda_j - \lambda_k + \lambda^{OS}_{j2} - \lambda^{OS}_{k2} + \lambda^S_1 + \lambda^S_2$$

where λ_j is the object parameter for the subject reference group, λ^{OS}_{js} is an interaction parameter between object j and subject level s and λ^S_s is the main effect of covariate S on level s (i.e. a nuisance parameter).

One numerical subject covariate (cf. Francis, Dittrich, Hatzinger, and Penn 2002):

$$\operatorname{logit}(\mu_{jk,i}) = \lambda_j - \lambda_k + x_i(\beta_j - \beta_k) + \lambda^M x_i$$

where x_i corresponds to the numerical covariate for individual i, β_j describes the effect of this covariate on object j and λ^M is the main effect of the numeric subject covariate M (i.e. a nuisance parameter).

One object-specific covariate (cf. Dittrich *et al.* 1998):

$$\operatorname{logit}(\mu_{jk,i}) = (x_j - x_k)\beta\ .$$

Here, the object parameters λ are replaced by $\lambda_j = x_j\beta$ where x_j is a covariate which describes a particular property of object j and β is an unknown object-specific parameter.

In all examples given we assumed that the precision parameter ϕ is constant for all observations, $g_2(\phi) = \gamma$. However, it is straightforward to specify a BBTR model for paired comparisons that allow the precision parameter to vary.

3. Application – Learning related emotions in mathematics

In the following we refer to achievement emotions (i.e. emotions that occur in achievement contexts) students typically experience when learning mathematics. Emotions are highly subjective experiences, sets of psychological processes for which it is difficult to come up with a reasonable measure (see also Pekrun, Goetz, Frenzel, Barchfeld, and Perry 2011).
In our study the emotions are the *objects* of interest for which we wanted to obtain an ordering on a continuum of frequency. In winter 2013 we carried out an online survey of students studying at the WU (Vienna university of economics and business) concerning learning related emotions in mathematics. By eliminating the response vector of a person with a missing value we received a sample size of N=111 (male=44, female=67).

In this study we considered five emotions taken from the study of Götz, Keller, and Martiny (2012): enjoyment, pride, anger, anxiety and boredom. We were interested if the ordering of learning related emotions on a frequency scale depends on particular subject variables. Several authors like Goetz, Pekrun, Zirngibl, Jullien, Kleine, vomHofe, and Blum (2004) and Frenzel, Pekrun, and Goetz (2007), for example, showed that the subject covariate gender, the individual achievement and the average class achievement have an influence on emotions experienced in mathematics. In this study we were interested if the gender and/or the students' *comparative ability* in mathematics (labelled `cab`), i.e. the self concept of ability compared to the perceived average of students' ability in mathematics, have an effect on learning related emotions in maths. The self concept of ability and the perceived students' ability were measured on a scale ranging from 0 to 100, where 0 indicated low and 100 high ability in mathematics. Comparing for each individual these two covariates we derived the subject covariate comparative ability (`cab`), where a negative sign indicates that student i has rated her/his ability in maths below the averaged perceived ability of other students (the social reference frame) and vice versa.

From this five learning related emotions we obtained ten emotion pairs. In each comparison students had to place a mark (with the cursor) on a bounded horizontal response scale (see Figure 1).

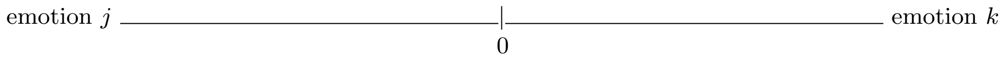

Figure 1: Assumed metric response scale with fixed bounds for the comparison (jk).

The given response corresponds to the degree of which an emotion is typically experienced more often than the other when learning maths. The response scale of each paired comparison had an arbitrary fixed length of 100 millimeters, i.e. 101 units including the response option in the middle of the scale (labelled with 0) which indicated a tie. The responses of the judges were scored by measuring the length or the units from the zero point (the middle of the scale) to the marked location. The random variables $Y_{jk,i}$ that are all associated with the first object in each of the $\binom{J}{2}$ comparisons can take on values in the interval $[-50, 50]$. An assigned value of $y_{jk,i} = -50$ indicates the most favourable response for the first emotion j in a given paired comparison (jk) and a value of $y_{jk,i} = 50$ the most favourable for the second emotion k. To avoid misunderstandings students were instructed how to use the response scale.
For the analysis we first squeezed the response variables $Y_{jk,i}$ with values in the interval $[-50, 50]$ into the interval $[0, 1]$ and then compressed the transformed variable $Y^{*}_{jk,i}$ so that the two times transformed variable $Y^{**}_{jk,i}$ assumes values in the standard interval $(0, 1)$ (see Section 2.1).
We started our model selection process by fitting a model with two main effects. One of them is the categorical subject covariate gender (`sex`) and the other the numerical subject covariate (`cab`). This main effects model gives a log likelihood of 141.4 (on 16 estimated parameters). For model comparison of nested BBTR models we used a likelihood ratio test. A reduction of the main effects model to either a model with the covariate gender (`sex`) or the covariate

comparative ability (`cab`) significantly worsened the model fit so that we preferred the main effects model (`sex`+`cab`). As models with constant precision parameter ϕ (as defined in Equation (5)) might be questionable, we also fitted a BBTR model that allows the precision parameter to vary, i.e. to depend on the subject covariates gender (`sex`) and comparative ability (`cab`). A general notation of the BBTR model with a variable precision parameter ϕ_i is given in Equation (2). We compared these two BBTR main effects models – one with a constant precision parameter (ϕ) for all observations i and the other one with `sex` and `cab` as additional regressors for the variable precision parameter (ϕ_i) in Equation (2) – by specifying for both models the log link function for the precision parameter. A likelihood ratio test gave evidence that the BBTR main effects model with a variable precision parameter significantly (p<0.001) improves the model fit and was therefore preferred for further analysis (see Table 2, main effects BBTR model).

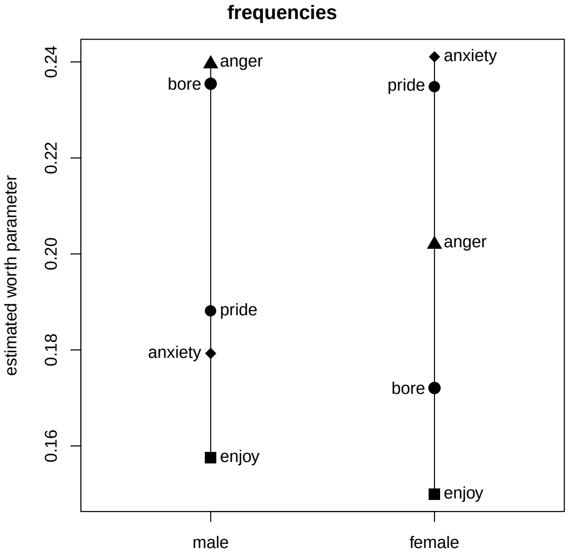

Figure 2: Plot of worth parameters for male and female students.

In Figure 2 the worth parameters of the BBTR model with the significant (p-value<0.001) covariate `sex` for both male and female students are shown. From this worth plot we can see that both groups differ in their ordering for the five emotions. Male students experience anger most often directly followed by boredom. For female students anxiety on the first place and pride on the second are the emotions most often experienced during learning mathematics. For both groups enjoyment is with distance the least experienced emotion. From the object parameter estimates (see Table 2, BBTR model covariate `sex`) of the BBTR model with covariate gender (`sex`) we noticed that female students experience the emotions pride and anxiety significantly (p-value=0.004 and p-value<0.001) more often than male students. From the fitted BBTR model with the significant (p-value<0.001) covariate comparative ability (`cab`) we can see in the worth plot (Figure 3) a tendency that the lower the comparative ability the more often anxiety is experienced. For students with relative low comparative ability (e.g. `cab` $= -60$) the distance between the first (anxiety) and second (anger) often experienced emotion is very large. This means that anxiety is with great distance the most often experienced emotion while learning maths. In Figure 3 we can see that there is a notable gap between the first three ranked emotions, i.e. anxiety, anger and pride. Whereas the worths of the emotions pride, boredom and enjoyment (the last three ranked emotions) are relative close together, which means that they are similar often experienced without big differences. We can see that the worth of all five emotions become more similar the closer the self concept of maths ability to the averaged perceived ability of other students. For students with high comparative ability enjoyment is ranked on the second place whereas for students who rate their ability lower than those of their colleagues enjoyment is the least experienced emotion. We can further see in Figure 3 that for the group of students with relative high comparative ability (e.g. `cab` $= 40$) the considerable distance between the three top ranked emotions, noticed from students with relative low comparative ability, diminished. For students with relative high comparative ability there is not one predominant emotion (compared to students with low comparative ability) but instead three emotions (i.e. pride, enjoyment and boredom) compete on the top of the most often experienced emotions. However, the gaps between the worths of the three top ranked emotions and the fourth ranked emotion (anger) and between anger and the least often experienced emotion pride widen for the group of students with high comparative ability. Let us now have a more detailed look at the estimated object parameters for students with `cab` $= -60$. The distance between the estimated parameters of anxiety and anger i.e. $\Delta_{anxiety,anger} = |(\hat{\lambda}_{o4}+\hat{\beta}_{o4}*-60)-(\hat{\lambda}_{o3}+\hat{\beta}_{o3}*-60)| = 0.660$ (see Table 2, BBTR model covariate `cab`). The distance between anger and pride $\Delta_{anger,pride} = 0.628$. For students with `cab` $= 40$ the distances between the first three ranked emotions are each much smaller than for students with low comparative ability (`cab` $= -60$), i.e. $\Delta_{pride,enjoyment} = 0.166$ and $\Delta_{enjoyment,boredom} = 0.100$.

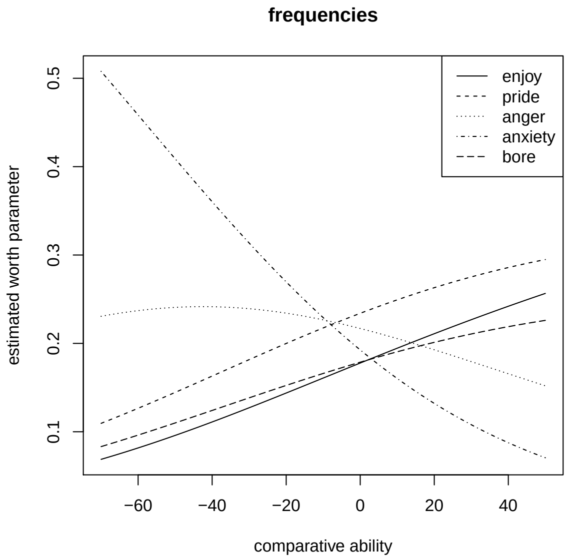

Figure 3: Plot of worth parameters for the subject covariate comparative ability in maths.

4. Discussion

In this article we suggested a simple beta Bradley-Terry regression (BBTR) model for modelling (assumed) metric paired comparison data. The structure of the BBTR model allows extensions for various variables, e.g. subject covariates or object-specific covariates. Model fitting can easily be done in a similar manner as for generalized linear models (GLMs) with the R-package `betareg` (Zeileis *et al.* 2013), provided that a corresponding design structure has been built up. Model selection of nested BBTR models can be done through likelihood ratio tests.

In our study we conducted a subjective self-rated survey which required both the ability and the willingness to deal with or to actually think about the experienced emotions and make them mentally present.

However, we selected and fitted a BBTR model with the two main effects gender (`sex`) and comparative ability (`cab`). A higher order model, i.e. an interaction model (`sex*cab`) would not significantly improve the model fit. In small samples the ML-parameter estimates may be biased, therefore we also fitted a BBTR model with bias correction (denoted by BC) and bias

Table 2: Estimates of nested BBTR models.

estimates	main effects BBTR model (s.e.)	BBTR model covariate `sex` (s.e.)	BBTR model covariate `cab` (s.e.)	BBTR 0-model (s.e.)
o1	0.434 (0.176)	0.402 (0.179)	0.005 (0.060)	0.005 (0.062)
o2	0.241 (0.148)	0.225 (0.151)	−0.270 (0.060)	−0.268 (0.062)
o3	−0.027 (0.126)	−0.017 (0.128)	−0.193 (0.060)	−0.218 (0.062)
o4	0.268 (0.112)	0.273 (0.114)	−0.075 (0.062)	−0.144 (0.063)
o5	0 (NA)	0 (NA)	0 (NA)	0 (NA)
o1:`sex2`	−0.318 (0.218)	−0.264 (0.223)	-	-
o2:`sex2`	−0.573 * (0.183)	−0.536 * (0.187)	-	-
o3:`sex2`	−0.083 (0.154)	−0.143 (0.157)	-	-
o4:`sex2`	−0.492 * (0.136)	−0.610 * (0.139)	-	-
o5:`sex2`	0 (NA)	0 (NA)	-	-
o1:`cab`	−0.005 (0.005)	-	−0.003 (0.005)	-
o2:`cab`	−0.003 (0.004)	-	0.000 (0.005)	-
o3:`cab`	0.010 * (0.004)	-	0.012 * (0.004)	-
o4:`cab`	0.022 * (0.003)	-	0.025 * (0.003)	-
o5:`cab`	0 (NA)	-	0 (NA)	-
`sex1` (male)	−0.096 (0.089)	−0.102 (0.091)	-	-
`sex2` (female)	−0.203 (0.065)	−0.188 (0.068)	-	-
`cab`	−0.002 (0.003)	-	−0.001 (0.003)	-
precision submodel:				
intercept	0.960 (0.059)	0.876 (0.058)	1.182 (0.038)	1.070 (0.037)
`sex2`	0.511 (0.078)	0.438 (0.076)	-	-
`cab`	0.004 (0.002)	-	0.001 (0.002)	-
log-likelihood	163.5	96	116.2	56.39
number of estimated parameters	18	12	11	5

The five emotions are shortnamed as follows: o1=enjoyment, o2=pride, o3=anger, o4=anxiety, o5=boredom and the numeric subject covariate comparative ability is labelled `cab`. Note that the worth parameters illustrated in Figure 2 and 3 were obtained from reversed object parameter estimates so that higher values indicate a more frequent experience.

reduction (denoted by BR) of the maximum likelihood (ML) parameter estimates using the `betareg`-package (see Grün, Kosmidis, and Zeileis 2012). We obtained very similar parameter estimates i.e. the differences in the parameter estimates were not worth mentioning.

The results of the BBTR model with the subject covariate gender (`sex`) indicate that male and female students differ in their ordering of learning related emotions. Male students typically experience most often anger while learning maths. Female students place anxiety on the first place of frequent experienced emotions. This outcome could be due to the fact that female students may fear (failure in) solving difficult maths tasks as they might have a tendency to underestimate their abilities in maths for various reasons (e.g. stereotype thinking).

One aim in the future is to define a latent class BBTR model for being able to model possible effects of latent (unobservable) subgroups.

Acknowledgement

We would like to thank Walter Katzenbeisser and Brian Francis for discussions and helpful comments.

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Austrian Journal of Statistics `http://www.ajs.or.at/`
published by the Austrian Society of Statistics `http://www.osg.or.at/`
Volume 44 *Submitted:* 2014-04-20
January 2015 *Accepted:* 2014-07-29

Austrian Journal of Statistics
January 2015, Volume 44, 17–24.
http://www.ajs.or.at/

Statistics and Probability at Secondary Schools in the Federal State of Salzburg: An Empirical Study

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Abstract

Knowledge about the practical use of statistics and probability in today's mathematics instruction at secondary schools is vital in order to improve the academic education for future teachers. We have conducted an empirical study among school teachers to inform towards improved mathematics instruction and teacher preparation. The study provides a snapshot into the daily practice of instruction at school. Centered around the four following questions, the status of statistics and probability was examined. Where did the current mathematics teachers study? What relevance do statistics and probability have in school? Which contents are actually taught in class? What kind of continuing education would be desirable for teachers? The study population consisted of all teachers of mathematics at secondary schools in the federal state of Salzburg.

Keywords: high school, mathematics education, secondary school, teacher.

1. Introduction and motivation

The Department of Mathematics at the University of Salzburg offers, in addition to Bachelor's, Master's, and PhD degrees in Mathematics, also a study program for secondary school mathematics instruction, resulting in Bachelor's and Master's degrees in Mathematics Education. For this program, courses in probability and statistics have been in existence for several years, specially designed with the intention of providing effective and up-to-date teacher training. As school curricula are being adjusted, also university curricula for teacher education need to adapt. A major change in Austrian secondary school education is the current introduction of a central school leaving examination (Matura), combined with the call for the formulation of competence oriented learning objectives in school. At the same time, the role and importance of statistics in secondary school mathematics classes have gradually increased over the last years. Universities offering mathematics teacher training thus need to adapt to these changes occurring in the secondary education landscape. However, once teachers leave the university training, they are generally not available to provide feedback to their *alma mater*, as continuing education for Austrian school teachers is typically not provided by the universities, but by a system of teacher training colleges (*Pädagogische Hochschulen*). While much data is collected regularly based on surveying students, a comprehensive survey of mathematics school teachers in Austria, not to mention an up-to-date one, is not known to the authors.

Thus, the university departments are lacking up-to-date information on the actual demands and practice in school instruction, and this has motivated the present empirical study involving secondary school mathematics teachers in the Austrian Federal State of Salzburg. The teachers were contacted to provide feedback on issues centered around the following key questions.

1. At which universities or colleges, and with which perceived quality of preparation, did current Mathematics teachers in Salzburg receive their training?
2. What role do probability and statistics currently play in school teaching?
3. What specific (probabilistic/statistical) content is actually covered in secondary school mathematics instruction?
4. What are the continuing education needs of school teachers with regard to probability and statistics?

These questions make it clear that the goals of the study were different from those trying to assess knowledge or professionalism of mathematics teachers (see, *e.g.* Blömeke, Kaiser, and Lehmann 2008; Krauss, Brunner, Kunter, Baumert, Blum, Neubrand, and Jordan 2008, and references cited therein). Simply said, in the present study, the questions were not centered around “what do mathematics teachers know?”, but rather “what do mathematics teachers teach, and do they feel prepared for teaching it?”. The authors were mostly interested in the subject of statistics and how it is taught in school, but statistics and probability are very closely related in secondary school mathematics in German speaking countries. Therefore, both topics were explicitly included in the study.

In the following, the results of the empirical study are presented, along with several details on how it was performed. The survey and all communication with teachers and school administrators was conducted in German. Survey responses or answer categories quoted below have been translated for this article. The original answer categories in German can be obtained from the first author upon request.

2. Conducting the survey

The survey was conducted from January to June 2013 and evaluated as part of the diploma thesis of the first author (Voit 2013). All mathematics teachers at the 66 secondary schools in the Federal State of Salzburg were contacted and asked to fill out the survey. There are currently about 250 mathematics teachers in Salzburg. The exact number of people teaching mathematics in any given year is not known, as not everyone who has studied mathematics education actually teaches this subject every year. Typically secondary school teachers are trained to teach two subjects. Some are in practice actually only teaching one of those, and some are mostly teaching one subject, and rarely the other. The estimate of 250 is calculated as follows: the federal statistics agency *Statistik Austria* publishes the number of teachers per school type for each state (Statistik Austria 2012a). For each school type, the proportion of mathematics instruction hours as part of the total hours of instruction is known, based on standard curricula (bmukk 2003). Assuming that the proportion of mathematics teachers among all teachers corresponds roughly to the proportion of mathematics instruction hours among all hours, the above estimate is obtained. This approach was validated using the course schedule of two schools known to the first author: in one of these schools, 40 out of 370 total instruction hours were mathematics, and 8 of 74 teachers were mathematics teachers. Both proportions are equal (10.8%). In the other school, 21 of 415 hours (5.0%), and 2 of 38 teachers (5.3%) correspond to mathematics. Both examples suggest that the used method of approximating the number of mathematics teachers per school is reasonable.

In order to obtain a large response rate, a combination of different strategies was used. The respective school supervisory board members as well as school directors were asked for permission and support. Further, most schools in the city of Salzburg were visited in person, and paper surveys were hand-delivered. Schools in the countryside were contacted via email, but it was requested that the surveys were printed and handed to the mathematics teachers in paper form. The decision to use paper surveys stems from the fact that teachers nowadays often get inundated with email requests for participation in surveys which may obviously get deleted and forgotten quickly. In order to differentiate the effect of personal visit vs. email request on response rate from the possible effects of urban vs. rural setting, a few schools in the city served as a control group and received the survey electronically, in the same way as all the rural schools. Survey results were picked up personally in the respective schools in the city, otherwise they were mailed in, with a few exceptions where surveys were scanned and emailed. In order to provide a serious incentive for participation, the Department of Mathematics agreed to indeed organize continuing education courses if a need or desire became apparent, and with the topics deemed most needed by the school teachers. No monetary incentives or raffles were provided.

The survey itself had 25 questions on two A4 pages, accompanied by a letter. The 25 questions resulted in 76 individual variables which were entered into and analyzed using SPSS 20 (IBM Corp 2011).

Table 1 summarizes the responses per school type. Austrian higher secondary schools are divided into two main types. There are the common secondary high schools with an education preparing students for possible further studies at universities and applied universities (AHS, *Allgemeinbildende Höhere Schule*), as well as professionally oriented secondary high schools (BHS, *Berufsbildende Höhere Schule*) that offer a dual education system combining common topics and apprenticeship in prepration for special professions in technical (HTL) or commercial (HAK) subjects, and also for tourism and home economics (HLW).

Altogether, the response rate across all school types was almost 60%, indicating a strong interest of mathematics teachers in the survey and in the issues surrounding it. However, among the technical secondary schools *HTL*, the response rate was at less than 10% (only three surveys returned) strikingly low and thus they were removed from some of the further analysis. The technical secondary schools are typically relatively large in size. The authors suspect that the dimensions and buerocracy at these large schools were a prohibitive factor. When reducing the study to the non-technical secondary schools, the response rate was at two thirds (67.3%) of the estimated study population. Comparison of the city schools showed a clearly higher response rate among those schools that were visited personally, as compared to those that only received an electronic invitation. It is evident that the time-consuming delivery and pick-up of surveys were important factors in achieving the high response rate.

Table 1: Response numbers and rates per school type. For the school type abbreviations, see Section 2.

School Type	Estimated Population Size	Number of Responses	Response Rate [%]
All types	247	143	58
AHS	150	102	68
BHS	97	41	42
HTL	39	3	8
BHS other	58	38	66
All without HTL	208	140	67

3. Survey results

Technically, all the mathematics teachers in the state were attempted to be contacted. Thus, the survey may be regarded as a census with nonresponse, rather than a random sample. In order to correct for the effects of about one third nonresponse, respondents could be assigned weights according to their school type. *A priori*, there is no justification to assume that age or gender distributions of mathematics teachers are to resemble those of the general teacher population in the state. Thus, school type is the only stratifying variable for which population totals can be convincingly estimated, as described in the beginning of Section 2. The response rates of 68% for *AHS* teachers, and 66% for *BHS* teachers (without technical schools) are almost identical. Consequently, the respective weights would be quasi identical, as well. Therefore, the authors have decided to forego any response weighting for this survey, considering also that the actual response counts are presented and interpreted in a more straightforward manner without weighting.

3.1. Demographics

Table 2: Age distribution per school type. For the school type abbreviations, see Section 2. The table shows total counts and column percentages (in parentheses). Some teachers are shared between different schools and the respective proportions are considered at each table entry, resulting in rounding differences for the totals. Three teachers did not provide their age.

	Mathematics Teachers (Sample)			All Teachers (Statistik Austria 2012b)		
Age Group [y]	AHS	BHS	All Schools	AHS	BHS	All Schools
< 30	19 (19%)	4 (10%)	23 (17%)	125 (9%)	107 (6%)	232 (7%)
30-39	17 (17%)	10 (26%)	27 (19%)	302 (21%)	384 (22%)	686 (21%)
40-49	25 (25%)	11 (28%)	36 (26%)	394 (27%)	597 (34%)	991 (31%)
50-59	34 (34%)	11 (28%)	45 (32%)	535 (37%)	583 (33%)	1118 (35%)
60-	6 (6%)	3 (8%)	9 (6%)	84 (6%)	89 (5%)	173 (5%)
Total	101	39	140	1440	1758	3200

Table 2 shows the age structure of the sample, compared to that of the total teacher population (not only mathematics teachers) at secondary schools in the State of Salzburg (Statistik Austria 2012b). The only noticeable difference is with regard to the group of under 30 year old teachers, which appear to be overrepresented in the sample. A possible explanation could be the recent lack of mathematics teachers, resulting in recent mathematics graduates being employed at higher proportions than graduates of other subjects. Also, recent graduates from the University of Salzburg may have been more inclined to respond to the survey, as they may still feel a stronger connection to their *alma mater*. Among all respondents were 57% women. Among those younger than 40 years of age, even 33 out of 50 (66%) were female.

3.2. Where did they study – and did it prepare them well for their job?

Table 3: Number of responding teachers by university where teaching degree was obtained. Four teachers did not provide this information.

	Number	Percentage
Salzburg	114	82
Innsbruck	2	1
Linz	6	4
Graz	5	4
Wien (U)	6	4
Wien (TU)	3	2
not in Austria	3	2
Total	139	100

As Table 3 shows, more than 80% of the current mathematics teachers responding to the survey have studied at the University of Salzburg. Clearly, this indicates low incoming mobility of teachers. It would be interesting to compare these numbers to those in other Austria states in order to see whether there is outgoing mobility from Salzburg, or whether generally students strongly tend to become mathematics teachers in the same states where they have studied. Due to the low number of graduates from other institutions in the survey, comparisons of the different universities with regard to their mathematics teacher preparation were not conducted. Also, it did not seem advisable to aggregate graduates from all universities other than Salzburg for comparison purposes.

Table 4 shows to which degree current mathematics teachers considered their university education with regard to statistics and probability *meaningful* or *sufficient.*

Table 4: Degree to which respondents found university preparation regarding statistics and probability meaningful or sufficient (counts). Ten teachers did not provide an answer regarding "meaningful", eight did not provide an answer regarding "sufficient". In total, twelve teachers did not provide both answers.

Number of teachers		University preparation sufficient					Total
		yes	somewhat	don't know	not so much	no	
University preparation meaningful	yes	24	11	0	8	0	43
	somewhat	3	8	6	12	2	31
	don't know	0	3	3	3	3	12
	not so much	4	1	3	11	4	23
	no	1	1	1	2	17	22
Total		32	24	13	36	26	131

Altogether, the university preparation was considered meaningful or somewhat meaningful by a majority (56%) of respondents. However, answers regarding usefulness of the preparation were not at all unequivocal, with 43% responding positively (yes or somewhat) and 47% answering negatively (no or not so much). The Spearman rank correlation between both variables ($\hat{\rho} = 0.627$), indicates that generally respondents who considered the university preparation sufficient were also more likely to consider it meaningful.

3.3. How important are statistics, probability, and computers in today's mathematics teaching?

More than 70% of respondents stated that they knew which mathematics standards were required for the 8th school level. This is important, as mathematics instruction in the following years builds upon these standards. Stratifying by school type differentiated this picture: in the higher vocational schools with business (HLW, *Höhere Lehranstalt für Wirtschaftliche Berufe*) or tourism focus, only 5 out of 12 responding teachers (42%) knew the 8th grade standards. In these school types, mathematics was generally not considered as important as in other schools, and it had not been a subject on the school leaving examination. However, this will now change because of the new central school leaving examination (Matura) making mathematics an exam subject for all secondary schools. Altogether, 60% of respondents said that they new the requirements for the new Matura, and about the same proportion stated that the new Matura would increase the importance of probability and statistics in mathematics instruction.

The new central school leaving examination assumes that pupils will have obtained experience in technology (computer) use before they graduate. Therefore, a closely related question is whether computers are actually used in mathematics instruction. A majority of AHS teachers (52%) responded positively (see Table 5). Strikingly low was however the proportion of positive responses among BHS teachers (22%). This appears particularly surprising as the BHS is considered more practically oriented and less academic. In this context, more data from the technical schools, which had to be excluded from some of the analyses due to their non-representative low response rates, would be interesting. Based on personal experience, the authors conjecture that computer technology is frequently used in mathematics instruction in these schools, and 3 out of the 4 responses from technical schools were affirmative.

Table 5: Use of computer in class instruction (counts). Technical schools excluded (four teachers). Three teachers did not answer the question regarding computer use in the classroom.

Number		School type		Total
		AHS	BHS	
Computer use	yes	53	8	61
in classroom	no	48	27	75
Total		101	35	136

3.4. What is actually taught in statistics and probability school lessons?

We asked the mathematics teachers to check on a list of more than 20 topics taken from mathematics curricula which ones are covered in their classes. There was a large spread in responses with some topics only being taught by a few (as few as 4), while others were taught by almost everyone (as many as 128). Also, 27 respondents found that there was not enough time for a comprehensive treatment of probability and statistics in class.

Specifically, about 90% of respondents said that they covered descriptive statistics, and elementary probability, while combinatorics and conditional probability were each taught by about 70%. Normal and binomial distribution were each introduced by more than 85%, but less than 15% covered the Poisson distribution. About 40% taught confidence intervals and 30% hypotheses tests, but only about 12% said that they covered p-values, and about the same number (13%) taught sampling distributions. The normal approximation was introduced by about 60% of respondents, whereas 20% mentioned that they specifically covered the central limit theorem. Correlation and linear regression were each covered by about 30% of respondents (on the survey, these options did not appear directly adjacent). Less than 5% covered analysis of variance.

3.5. Continuing education for mathematics teachers

Perusing continuing education catalogues of the teacher training colleges (*Pädagogische Hochschulen*) revealed only a small number of events regarding probability and statistics. Thus, it is perhaps not surprising that about two thirds of respondents (see Table 6) expressed a desire to have continuing education events in statistics and probability, that are offered by the Department of Mathematics at the University of Salzburg (which includes the statistics and probability group). Specific topics desired most frequently were those of statistical inference, namely estimation and hypothesis testing.

Table 6: Should the Department of Mathematics at the University of Salzburg offer continuing education for school teachers? One person did not provide an answer.

	Number	Percentage
yes	42	30
rather yes	54	38
don't know	23	16
rather not	12	8
no	11	8
total	142	100

While there was a general concern about the lack of sufficient continuing education opportunities offered by the teacher training colleges, also two thirds of respondents said that they generally did not take advantage of the teacher training colleges' continuing education offers. Among the teachers at business colleges (*Handelsakademie*), even as many as 94% stated that they did not use these opportunities, indicating a possible disconnect between desired and offered course topics.

The request for continuing education to be offered by the university department did not differ much between school types and was larger than 60% for each school type. This was communicated to the Department of Mathematics at the University of Salzburg and has already resulted in the planning of such events. For practical reasons, these should be offered in cooperation with the teacher training colleges. In fact, only 7% of respondents explicitly stated that they would like for the teacher training colleges not to be involved in such events.

4. Discussion and outlook

For a large part, the results summarized above are not surprising. Nevertheless, they provide a glimps at daily work in Salzburg's schools. For example, while just a few years ago, mathematics was a subject taught mostly by men, today almost 60% of mathematics teachers are female. The age structure of mathematics teachers in Salzburg corresponds closely to that of all teachers in the state. Most of those currently teaching mathematics have obtained their university degree at the University of Salzburg.

Statistics and probability are currently gaining in importance, also reinforced by the requirements of the new central Austrian school leaving examination. Mathematics teachers would like to see continuing education opportunities in statistics and probability, and these are not yet sufficiently provided by the teacher training colleges. Here, the university departments of mathematical sciences, not only in Salzburg, can position themselves as competent partners in providing comprehensive and competent continuing education opportunities for secondary school teachers, ideally in cooperation with nearby teacher training colleges.

Most teachers responding to the survey discussed basic aspects of statistics and probability in their mathematics school lessons, while advanced material such as the sampling distributions, p-values, and the central limit theorem was taught by less than a quarter, and special topics such as analysis of variance only by very few. Several teachers mentioned lacking sufficient time for an adequate coverage of statistics and probability in class.

It would be interesting to conduct a similar study in other Austrian states or even beyond the Austrian borders, and compare the results, also with regard to the universities preparing students to become mathematics teachers. However, in light of the current structural reforms affecting secondary education in Austria, the results of such studies would certainly depend on the degree with which changes have been implemented and thus they are expected to change over the coming years.

Acknowledgement

The study was supported by a large majority of mathematics teachers at the secondary schools in the Austrian Federal State of Salzburg. These mathematics teachers have contributed greatly to its success, and the authors would like to express their sincere gratitude to all participants, as well as to the administrators who have provided support at different stages of the study. Furthermore, several helpful and constructive suggestions by Prof. Andreas Quatember (JKU Linz, Department of Applied Statistics) and by the AJS review team have resulted in a much improved manuscript.

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Austrian Journal of Statistics http://www.ajs.or.at/
published by the Austrian Society of Statistics http://www.osg.or.at/
Volume 44 *Submitted:* 2014-02-10
January 2015 *Accepted:* 2014-07-17

Austrian Journal of Statistics
January 2015, Volume 44, 25–32.
http://www.ajs.or.at/

Measurement of Continuous Quantities and their Statistical Evaluation

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Abstract

Measurement results of continuous quantities are always more or less imprecise. This imprecision is different from errors. The most suitable mathematical model to describe imprecision is by special fuzzy subsets of the set of real numbers $I\!R$, called characterizing functions. The statistical analysis of fuzzy measurement data is subject of this paper.

Keywords: characterizing function, data uncertainty, fuzzy models, fuzzy vectors, non-precise numbers.

1. Introduction

Usually the analysis of measurement uncertainty is done by stochastic models and statistical methods. This is also realized in norms like DIN EN ISO 20988, VDI 4219(E), and GUM EVN (1999).

More recent methods for the analysis of measurement results of continuous quantities are using a combination of fuzzy models and stochastic models. The fuzziness of individual measurement results is described by so-called *fuzzy numbers*, and the variability by stochastic models. Based on that the analysis of repeated measurements is possible by suitably generalized statistical methods.

2. Measurement uncertainty

Measurement results of continuous quantities are connected with unavoidable uncertainties. The best one can do is to try keeping the uncertainty as small as possible, and to make reasonable (reliable) assessments (estimates) for the considered quantities.

The most important measurement uncertainties are errors and the fuzziness of individual measurement results. Errors are devided into systematic errors and random errors.

Usually measurement results of one-dimensional quantities are described by real numbers x_i. Moreover it is assumed that the measurement result x_i is the sum of the so-called *true value* x, a systematic error y, and a random error ϵ_i, i.e.

$$x_i = x + y + \epsilon_i \quad \text{with} \quad x_i \in I\!R.$$

For multiple measurements ϵ_i is assumed to be the realisation of a one-dimensional stochastic quantity with expectation 0.

The distribution of the stochastic quantity $\widetilde{\epsilon}$, which is modelling the random error, can be estimated from repeated measurements of the observed quantity.

The analysis of the systematic error is more complicated and needs a deep scientific analysis of the measurement procedure.

The above assumption that measurement results are real numbers is not realistic because a real number is determined by all its infinitely many decimals which never can be known (except in trivial cases). Therefore it turns out that measurement results are better described by so-called *fuzzy numbers.*

3. Mathematical description of fuzziness

What is the result of an individual measurement of an one-dimensional continuous quantity like length, time, concentration etc.? For digital measurement equipments the measurement result x of an individual measurement is a decimal number with finitely many digits. Concerning the missing infinitely many decimals almost nothing is known. Therefore all possible digits from 1 to 9 are possible for the remaining decimals. In this case the measurement result is an interval $[\underline{x};\overline{x}]$, where $\underline{x}$ is the real number which is generated if all unknown decimals of x are assumed to be 0. The real number $\overline{x}$ is generated when all unknown decimals of x are set to be 9.

Therefore the measurement result is an *interval*, i.e. a subset of the set of real numbers $I\!R$.

The analysis of multiple measurements of the same quantity is than a problem of interval analysis.

The situation is more general in case of using an analog measurement equipment like measurement rods, pointers, and imaging systems like oscilloscopes. In this case individual measurement results are pictures, light points, functions or colour intensity pictures. In order to describe mathematically this kind of measurement results so-called *fuzzy numbers* $x^\star$are used. These are special fuzzy subsets of the set $I\!R$.

Coming back to the measurement result $[\underline{x};\overline{x}]$ in case of a digital measurement equipment, this interval can be characterized logically equivalent by its *indicator function* $I_{[\underline{x};\overline{x}]}(\cdot)$. The indicator function is defined by its values

$$I_{[\underline{x};\overline{x}]}(x) := \left\{ \begin{array}{lll} 1 & \text{for} & \underline{x} \leq x \leq \overline{x} \\ 0 & \text{otherwise} & \end{array} \right\} \quad \forall \quad x \in I\!R.$$

Indicator functions can be identified with subsets of general sets M.

In reality the boundaries of subsets can be difficult. Therefore in 1951 K. Menger Menger (2003) published a generalization of indicator functions describing generalized sets called *ensembles flous.* These generalized indicator functions where later called *membership functions* $\zeta(\cdot)$, which can assume all values from the unit interval $[0;1]$, i.e. $\zeta : M \to [0;1]$. These generalized sets where called *fuzzy sets* in a paper by L. Zadeh in 1965. Taking care of a continuum of truth values mathematical methods connected with fuzzy sets are called *fuzzy logic.*

If $A^\star$ is a fuzzy subset of an arbitrary set M, having membership function $\zeta(\cdot)$, for elements $x \in M$ the value $\zeta(x)$ is called *degree of membership* of x in $A^\star$.

For the description of measurement results of one-dimensional quantities by analog measurement equipments special fuzzy subsets of $I\!R$, so-called *fuzzy numbers* are most suitable. The correspondig membership function $\zeta(\cdot)$ must meet the following conditions:

(1) $\mathit{supp}\,[\zeta(\cdot)] = \{x \in I\!R : \zeta(x) > 0\}$ is a bounded subset of $I\!R$

(2) $\forall \delta \in (0;1]$ the so-called *δ-cut* $C_\delta[\zeta(\cdot)] := \{x \in I\!R : \zeta(x) \geq \delta\}$ must be non-empty and a finite union of compact intervals, i.e.

$$C_\delta[\zeta(\cdot)] = \bigcup_{j=1}^{k_\delta} [a_{\delta,j}; b_{\delta,j}] .$$

Membership functions obeying (1) und (2) are called *characterizing functions.*

Characterizing functions of fuzzy numbers will be denoted by $\xi(\cdot)$ in the following.

In figure 1 some characterizing functions are depicted.

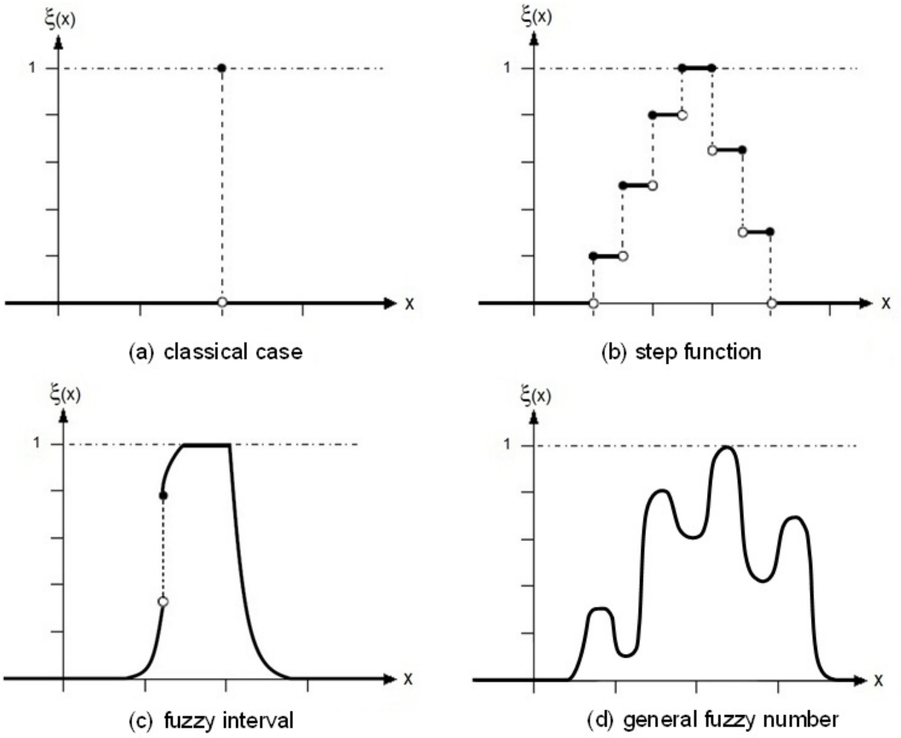

Figure 1: Examples of characterizing functions

4. Analysis of fuzzy measurement results

Based on the description of measurement results in section 3 a measurement result is a fuzzy number $x_i^\star$, whose characterizing function $\xi_i(\cdot)$ in case of a colour intensity picture is a function of the colour intensity $h(x)$, $x \in I\!R$ and its derivative $h'(\cdot)$. The values of the characterizing function are obtained in the following way:

$$\xi_i(x) = \frac{|h'(x)|}{\sup\{\,|h'(x)| : x \in I\!R\}} \quad \forall \quad x \in I\!R$$

Considering errors we have $x_i^\star = x + y + \epsilon_i$, which means, that at least one of the quantities x, y, and ϵ_i is fuzzy. The systematic error is assumed to be a real number, therefore one of the quantities x and ϵ_i must be fuzzy. Based on the standard statistical analysis ϵ_i is the realization of a one-dimensional stochastic quantity. Therefore x must be fuzzy, which means the so-called *true value* is a fuzzy number $x^\star$. It should be noted that the systematic error y can practically only be determined as fuzzy number.

Assuming a measurement procedure has no systematic error, i.e.

$$x_i^\star = x^\star + \epsilon_i, \quad i = 1(1)n,$$

the determination, i.e. estimation of $x^\star$ based on fuzzy data $x_1^\star, \cdots, x_n^\star$ is possible.

In generalization of the classical mean value the mean value (which has to be defined) of the fuzzy observations has to be calculated:

$$\overline{x}_n^\star = \frac{1}{n}(x_1^\star \oplus \cdots \oplus x_n^\star).$$

The generalized addition operation $\oplus$ of fuzzy numbers is based on the so-called *extension principle* from the theory of fuzzy sets. $\overline{x}_n^\star$ is a fuzzy number whose characterizing function $\psi(\cdot)$ can be calculated from the characterizing functions $\xi_1(\cdot), \cdots, \xi_n(\cdot)$ of the fuzzy numbers $x_1^\star, \cdots, x_n^\star$. The fuzzy number $\overline{x}_n^\star$ is the best information which can be obtained concerning the true value $x^\star$, based on the fuzzy sample $x_1^\star, \cdots, x_n^\star$.

In case of continuous quantities the unavoidable fuzziness of measurements $x_i^\star$ appears in the estimation of $x^\star$.

5. Calculation of the characterizing function of the fuzzy sample mean

In order to determine the characterizing function $\psi(\cdot)$ of $\overline{x}_n^\star$ it is necessary to combine the fuzzy numbers $x_1^\star, \cdots, x_n^\star$, having characterizing functions $\xi_1(\cdot), \cdots, \xi_n(\cdot)$, into a so-called *fuzzy vector* $\underline{x}^\star$ of the n-dimensional Euclidean space $\mathbb{R}^n$, in order to apply the so-called *extension principle* of the theory of fuzzy sets. This combination is done by the so-called *minimum-t-norm* (compare Viertl and Hareter (2006)) in the following way.

A fuzzy vector $\underline{x}^\star$ is defined by its so-called *vector-characterizing function* $\zeta(\cdot, \cdots, \cdot)$, which has to obey the following:

($\underline{1}$) $\zeta : \mathbb{R}^n \longrightarrow [0; 1]$

($\underline{2}$) $supp\Big[\zeta(\cdot, \cdots, \cdot)\Big]$ is a bounded subset of $\mathbb{R}^n$

($\underline{3}$) $\forall \delta \in (0; 1]$ the δ-cut $C_\delta[\zeta(\cdot, \cdots, \cdot)] := \{\underline{x} \in \mathbb{R}^n : \zeta(\underline{x}) \geq \delta\}$ has to be a non-empty, closed and bounded subset of $\mathbb{R}^n$, which is a finite union of simply connected sets.

In order to extend functions $g : \mathbb{R}^n \longrightarrow \mathbb{R}$ to the situation of fuzzy arguments $\underline{x}^\star$, the so-called *extension principle* is used.

Let M and N be arbitrary sets and $g\colon M \to N$ an arbitrary function. The function $g(\cdot)$ can be extended to fuzzy argument values $x^\star$ of M, where $\zeta(\cdot)$ is the membership function of $x^\star$.

The value $g(x^\star)$ is a fuzzy subset of N, whose membership function $\psi(\cdot)$ is defined in the following way:

$$\psi(y) := \left\{ \begin{array}{lll} \sup\{\zeta(x) : x \in M,\ g(x) = y & \text{if} & g^{-1}(\{y\}) \neq \emptyset \\ 0 & \text{if} & g^{-1}(\{y\}) = \emptyset \end{array} \right\} \quad \forall \quad y \in N$$

In order to apply the extension principle to the function $g(x_1, \cdots, x_n) = \frac{1}{n} \sum\limits_{i=1}^{n} x_i$ for fuzzy numbers $x_1^\star, \cdots, x_n^\star$ these fuzzy numbers have to be combined into a n-dimensional fuzzy vector. This is done by application of the minimum-t-norm, which determines the vector-characterizing function $\zeta(\cdot, \cdots, \cdot)$ in the following way:

$$\zeta(x_1, \cdots, x_n) := \min\{\xi_1(x_1), \cdots, \xi_n(x_n)\} \quad \forall (x_1, \cdots, x_n) \in \mathbb{R}^n$$

The reason for using the minimum-t-norm is explained in the following proposition.

Proposition 1: Let n fuzzy numbers $x_1^\star, \cdots, x_n^\star$ be combined by the minimum-t-norm, then the resulting function $\zeta\colon \mathbb{R}^n \to [0; 1]$ is a vector-characterizing function.

Proof: Condition (1) is trivially fulfilled.

Condition (2) follows from the fact that all supports of $\xi_i(\cdot)$ are bounded and the following holds: $supp[\zeta(\cdot,\cdots,\cdot)] = \mathop{\times}\limits_{i=1}^{n} supp[\xi_i(\cdot)]$.

Condition (3) follows from the fact that the δ-cuts of $\zeta(\cdot,\cdots,\cdot)$ are the Cartesian products of the δ-cuts of $\xi_i(\cdot)$.

Based on this combined fuzzy vector the characterizing function of the fuzzy arithmetic mean of n fuzzy numbers can be determined.

Proposition 2: The arithmetic mean of n fuzzy numbers with corresponding characterizing functions $\xi_1(\cdot),\cdots,\xi_n(\cdot)$ is a fuzzy number, whose characterizing function $\psi(\cdot)$ is given by its values $\psi(x)\ \forall x \in I\!R$ in the following way:

$$\psi(x) = \sup\left\{\min\left\{\xi_1(x_1),\cdots,\xi_n(x_n)\right\} : \text{for} \frac{1}{n}\sum_{i=1}^{n} x_i = x\right\}$$

The δ-cuts $C_\delta\left[\psi(\cdot)\right]$ for all $\delta \in (0;1]$ are finite unions of compact intervals given by

$$C_\delta[\psi(\cdot)] = \bigcup_{\sum\limits_{i=1}^{n} x_i = x} \left\{\frac{x_1+\cdots+x_n}{n} : x_i \in C_\delta\left[\xi_i(\cdot)\right]\right\}.$$

Proof: The δ-cuts of the combined fuzzy vector $\underline{x}^\star$ are finite unions of simply connected and compact subsets of $I\!R^n$. By the continuity of the function $g(x_1,\cdots,x_n) = \frac{1}{n}\sum\limits_{i=1}^{n} x_i$ also the δ-cuts of $g(\underline{x}^\star)$ from the extension principle are finite unions of compact simply connected subsets of $I\!R$, and therefore finite unions of compact intervals. By that reason the fuzzy mean value is a fuzzy number. For the δ-cuts of the fuzzy mean value the following holds:

$$C_\delta\left[\psi(\cdot)\right] = g\Big(C_\delta\left[\xi_1(\cdot)\right],\cdots,C_\delta\left[\xi_n(\cdot)\right]\Big) = \left\{\frac{1}{n}\sum_{i=1}^{n} x_i : x_i \in C_\delta\left[\xi_i(\cdot)\right]\ \forall\ i = 1(1)n\right\}$$

The result of measurements of continuous quantities are fuzzy numbers and the measurement uncertainty of the calculated value is presented by the characterizing function of the fuzzy mean value.

Example: Let 5 measurement results be given as fuzzy numbers with characterizing functions $\xi_1(\cdot),\cdots,\xi_5(\cdot)$ which are depicted in figure 2. Then the characterizing function $\psi(\cdot)$ of the fuzzy arithmetic mean is determined with the help of δ-cuts.

The characterizing function $\psi(\cdot)$ of the arithmetic mean of the fuzzy observations is depicted in figure 3.

Remark: The characterizing function $\psi(\cdot)$ is the most realistic information concerning the measured quantiy. The area under the function $\psi(\cdot)$ is characteristic for the measurement uncertainty.

It is possible to analyse the dispersion of the fuzzy measurements. In order to do that the generalized sample spread of the fuzzy measurement results can be calculated. This is the fuzzy quantity $s_n^\star$, whose characterizing function can be calculated by application of the extension principle to the classical sample spread

$$s_n = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x}_n)^2\right]^{1/2}.$$

For this the extension principle is applied to the function

$$g(x_1,\cdots,x_n) = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x}_n)^2\right]^{1/2}.$$

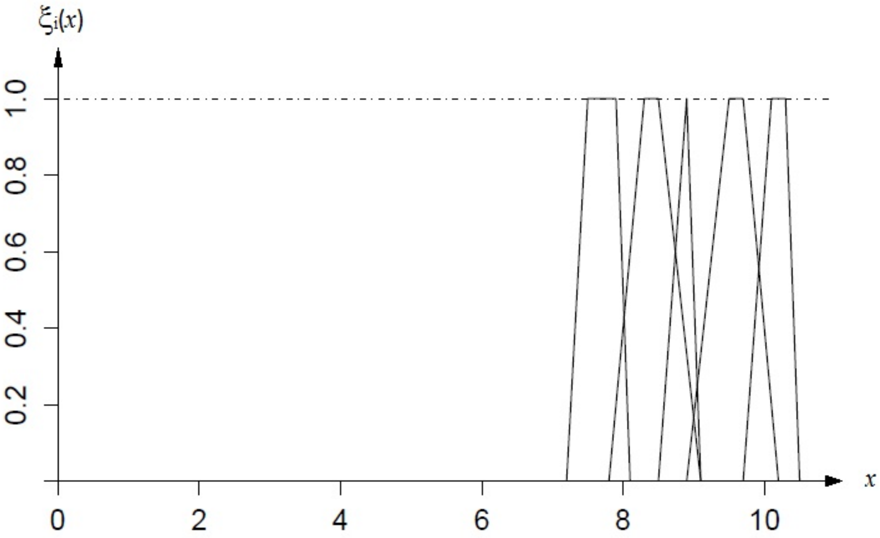

Figure 2: Fuzzy measurement results

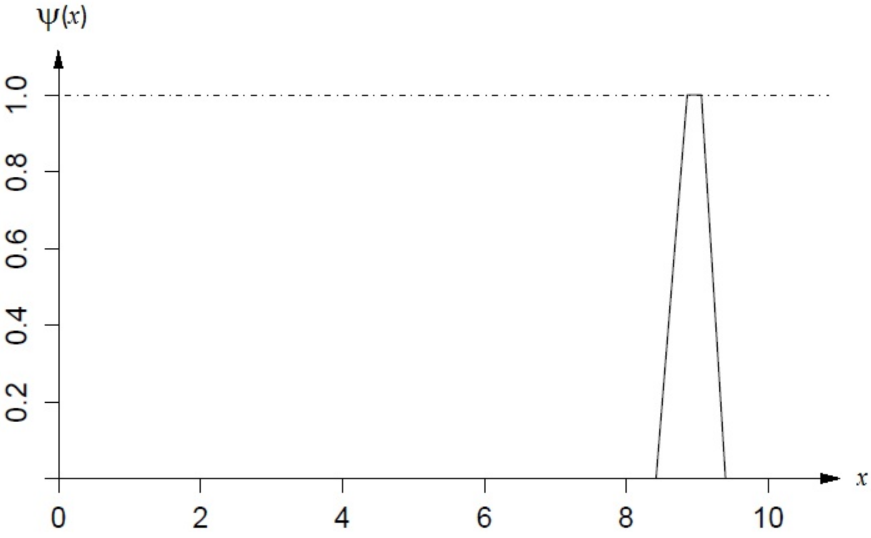

Figure 3: Fuzzy arithmetic mean

The δ-cuts $C_\delta\left[s_n^\star\right]$ of the generalized (fuzzy) estimate $s_n^\star$ of the generalized standard deviation can be obtained by the following equation:

$$C_\delta\left[s_n^\star\right] = \left[\min_{(x_1,\cdots,x_n)\in C_\delta[x_1^\star]\times\cdots\times C_\delta[x_n^\star]} g(x_1,\cdots,x_n);\ \max_{(x_1,\cdots,x_n)\in C_\delta[x_1^\star]\times\cdots\times C_\delta[x_n^\star]} g(x_1,\cdots,x_n)\right]$$

The characterizing function $\varphi(\cdot)$ of $s_n^\star$ can be obtained by the so-called *characterization lemma for characterizing functions* (compare Viertl (2011)):

$$\varphi(x) = \max\left\{\delta\cdot \mathbb{1}_{C_\delta[s_n^\star]}(x):\ \delta\in[0;1]\right\}\quad \forall\, x\in I\!R.$$

For the fuzzy sample $x_1^\star,\cdots,x_5^\star$ from figure 2 the characterizing function of the fuzzy estimate $s_n^\star$ of the generalized standard deviation is depicted in figure 4.

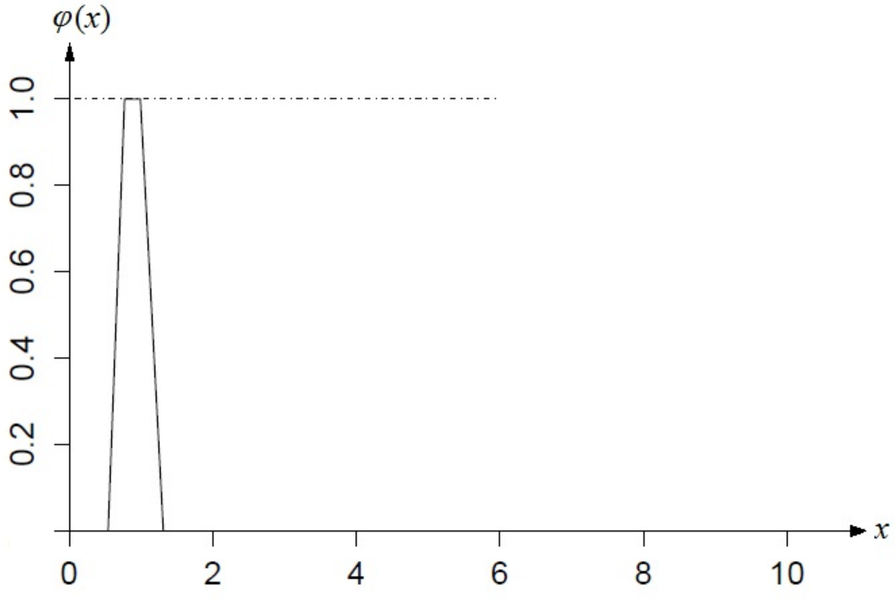

Figure 4: Fuzzy sample standard deviation

Details on the algorithmic realisation of the calculation of $s_n^\star$ are given in Viertl and Hareter (2006).

6. Summary and Outlook

In the paper the mathematical description of measurement results of one-dimensional continuous quantities is explained. This is possible by so-called fuzzy numbers. Repeating the measruement yields a finite sequence of observations in form of fuzzy numbers $x_1^\star,\cdots,x_n^\star$ with corresponding characterizing functions $\xi_1(\cdot),\cdots,\xi_n(\cdot)$. The generalization of averaging the sample is explained, and a generalized fuzzy estimator for the standard deviation of the quantity is given. For multivariate continuous quantities the measurement results are also more or less imprecise. The results could be modelled by so-called k-dimensional fuzzy vectors which are special fuzzy subsets of the k-dimensional Euclidean space $I\!R^k$. The statistical analysis of multivariate fuzzy data is an interesting topic for future research.

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Austrian Journal of Statistics http://www.ajs.or.at/
published by the Austrian Society of Statistics http://www.osg.or.at/
Volume 44 *Submitted:* 2013-02-18
January 2015 *Accepted:* 2014-05-05

Austrian Journal of Statistics
January 2015, Volume 44, 33–46.
http://www.ajs.or.at/

Estimation of Finite Population Ratio When Other Auxiliary Variables are Available in the Study

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Abstract

The estimation of the population total t_y, by using one or more auxiliary variables, and the population ratio $\theta_{xy} = t_y/t_x$, t_x is the population total for the variable X, for a finite population are heavily discussed in the literature. In this paper, the idea of estimating the finite population ratio θ_{xy} is extended to use the availability of auxiliary variable Z in the study. The availability of such variable can be used to increase the precision of estimating the population ratio θ_{xy}. Our idea is supported by the fact that the variable Z may be more correlated with the variable Y than the correlation between the variables X and Y. To our knowledge, this idea is not discussed in the literature, and may be extended to use the availability of p auxiliary variables.

The bias, variance and the mean squares error are given for our approach. Simulation from real data set, the empirical relative bias and the empirical relative mean squares error are computed for our approach and for different estimators proposed in the literature for estimating the population ratio θ_{xy}. Analytically and the simulation results show that, by suitable choices, our approach gives negligible bias and has less mean squares error.

Further, under simple random sampling without replacement, the population variances of the estimators that are used in this paper are computed. Based on the random samples, that are used for estimating the population ratio θ_{xy}, the sample variances for the different estimators that are used in our approach are compared with the population variances for each estimators i.e. the empirical mean, the empirical relative bias, and the empirical relative mean squares error for the sample variances are reported. As a result of this simulation study, our approach is more efficient than other estimators proposed in the literature.

Keywords: population ratio, auxiliary variables, bias, mean squared error, general sampling design, mean, variance.

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1. Introduction

Consider a finite population U of N units indexed by the set $\{1, 2, \cdots, N\}$. For the ith unit, let y_i, and x_i be the values of the variables Y and X, respectively. One of the main interest in survey sampling is to estimate the population ratio $\theta_{yx} = t_y/t_x$, where $t_y = \sum_{i \in U} y_i$ be the population total for the variable Y, and $t_x = \sum_{i \in U} x_i$ be the population total for the variable X. In the literature, there are different ideas for estimating the population ratio θ_{yx}. To our knowledge, none of them used the availability of another auxiliary variable Z in the study.

The availability of such auxiliary variable can be used to improve the precision of estimating θ_{yx}. Our idea is to use the auxiliary variable Z to improve the precision of the estimator of θ_{yx}.

Under simple random sampling without replacement (srs) design, Hartley and Ross (1954) proposed an exactly unbiased estimator for θ_{yx}. The proposed estimator is given by

$$\hat{\theta}_{HR} = \bar{r}_s + \frac{n(N-1)}{N(n-1)\bar{x}_u}(\bar{y}_s - \bar{r}_s\bar{x}_s), \tag{1}$$

where, $\bar{y}_s = \sum_{i\in s} y_i/n$, $\bar{r}_s = \sum_{i\in s} r_i/n$, $r_i = y_i/x_i$, $\bar{x}_s = \sum_{i\in s} x_i/n$, and $\bar{x}_u = t_x/N$. This estimator can be rewritten under general sampling design $p(\cdot)$. In this case, this estimator is no longer unbiased but still with negligible bias (Al-Jararha 2012).

Under general sampling design, Al-Jararha and Al-Haj Ebrahem (2012) proposed an estimator for estimating the population ratio θ_{yx}. This estimator, has negligible relative bias especially for small sample sizes n and approaches zero with increasing n. Under srs, and based on simulation results, the performance of this estimator is better than Hartley and Ross (1954) estimator. Their estimator is defined by

$$\hat{\theta}_{JM} = \bar{r}_s + \frac{1}{\bar{x}_u}(\bar{y}_s - \bar{r}_s\bar{x}_s). \tag{2}$$

Under General sampling design, Al-Jararha (2012) obtained an exactly unbiased estimator for the population ratio θ. This estimator, under srs design, gives the Hartley and Ross (1954) estimator. Further, the variance and an unbiased estimator of the variance of such estimator were obtained. This estimator, works well in stratified sampling designs.

Define π_i, the first order inclusion probability, by

$$\pi_i = Pr\left(i^{th} \text{ element} \in s\right) = \sum_{s\ni i} p(s).$$

For $i \neq j$, the second order inclusion probability is defined by

$$\pi_{ij} = Pr\left(i^{th} \text{ and } j^{th} \text{ elements} \in s\right) = \sum_{s\ni i,\, j} p(s).$$

The Horvitz and Thompson (1952) estimator of the population total $t_y = \sum_{i\in U} y_i$ is defined by

$$\hat{t}_{y\pi} = \sum_{i\in U} y_i \frac{I_{\{i\in s\}}}{\pi_i},$$

where $I_{\{i\in s\}}$ is one if $i \in s$ and zero otherwise. Further,

$$\bar{y}_s = \frac{1}{N}\hat{t}_{y\pi},$$

can be used to estimate the population mean $\bar{y}_u = \frac{1}{N}t_y$. It can be noted that $\hat{t}_{y\pi}$ and $\bar{y}_s$ are unbiased estimators for t_y, and $\bar{y}_u$ respectively. However, $\hat{t}_{y\pi}$ and $\bar{y}_s$ do not use the availability of auxiliary variables in the study. In similar way,

$$\bar{x}_s = \frac{1}{N}\hat{t}_{x\pi}, \quad \text{and} \quad \bar{r}_s = \frac{1}{N}\hat{t}_{r\pi}$$

are unbiased estimators for $\bar{x}_u$ and $\bar{r}_U$ respectively.

The availability of more than one auxiliary variable is used in literature for estimating the finite population total t_y, or finite population mean $\bar{y}_u$.

Under srs, Olkin (1958) was the first one who deals with the problem of estimating the population mean using more than one auxiliary variables. His estimator is given by

$$\hat{\bar{y}}_u = \sum_{i=1}^{p} w_i \bar{x}_{iu} \hat{\theta}_{yx_i},$$

where p is the number of the auxiliary variables, $\hat{\theta}_{yx_i} = \bar{y}_s/\bar{x}_{is}$, w_i is the weight of the ith auxiliary variable such that $\sum_{i=1}^{p} w_i = 1$, $\bar{y}_s$ is the sample mean of Y and $\bar{x}_{iu}$, $\bar{x}_{is}$ are the population mean and the sample mean of X_i, respectively, for $i = 1, \ldots, p$.

Singh and Chaudhary (1986) proposed the following estimator

$$\hat{\bar{y}}_u = \bar{y}_s \left(w_1 \frac{\bar{x}_{1u}}{\bar{x}_{1s}} + w_2 \frac{\bar{x}_{2u}}{\bar{x}_{2s}} \right)$$

for estimating the population mean $\overline{y}_u$, where $w_1 + w_2 = 1$.

Abu-Dayyeh, Ahmad, Ahmad, and Hassen (2003) studied the general form of Singh and Chaudhary (1986) estimator. They proposed two classes of estimators using two auxiliary variables to estimate the population mean for the variable of interest Y.

Kadilar and Cingi (2004) suggested a new multivariate ratio estimator using the regression estimator instead of $\bar{y}_s$ which used in Singh and Chaudhary (1986) estimator. Their estimator is given by

$$\bar{y}_{pr} = \sum_{i=1}^{2} w_i \frac{\bar{y}_s + b_i \left(\bar{x}_{iu} - \bar{x}_{is} \right)}{\bar{x}_{is}} \bar{x}_{iu},$$

where b_i, $i = 1, 2$ are the regression coefficients. Based on the mean squares error (MSE), they found that their estimator is more efficient than Singh and Chaudhary (1986) estimator when

$$MSE\left(\bar{y}_{pr}\right) < MSE\left(\bar{y}_u\right),$$

where $MSE\left(\bar{y}_{pr}\right)$, and $MSE\left(\bar{y}_u\right)$ are defined by Equations (2.4), and (1.2) of Kadilar and Cingi (2004), respectively.

Other authors are using different ideas for estimating the population mean $\bar{y}_u$. On the other side, there are different ideas for estimating θ_{yx}, to our knowledge, none of them discussed the idea of using the availability of other auxiliary variable Z for estimating the population ratio θ_{yx}. In this article, under general sampling design, a family of estimators is adopted for estimating the population ratio θ_{yx}. For such family, the bias, variance, MSE are given. Based on simulation from real data set, we will compare between given estimators for θ_{yx}, proposed in the literature and our approach.

2. Proposed Family

The existence of one or more auxiliary variables can be used to improve the estimate of θ_{yx}. In our approach, for the ith unit, let y_i, x_i and z_i be the values of the variable of interest Y, and the auxiliary variables X, and Z respectively. Our goal is to estimate the population ratio $\theta_{yx} = t_y/t_x$ when the auxiliary variable Z is available in the study.

Our approach is summarized by rewriting the definition of θ_{yx} as

$$\theta_{yx} = \lambda \theta_{yx} + (1 - \lambda) \theta_{zx} \theta_{yz}, \tag{3}$$

for given λ and $\theta_{zx} = t_z/t_x$. Usually, t_x and t_z are assumed to be known; therefore, we assume θ_{zx} to be known. Based on this, estimate θ_{yx} by

$$\tilde{\theta}_{yx} = \lambda \hat{\theta}_{yx} + (1 - \lambda) \theta_{zx} \breve{\theta}_{yz}. \tag{4}$$

Remark 2.1. *The estimators $\hat{\theta}_{yx}$, and $\breve{\theta}_{yz}$ can be computed from proposed estimators for the population ratio in the literature. Both, $\hat{\theta}_{yx}$, $\breve{\theta}_{yz}$ can be computed from the same estimator of the population ratio, or from different estimators.*

From Equation(4), take the expectation of $\tilde{\theta}_{yx}$, we have

$$E\left(\tilde{\theta}_{yx}\right) = \lambda E\left(\hat{\theta}_{yx}\right) + (1-\lambda)\,\theta_{zx} E\left(\breve{\theta}_{yz}\right). \tag{5}$$

Therefore,

$$bias\left(\tilde{\theta}_{yx}\right) = \lambda bias\left(\hat{\theta}_{yx}\right) + (1-\lambda)\,\theta_{zx} bias\left(\breve{\theta}_{yz}\right). \tag{6}$$

Remark 2.2. *From Equation(6), $\tilde{\theta}_{yx}$ is unbiased or asymptotically unbiased is achieved by choosing $\hat{\theta}_{yx}$, and $\breve{\theta}_{yz}$ to be unbiased or asymptotically unbiased.*

From Equation(4), the variance of $\tilde{\theta}_{yx}$ is

$$var\left(\tilde{\theta}_{yx}\right) = \lambda^2 var\left(\hat{\theta}_{yx}\right) + (1-\lambda)^2\,\theta_{zx}^2 var\left(\breve{\theta}_{yz}\right) + 2\lambda\,(1-\lambda)\,\theta_{zx} cov\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right). \tag{7}$$

From Equations (6), and (7), the MSE of $\tilde{\theta}_{yx}$ is

$$MSE\left(\tilde{\theta}_{yx}\right) = var\left(\tilde{\theta}_{yx}\right) + bias^2\left(\tilde{\theta}_{yx}\right). \tag{8}$$

Assume that $\tilde{\theta}_{yx}$ to be unbiased or asymptotically unbiased, by choosing $\hat{\theta}_{yx}$, and $\breve{\theta}_{yz}$ to be unbiased or asymptotically unbiased. In this case, $MSE\left(\tilde{\theta}_{yx}\right) = var\left(\tilde{\theta}_{yx}\right)$. The optimal value of λ, can be obtained by differentiating the right hand side of Equation(8) with respect to λ, equate to zero, and solve for λ we have

$$\lambda_{opt} = \frac{1}{1+\lambda^*}, \tag{9}$$

where

$$\lambda^* = \frac{var\left(\hat{\theta}_{yx}\right) - \theta_{zx} cov\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)}{\theta_{zx}^2 var\left(\breve{\theta}_{yz}\right) - \theta_{zx} cov\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)}. \tag{10}$$

From Equations (4) and (9) the optimal estimator for θ_{yx} is

$$\tilde{\theta}_{yx} = \lambda_{opt}\hat{\theta}_{yx} + (1-\lambda_{opt})\,\theta_{zx}\breve{\theta}_{yz}. \tag{11}$$

Remark 2.3. *In general, the transformation given by Equation (11) is not a convex transformation. However, the transformation is a convex transformation when $0 \leq \lambda_{opt} \leq 1$, this condition holds if $\lambda^* \geq 0$. In this case, the numerator and the denominator of λ^* should be positive; equivalently, from Equation (10), if*

$$\rho\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right) \leq min\left\{\frac{1}{\theta_{zx}} \cdot \sqrt{\frac{var\left(\hat{\theta}_{yx}\right)}{var\left(\breve{\theta}_{yz}\right)}}, \theta_{zx} \cdot \sqrt{\frac{var\left(\breve{\theta}_{yz}\right)}{var\left(\hat{\theta}_{yx}\right)}}\right\} \quad for \quad \theta_{zx} > 0,$$

where $\rho\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)$ is the correlation between $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$.

In real applications, λ_{opt} is unknown; however, λ_{opt} can be estimated from random sample. Under general sampling design $p(\cdot)$, draw the random sample S, estimate λ_{opt} by

$$\hat{\lambda}_{opt} = \frac{1}{1+\hat{\lambda}^*}, \tag{12}$$

where

$$\hat{\lambda}^* = \frac{\widehat{var}\left(\hat{\theta}_{yx}\right) - \theta_{zx}\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)}{\theta_{zx}^2\widehat{var}\left(\breve{\theta}_{yz}\right) - \theta_{zx}\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)}. \tag{13}$$

From Equation (11), $\tilde{\theta}_{yx}$ is computed from

$$\tilde{\theta}_{yx} = \hat{\lambda}_{opt}\hat{\theta}_{yx} + \left(1 - \hat{\lambda}_{opt}\right)\theta_{zx}\breve{\theta}_{yz}. \tag{14}$$

In the next section, we describe how we can apply our approach. In most applicable cases, t_x and t_z are known from previous studies or from a pilot study. However, the worst scenario happens when $\theta_{zx} = t_z/t_x$ is unknown. In this case, estimate θ_{yx} by

$$\tilde{\theta}_{yx} = \lambda\hat{\theta}_{yx} + (1-\lambda)\acute{\theta}_{zx}\breve{\theta}_{yz}, \tag{15}$$

where $\acute{\theta}_{zx}$ is an estimate for θ_{zx}. Our goal is to find the bias, variance, and the MSE of $\tilde{\theta}_{yx}$. As it is clear from Equation (15), $\tilde{\theta}_{yx}$ is not a linear function in $\acute{\theta}_{zx}$, and $\breve{\theta}_{yz}$, and to avoid the 3rd and 4th order inclusion probabilities, to first order and by using Taylor expansion, expand the right hand side of Equation(15), we have

$$\tilde{\theta}_{yx} \cong \lambda\hat{\theta}_{yx} + (1-\lambda)\left\{\theta_{yx} + \theta_{zx}\left(\breve{\theta}_{yz} - \theta_{yz}\right) + \theta_{yz}\left(\acute{\theta}_{zx} - \theta_{zx}\right)\right\}. \tag{16}$$

Remark 2.4. *The first order linearization is widely used in survey practice, but that in general it is very difficult to evaluate the quality of approximation analytically. Therefore, simulations are presented that show reasonable results at least in the particular case described.*

From Equation(16), the bias of $\tilde{\theta}_{yx}$ is

$$\text{bias}\left(\tilde{\theta}_{yx}\right) \cong \lambda\text{bias}\left(\hat{\theta}_{yx}\right) + (1-\lambda)\left\{\theta_{zx}\text{bias}\left(\breve{\theta}_{yz}\right) + \theta_{yz}\text{bias}\left(\acute{\theta}_{zx}\right)\right\}. \tag{17}$$

The variance of $\tilde{\theta}_{yx}$ is

$$\begin{aligned}\text{var}\left(\tilde{\theta}_{yx}\right) &\cong \lambda^2\text{var}\left(\hat{\theta}_{yx}\right) + (1-\lambda)^2\left\{\theta_{zx}^2\text{var}\left(\breve{\theta}_{yz}\right) + \theta_{yz}^2\text{var}\left(\acute{\theta}_{zx}\right)\right.\\ &+ \left. 2\theta_{yx}\text{cov}\left(\breve{\theta}_{yz}, \acute{\theta}_{zx}\right)\right\} + 2\lambda(1-\lambda)\left\{\theta_{zx}\text{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)\right.\\ &+ \left.\theta_{yz}\text{cov}\left(\hat{\theta}_{yx}, \acute{\theta}_{zx}\right)\right\},\end{aligned} \tag{18}$$

From Equations (17) and (18), the MSE of $\tilde{\theta}_{yx}$ is

$$\text{MSE}\left(\tilde{\theta}_{yx}\right) = \text{var}\left(\tilde{\theta}_{yx}\right) + \text{bias}\left(\tilde{\theta}_{yx}\right)^2. \tag{19}$$

Remark 2.5. *From the right hand side of Equation(17), it is clear that the need of using unbiased or asymptotically unbiased estimators for estimating θ_{yx}, θ_{zx}, and θ_{yz}. In this case, bias $\left(\tilde{\theta}_{yx}\right)$ is zero or asymptotically zero i.e. $\tilde{\theta}_{yx}$ is unbiased or asymptotically unbiased estimator for θ_{yx}. As a result of this,*

$$MSE\left(\tilde{\theta}_{yx}\right) \cong var\left(\tilde{\theta}_{yx}\right). \tag{20}$$

Under the assumption $\hat{\theta}_{yx}$, $\acute{\theta}_{zx}$, and $\breve{\theta}_{yz}$ are unbiased (or asymptotically unbiased) estimator for θ_{yx}, θ_{zx}, and θ_{yz}, respectively. The optimum value of λ which is minimizing the right hand side of Equation(19) is

$$\lambda_{opt} = \frac{\mathrm{var}\left(\theta_{zx}\breve{\theta}_{yz} + \theta_{yz}\acute{\theta}_{zx}\right) - \theta_{zx}\mathrm{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right) - \theta_{yz}\mathrm{cov}\left(\hat{\theta}_{yx}, \acute{\theta}_{zx}\right)}{\mathrm{var}\left(\theta_{zx}\breve{\theta}_{yz} + \theta_{yz}\acute{\theta}_{zx} - \hat{\theta}_{yx}\right)} \tag{21}$$

In real applications, λ_{opt} needs to be estimated from random sample. In this case, the estimate value of λ_{opt} is

$$\hat{\lambda}_{opt} = \frac{4\widehat{\mathrm{var}}\left(\breve{\theta}_{yz}\acute{\theta}_{zx}\right) - \acute{\theta}_{zx}\widehat{\mathrm{cov}}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right) - \breve{\theta}_{yz}\widehat{\mathrm{cov}}\left(\hat{\theta}_{yx}, \acute{\theta}_{zx}\right)}{\widehat{\mathrm{var}}\left(2\acute{\theta}_{zx}\breve{\theta}_{yz} - \hat{\theta}_{yx}\right)} \tag{22}$$

Remark 2.6. *Insert $\hat{\lambda}_{opt}$ into Equation(15), we have the optimal choice of estimating θ_{yx} i.e. estimate θ_{yx} by*

$$\tilde{\theta}_{yx} = \hat{\lambda}_{opt}\hat{\theta}_{yx} + \left(1 - \hat{\lambda}_{opt}\right)\acute{\theta}_{zx}\breve{\theta}_{yz}. \tag{23}$$

In real application, the first case, $\theta_{zx} = t_z/t_x$ is known, is more applicable than the second case, $\theta_{zx} = t_z/t_x$ is unknown. Therefore, in the next section, we will describe how we can apply the first approach. However, the second approach can be used in similar way as the first one.

3. Applying Our Approach

In this section, we will apply the first case, $\theta_{zx} = t_z/t_x$ is known. However, the second approach, $\theta_{zx} = t_z/t_x$ is unknown, can be used in similar way as the first one. Based on Remark(2.2), we restrict ourselves to the estimation of θ_{yx}, and θ_{yz}, by unbiased or asymptotically unbiased estimators from the literature. In this paper, we will use the classical ratio estimator, and the estimators given by Equations (1) and (2).

3.1. Classical Ratio Estimator

In this subsection, we will compute $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$ from the usual classical ratio estimator, i.e. $\hat{\theta}_{yx}$, and $\breve{\theta}_{yz}$ are computed from

$$\hat{\theta}_{yx} = \frac{\hat{t}_{y\pi}}{\hat{t}_{x\pi}} \tag{24}$$

and

$$\breve{\theta}_{yz} = \frac{\hat{t}_{y\pi}}{\hat{t}_{z\pi}}, \tag{25}$$

respectively. In this case,

$$\widehat{var}\left(\hat{\theta}_{yx}\right) = \sum_{ij\in S}\frac{\hat{w}_i}{\pi_i}\frac{\hat{w}_j}{\pi_j}\frac{\Delta_{ij}}{\pi_{ij}}, \tag{26}$$

$$\widehat{var}\left(\breve{\theta}_{yz}\right) = \sum_{ij\in S}\frac{\breve{w}_i}{\pi_i}\frac{\breve{w}_j}{\pi_j}\frac{\Delta_{ij}}{\pi_{ij}}, \tag{27}$$

$$\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right) = \sum_{ij\in S}\frac{\hat{w}_i}{\pi_i}\frac{\breve{w}_j}{\pi_j}\frac{\Delta_{ij}}{\pi_{ij}}, \tag{28}$$

respectively. Where

$$\hat{w}_i = \left(y_i - \hat{\theta}_{yx} x_i \right) / N \bar{x}_u, \tag{29}$$

and

$$\breve{w}_i = \left(y_i - \breve{\theta}_{yz} z_i \right) / N \bar{z}_u. \tag{30}$$

For more details, see Al-Jararha and Al-Haj Ebrahem (2012).

In order to use Equation (14), insert the estimators in Equations (26), (27), and (28) into Equation (13) to compute $\hat{\lambda}^*$, use the result in Equation (12). Now Equation (14) is ready to be used.

3.2. Hartley and Ross Estimator

Under srs sampling design, Hartley and Ross (1954) proposed an exactly an unbiased estimator for estimating the population ratio. This estimator can be rewritten under general sampling design (Al-Jararha 2012). In this case, $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$ are computed from

$$\hat{\theta}_{yx} = \bar{r}_{yxs} + \frac{n\,(N-1)}{N\,(n-1)\,\bar{x}_u} \left(\bar{y}_s - \bar{r}_{yxs} \bar{x}_s \right) \tag{31}$$

and

$$\breve{\theta}_{yz} = \bar{r}_{yzs} + \frac{n\,(N-1)}{N\,(n-1)\,\bar{z}_u} \left(\bar{y}_s - \bar{r}_{yzs} \bar{z}_s \right), \tag{32}$$

respectively. To compute $\widehat{var}\left(\hat{\theta}_{yx}\right)$, $\widehat{var}\left(\breve{\theta}_{yz}\right)$, and $\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)$ reuse Equations (26), (27), and (28) but with the following definitions

$$\hat{w}_i = \frac{n\,(N-1)}{N^2\,(n-1)\,\bar{x}_u} \left(y_i - \bar{r}_{yxs} x_i \right) - \frac{N-n}{N^2\,(n-1)} r_{i_{yx}}, \tag{33}$$

and

$$\breve{w}_i = \frac{n\,(N-1)}{N^2\,(n-1)\,\bar{z}_u} \left(y_i - \bar{r}_{yzs} z_i \right) - \frac{N-n}{N^2\,(n-1)} r_{i_{yz}}. \tag{34}$$

For more details, see Al-Jararha and Al-Haj Ebrahem (2012).

3.3. Al-Jararha and Al-Haj Ebrahem Estimator

Under general sampling design, Al-Jararha and Al-Haj Ebrahem (2012) proposed an asymptotic unbiased estimator for estimating the population ratio. This estimator is working better than Hartley and Ross (1954). In this case, $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$ are computed from

$$\hat{\theta}_{yx} = \bar{r}_{yxs} + \frac{1}{\bar{x}_u} \left(\bar{y}_s - \bar{r}_{yxs} \bar{x}_s \right) \tag{35}$$

and

$$\breve{\theta}_{yz} = \bar{r}_{yzs} + \frac{1}{\bar{z}_u} \left(\bar{y}_s - \bar{r}_{yzs} \bar{z}_s \right), \tag{36}$$

respectively. To compute $\widehat{var}\left(\hat{\theta}_{yx}\right)$, $\widehat{var}\left(\breve{\theta}_{yz}\right)$, and $\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)$ reuse Equations (26), (27), and (28) but with the following definitions

$$\hat{w}_i = \left(y_i - \bar{r}_{yxs} x_i\right) / N\bar{x}_u, \tag{37}$$

and

$$\breve{w}_i = \left(y_i - \bar{r}_{yzs} z_i\right) / N\bar{z}_u. \tag{38}$$

For more details, see Al-Jararha and Al-Haj Ebrahem (2012).

Remark 3.1. *In order to compute the* $\widehat{cov}\left(\hat{\theta}_{yx}, \breve{\theta}_{yz}\right)$ *when* $\hat{\theta}_{yx}$ *and* $\breve{\theta}_{yz}$ *are to be computed from different estimators, for example,* $\hat{\theta}_{yx}$ *is computed from Equation (24), and* $\breve{\theta}_{yz}$ *is computed from Equation (32); in this case, use Equation (28) with the definition of* $\hat{w}_i$ *as given in Equation (29), and* $\breve{w}_i$ *as given in Equation (34).*

4. Simulation Studies and Conclusions

4.1. Estimation the Population Ratio θ_{yx}

Consider the real data set FEV: Forced Expiratory Volume. FEV is an index of pulmonary function that measures the volume of air expelled after one second of constant effort. This data is downloaded from http://www.amstat.org/publications/jse/datasets/fev.dat.txt. The FEV data set was taken from a study conducted in East Boston, Massachusetts, 1980, on 654 children aged from 3 to 19 years who were seen in the childhood respiratory disease (CRD). The variable of interest is Y: Forced expiratory volume, and the auxiliary variables are X: Children age, from 3-19 years age, and Z: Children height in inches. For this data set, $t_y = 1724$, $t_x = 6495$, and $t_z = 39988$. In this section, we will assume that $t_x = 6495$, and $t_z = 39988$ are known.

In this section, our main goal is to estimate the population ratio $\theta = t_y/t_x = 0.2655$ by using our approach i.e. by using Equation (14) and the three estimators given by Equations (24), (31), and (35) under different sampling designs i.e. under srs, probability proportional to size and without replacement πps; in this case, the size variable will be the age, and stratified sampling design; in this case, the FEV data set will be divided into $H = 2$ non-overlapping strata according to the variable sex.

The empirical mean (EM) of the estimator $\tilde{\theta}$ of θ is defined by

$$\text{EM}\left(\tilde{\theta}\right) = \frac{1}{m}\sum_{i=1}^{m}\tilde{\theta}_i, \tag{39}$$

where $\tilde{\theta}_i$ is the estimate of θ based on the i^{th} simulated random sample, and m is the number of simulated random samples under different random sampling designs. The empirical relative bias (ERB) of $\tilde{\theta}$ is defined by

$$\text{ERB}\left(\tilde{\theta}\right) = \frac{\frac{1}{m}\sum_{i=1}^{m}\tilde{\theta}_i - \theta}{\theta} \times 100\%. \tag{40}$$

The empirical mean squares error (EMSE) of $\tilde{\theta}$ is defined by

$$\text{EMSE}\left(\tilde{\theta}\right) = \frac{1}{m}\sum_{i=1}^{m}\left(\tilde{\theta}_i - \theta\right)^2, \tag{41}$$

and the empirical relative mean squares error (RE) of the estimator $\tilde{\theta}$ is defined by

$$\mathrm{RE}\left(\tilde{\theta}\right) = \frac{\frac{1}{m}\sum_{i=1}^{m}\left(\tilde{\theta}_i - \theta\right)^2}{\frac{1}{m}\sum_{i=1}^{m}\left(\hat{\theta}_i - \theta\right)^2} = \frac{\mathrm{EMSE}\left(\tilde{\theta}\right)}{\mathrm{EMSE}\left(\hat{\theta}\right)}, \tag{42}$$

where $\hat{\theta}$ is another estimator for θ.

From Equation (14), recall our approach,

$$\tilde{\theta}_{yx} = \hat{\lambda}_{opt}\hat{\theta}_{yx} + \left(1 - \hat{\lambda}_{opt}\right)\theta_{zx}\breve{\theta}_{yz}, \tag{43}$$

to make the notations clear, consider the following

	$\hat{\theta}_{yx}$ is computed from	$\breve{\theta}_{yz}$ is computed from Eq(25)	$\breve{\theta}_{yz}$ is computed from Eq(32)	$\breve{\theta}_{yz}$ is computed from Eq(36)
group I	Eq(24)	$\tilde{\theta}_{yx.RR}$	$\tilde{\theta}_{yx.RH}$	$\tilde{\theta}_{yx.RJ}$
group II	Eq(31)	$\tilde{\theta}_{yx.HR}$	$\tilde{\theta}_{yx.HH}$	$\tilde{\theta}_{yx.HJ}$
group III	Eq(35)	$\tilde{\theta}_{yx.JR}$	$\tilde{\theta}_{yx.JH}$	$\tilde{\theta}_{yx.JJ}$

Further, for group I, compute $\hat{\theta}_{RR}$ from Equation(24), for group II, compute $\hat{\theta}_{HH}$ from Equation(31), and for group III, compute $\hat{\theta}_{JJ}$ from Equation(35). We can see that the computation of $\hat{\theta}_{RR}$, $\hat{\theta}_{HH}$, and $\hat{\theta}_{JJ}$ depend on the variable of interest Y and the auxiliary variable X only. In order to use Equation (42), and for the ith group, compute EMSE $\left(\tilde{\theta}\right)$ for the estimators in this group and the EMSE $\left(\hat{\theta}\right)$ for its corresponding group.

From the described population, simulate $m = 3,000$ samples under different sampling designs i.e. srs, πps, and stratified sampling design, when the sample size $n = 20$, 30, 40, 50 and 60. Sampling from the population will be achieved by using procedure `surveyselect` of SAS Institute, and the computations are computed by using a macro written in SAS. For a given sample of size n, and based on each sample, compute the estimators $\tilde{\theta}_{yx}$, and $\hat{\theta}_{ww}$, $w = R, H, J$, as they described above.

4.2. Variance Estimation of the $\tilde{\theta}_{yx}$

In this section, under srs, our main goal is to compute the population variances for the 12 estimators described in the Subsection (4.1). Further, we will compute the empirical sample mean, relative bias, and the MSE for the sample variances computed from the random samples simulated in the Subsection (4.1).

Recall that $\hat{t}_{y\pi} = \sum_{i\in U} y_i \frac{I_{\{i\in s\}}}{\pi_i}$, the Horvitz and Thompson (1952) estimator of the population total $t_y = \sum_{i\in U} y_i$. Under srs (Särndal, Swensson, and Wretman 1992),

$$var_{srs}\left(\hat{t}_{y\pi}\right) = N^2\frac{1-f}{n}S_{yu}^2, \tag{44}$$

and

$$\widehat{var}_{srs}\left(\hat{t}_{y\pi}\right) = N^2\frac{1-f}{n}s_{ys}^2, \tag{45}$$

where

$$S_{yu}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{y}_u\right)^2,$$

$$s_{ys}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}_s\right)^2,$$

and $f = n/N$. Similarly, the covariance between $\hat{t}_{y\pi}$ and $\hat{t}_{z\pi}$ is computed from

$$cov_{srs}\left(\hat{t}_{y\pi}, \hat{t}_{z\pi}\right) = N^2 \frac{1-f}{n} S_{yzu}, \tag{46}$$

which is estimated by

$$\widehat{cov}_{srs}\left(\hat{t}_{y\pi}, \hat{t}_{z\pi}\right) = N^2 \frac{1-f}{n} s_{yzs}, \tag{47}$$

where

$$S_{yzu} = \frac{1}{N-1} \sum_{i=1}^{N} \left(y_i - \bar{y}_u\right)\left(z_i - \bar{z}_u\right),$$

and

$$s_{yzs} = \frac{1}{n-1} \sum_{i=1}^{n} \left(y_i - \bar{y}_s\right)\left(z_i - \bar{z}_s\right)$$

Remark 4.1. *Since the 12 estimators discussed in the Subsection (4.1) are linearized to first order Taylor expansion (Al-Jararha and Al-Haj Ebrahem 2012), Equations (44)-(47) are ready to be used for such estimators. The computations in this part are similar to the computations as in Subsection (4.1), but for variances.*

The empirical mean (MV) of the $\widehat{var}_{srs}\left(\tilde{\theta}\right)$ of $var_{srs}\left(\tilde{\theta}\right)$ is

$$\mathrm{MV}\left(\tilde{\theta}\right) = \frac{1}{m} \sum_{i=1}^{m} \widehat{var}_{srs}\left(\tilde{\theta}\right)_i, \tag{48}$$

where $\widehat{var}_{srs}\left(\tilde{\theta}\right)_i$ is computed from the ith simulated random sample. The empirical relative bias (RBV) of $\widehat{var}_{srs}\left(\tilde{\theta}\right)$ is

$$\mathrm{RBV}\left(\tilde{\theta}\right) = \frac{\frac{1}{m}\sum_{i=1}^{m} \widehat{var}_{srs}\left(\tilde{\theta}\right)_i - var_{srs}\left(\tilde{\theta}\right)}{var_{srs}\left(\tilde{\theta}\right)} \times 100\%. \tag{49}$$

The empirical mean squares error (MSEV) of $\widehat{var}_{srs}\left(\tilde{\theta}\right)$ is

$$\mathrm{MSEV}\left(\tilde{\theta}\right) = \frac{1}{m} \sum_{i=1}^{m} \left(\widehat{var}_{srs}\left(\tilde{\theta}\right)_i - var_{srs}\left(\tilde{\theta}\right)\right)^2, \tag{50}$$

and the empirical relative mean squares error (REV) of the estimator $\widehat{var}_{srs}\left(\tilde{\theta}\right)$ is

$$\mathrm{REV}\left(\tilde{\theta}\right) = \frac{\frac{1}{m}\sum_{i=1}^{m}\left(\widehat{var}_{srs}\left(\tilde{\theta}\right)_i - var_{srs}\left(\tilde{\theta}\right)\right)^2}{\frac{1}{m}\sum_{i=1}^{m}\left(\widehat{var}_{srs}\left(\hat{\theta}\right)_i - var_{srs}\left(\hat{\theta}\right)\right)^2} = \frac{\mathrm{MSEV}\left(\widehat{var}_{srs}\left(\tilde{\theta}\right)\right)}{\mathrm{MSEV}\left(\widehat{var}_{srs}\left(\hat{\theta}\right)\right)}, \tag{51}$$

where $\widehat{var}_{srs}\left(\hat{\theta}\right)$ is another estimator for $var_{srs}\left(\hat{\theta}\right)$.

Under srs, population variances are computed for every estimator mentioned in Subsection (4.1). Further, based on every simulated sample used for estimating such estimators is also used to compute the sample variances for the 12 estimators. Results are reported in Table (5).

This Subsection is restricted to srs sampling design since there are difficulties to use other sampling designs. For example, under πps, procedure `surveyselect` gives the first and second order inclusion probabilities for the sample only. Even though, the computations under srs are not an easy task!

4.3. Results and Conclusions

The nine estimators, $\underbrace{\tilde{\theta}_{yx.RR},\ \tilde{\theta}_{yx.RH},\ \tilde{\theta}_{yx.RJ}}_{\text{group I}}$, $\underbrace{\tilde{\theta}_{yx.HR},\ \tilde{\theta}_{yx.HH},\ \tilde{\theta}_{yx.HJ}}_{\text{group II}}$, and $\underbrace{\tilde{\theta}_{yx.JR},\ \tilde{\theta}_{yx.JH},\ \tilde{\theta}_{yx.JJ}}_{\text{group III}}$, are used to estimate θ_{yx} based on our approach i.e. the estimators $\tilde{\theta}_{yx.wv}$, for $w,\ v = R,\ H,\ J$, are using the availability of another auxiliary variable Z in the study . However, the three estimators, $\hat{\theta}_{ww}$, for $w = R,\ H,\ J$, are not using the availability of Z.

From Tables (1), (2), (3), and (4), we can conclude the following:

1. The nine estimators, $\tilde{\theta}_{yx.wv}$, for $w,\ v = R,\ H,\ J$, have negligible empirical relative biased regardless the sample size n, and the group. This comes from the behavior of the estimators that are used in each group described above. In general, from Equation (6), the bias of $\tilde{\theta}_{yx}$ depends on the behavior of $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$; the estimators $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$ must be unbiased or asymptotically unbiased for θ_{yx} and θ_{yz}, respectively.

2. The use of the estimators, $\tilde{\theta}_{yx.wv}$, for $w,\ v = R,\ H,\ J$, perform much better than the estimators $\hat{\theta}_{ww}$, for $w = R,\ H,\ J$, from the empirical relative mean squares error point of view. In other words, the availability of auxiliary variable can be used to improve the precision of the estimation the population ratio θ_{xy}.

Population variances , the empirical means , relative bias, and relative mean squares error of the sample variances for the estimators discussed in the Subsection (4.1) are reported in Table (5). From this Table, we can see that all the discussed estimators have negligible relative biased. Further, in the meaning of the relative efficiency , the estimators based on our approach, $\tilde{\theta}_{yx.wv}$, for $w,\ v = R,\ H,\ J$, are more efficient than the proposed estimators $\hat{\theta}_{ww}$, for $w = R,\ H,\ J$. These results are true regardless the sample size n.

The absolute differences between the EV from Table(1), and the MV from Table(5) are summarized in Table (6). From Table (6), we can see that all the absolute differences are negligible regardless the sample size.

As a final remark, our approach can be adopted if we carefully choose the estimators $\hat{\theta}_{yx}$ and $\breve{\theta}_{yz}$ to be unbiased or asymptotically unbiased for θ_{yx} and θ_{yz}, respectively. In this case, our approach can be used to improve the precision of the estimation the population ratio θ_{xy}. Further, in similar steps our ideas can be extended to use more than one auxiliary variable.

Acknowledgement
The authors are grateful to the referee for his/her time reading this work, and for valuable comments and suggestions. In the same time, we are grateful to the editor for his valuable suggestions. Remark (2.4), is written exactly as the referee suggested.

n		$\hat{\theta}_{RR}$	$\tilde{\theta}_{yx.RR}$	$\tilde{\theta}_{yx.RH}$	$\tilde{\theta}_{yx.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{yx.HR}$	$\tilde{\theta}_{yx.HH}$	$\tilde{\theta}_{yx.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{yx.JR}$	$\tilde{\theta}_{yx.JH}$	$\tilde{\theta}_{yx.JJ}$
20	EM	0.266	0.266	0.266	0.266	0.266	0.265	0.266	0.265	0.266	0.266	0.266	0.266
	ERB	0.076	0.043	0.073	0.042	0.006	-0.003	0.027	-0.004	0.073	0.041	0.071	0.040
	EV	1.65E-4	1.42E-4	1.43E-4	1.43E-4	1.68E-4	1.43E-4	1.43E-4	1.43E-4	1.66E-4	1.42E-4	1.42E-4	1.42E-4
	RE	1.000	0.863	0.867	0.866	1.000	0.850	0.854	0.852	1.000	0.853	0.857	0.855
30	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.064	0.052	0.072	0.052	0.018	0.021	0.041	0.021	0.061	0.050	0.070	0.050
	EV	1.13E-4	9.61E-5	9.65E-5	9.64E-5	1.14E-4	9.64E-5	9.67E-5	9.66E-5	1.14E-4	9.59E-5	9.63E-5	9.62E-5
	RE	1.000	0.853	0.857	0.856	1.000	0.842	0.845	0.844	1.000	0.844	0.847	0.846
40	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.107	0.071	0.086	0.071	0.071	0.048	0.063	0.048	0.104	0.070	0.084	0.069
	EV	7.97E-5	6.86E-5	6.88E-5	6.88E-5	8.08E-5	6.86E-5	6.88E-5	6.87E-5	8.04E-5	6.83E-5	6.86E-5	6.85E-5
	RE	1.000	0.861	0.864	0.863	1.000	0.848	0.851	0.851	1.000	0.850	0.853	0.852
50	EM	0.266	0.266	0.266	0.266	0.265	0.265	0.265	0.265	0.266	0.266	0.266	0.266
	ERB	0.024	0.000	0.011	0.000	-0.002	-0.017	-0.006	-0.017	0.023	0.000	0.011	0.000
	EV	6.18E-5	5.40E-5	5.42E-5	5.42E-5	6.26E-5	5.39E-5	5.41E-5	5.41E-5	6.23E-5	5.38E-5	5.39E-5	5.39E-5
	RE	1.000	0.873	0.877	0.876	1.000	0.861	0.864	0.864	1.000	0.863	0.866	0.865
60	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.039	0.033	0.042	0.033	0.017	0.018	0.028	0.018	0.038	0.032	0.042	0.032
	EV	4.98E-5	4.41E-5	4.42E-5	4.42E-5	5.04E-5	4.40E-5	4.41E-5	4.41E-5	5.02E-5	4.39E-5	4.40E-5	4.40E-5
	RE	1.000	0.885	0.888	0.888	1.000	0.873	0.876	0.875	1.000	0.874	0.877	0.876

Table 1: Under srs: Comparisons between different estimators. $\mathrm{EV}\left(\tilde{\theta}\right) = \sum_{i=1}^{m}\left(\tilde{\theta}_i - \bar{\tilde{\theta}}\right)^2 / (m-1)$.

n		$\hat{\theta}_{RR}$	$\tilde{\theta}_{yx.RR}$	$\tilde{\theta}_{yx.RH}$	$\tilde{\theta}_{yx.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{yx.HR}$	$\tilde{\theta}_{yx.HH}$	$\tilde{\theta}_{yx.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{yx.JR}$	$\tilde{\theta}_{yx.JH}$	$\tilde{\theta}_{yx.JJ}$
20	EM	0.265	0.266	0.265	0.265	0.265	0.266	0.265	0.265	0.265	0.266	0.265	0.265
	ERB	-0.010	0.095	-0.013	-0.044	-0.075	0.052	-0.055	-0.086	-0.010	0.095	-0.013	-0.044
	RE	1.000	0.864	0.879	0.867	1.000	0.918	0.935	0.920	1.000	0.864	0.879	0.867
30	EM	0.265	0.266	0.265	0.265	0.265	0.266	0.265	0.265	0.265	0.266	0.265	0.265
	ERB	-0.027	0.045	-0.026	-0.046	-0.070	0.017	-0.054	-0.074	-0.027	0.045	-0.026	-0.046
	RE	1.000	0.826	0.835	0.830	1.000	0.861	0.870	0.864	1.000	0.826	0.835	0.830
40	EM	0.265	0.266	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.266	0.265	0.265
	ERB	-0.057	0.012	-0.062	-0.076	-0.088	-0.009	-0.082	-0.097	-0.057	0.012	-0.062	-0.076
	RE	1.000	0.791	0.802	0.799	1.000	0.820	0.830	0.826	1.000	0.791	0.802	0.799
50	EM	0.266	0.266	0.266	0.265	0.266	0.266	0.265	0.265	0.266	0.266	0.266	0.265
	ERB	0.032	0.105	0.008	-0.003	0.007	0.089	-0.008	-0.020	0.032	0.105	0.008	-0.003
	RE	1.000	0.695	0.717	0.716	1.000	0.722	0.741	0.739	1.000	0.695	0.717	0.716
60	EM	0.265	0.266	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.266	0.265	0.265
	ERB	-0.063	0.011	-0.067	-0.076	-0.083	-0.002	-0.080	-0.089	-0.063	0.011	-0.067	-0.076
	RE	1.000	0.696	0.711	0.710	1.000	0.718	0.732	0.730	1.000	0.696	0.711	0.710

Table 2: Under πps: Comparisons between different estimators.

n		$\hat{\theta}_{RR}$	$\tilde{\theta}_{yx.RR}$	$\tilde{\theta}_{yx.RH}$	$\tilde{\theta}_{yx.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{yx.HR}$	$\tilde{\theta}_{yx.HH}$	$\tilde{\theta}_{yx.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{yx.JR}$	$\tilde{\theta}_{yx.JH}$	$\tilde{\theta}_{yx.JJ}$
20	EM	0.266	0.266	0.266	0.266	0.266	0.265	0.266	0.265	0.266	0.266	0.266	0.266
	ERB	0.089	0.027	0.057	0.027	0.020	-0.018	0.012	-0.019	0.086	0.025	0.055	0.025
	RE	1.000	0.851	0.855	0.853	1.000	0.837	0.840	0.839	1.000	0.840	0.843	0.842
30	EM	0.266	0.266	0.266	0.266	0.266	0.265	0.266	0.265	0.266	0.266	0.266	0.266
	ERB	0.062	0.022	0.041	0.021	0.017	-0.008	0.012	-0.008	0.061	0.021	0.041	0.021
	RE	1.000	0.867	0.871	0.870	1.000	0.855	0.858	0.857	1.000	0.857	0.860	0.859
40	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.060	0.036	0.051	0.036	0.027	0.014	0.029	0.014	0.059	0.035	0.050	0.035
	RE	1.000	0.857	0.860	0.859	1.000	0.845	0.848	0.847	1.000	0.846	0.849	0.848
50	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.070	0.038	0.049	0.038	0.044	0.021	0.032	0.021	0.069	0.038	0.049	0.037
	RE	1.000	0.879	0.882	0.881	1.000	0.866	0.869	0.868	1.000	0.867	0.870	0.869
60	EM	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266	0.266
	ERB	0.049	0.035	0.044	0.035	0.026	0.020	0.029	0.020	0.047	0.034	0.043	0.034
	RE	1.000	0.851	0.854	0.853	1.000	0.839	0.842	0.841	1.000	0.840	0.843	0.842

Table 3: Stratified sampling design: Under srs, draw random sample of size n_h from each stratum and combined samples into one sample of size n.

n		$\hat{\theta}_{RR}$	$\tilde{\theta}_{yx.RR}$	$\tilde{\theta}_{yx.RH}$	$\tilde{\theta}_{yx.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{yx.HR}$	$\tilde{\theta}_{yx.HH}$	$\tilde{\theta}_{yx.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{yx.JR}$	$\tilde{\theta}_{yx.JH}$	$\tilde{\theta}_{yx.JJ}$
20	EM	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265
	ERB	-0.125	-0.181	-0.282	-0.298	-0.192	-0.203	-0.304	-0.320	-0.125	-0.181	-0.282	-0.298
	RE	1.000	0.954	0.980	0.961	1.000	0.997	1.028	1.005	1.000	0.954	0.980	0.961
30	EM	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265
	ERB	-0.086	-0.141	-0.213	-0.225	-0.128	-0.159	-0.231	-0.244	-0.086	-0.141	-0.213	-0.225
	RE	1.000	0.905	0.925	0.916	1.000	0.936	0.958	0.947	1.000	0.905	0.925	0.916
40	EM	0.266	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.266	0.265	0.265	0.265
	ERB	0.028	-0.044	-0.099	-0.108	-0.003	-0.057	-0.112	-0.122	0.028	-0.044	-0.099	-0.108
	RE	1.000	0.870	0.884	0.878	1.000	0.896	0.910	0.904	1.000	0.870	0.884	0.878
50	EM	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265
	ERB	-0.050	-0.082	-0.133	-0.142	-0.075	-0.095	-0.146	-0.155	-0.050	-0.082	-0.133	-0.142
	RE	1.000	0.778	0.791	0.789	1.000	0.797	0.810	0.807	1.000	0.778	0.791	0.789
60	EM	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265	0.265
	ERB	-0.143	-0.168	-0.213	-0.220	-0.164	-0.178	-0.223	-0.231	-0.143	-0.168	-0.213	-0.220
	RE	1.000	0.795	0.806	0.804	1.000	0.812	0.823	0.821	1.000	0.795	0.806	0.804

Table 4: Stratified sampling design: Under πps, draw random sample of size n_h from each stratum and combined samples into one sample of size n.

n		$\hat{\theta}_{RR}$	$\tilde{\theta}_{.RR}$	$\tilde{\theta}_{.RH}$	$\tilde{\theta}_{.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{.HR}$	$\tilde{\theta}_{.HH}$	$\tilde{\theta}_{.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{.JR}$	$\tilde{\theta}_{.JH}$	$\tilde{\theta}_{.JJ}$
	var	1.66E-4	1.69E-4	1.68E-4	1.43E-4	1.43E-4	1.43E-4	1.43E-4	1.44E-4	1.43E-4	1.42E-4	1.43E-4	1.42E-4
20	MV	1.65E-4	1.70E-4	1.68E-4	1.34E-4	1.34E-4	1.34E-4	1.34E-4	1.35E-4	1.35E-4	1.34E-4	1.34E-4	1.34E-4
	RBV	-7.51E-3	1.08E-3	1.76E-3	-6.15E-2	-6.21E-2	-6.17E-2	-6.12E-2	-6.18E-2	-6.14E-2	-6.00E-2	-6.06E-2	-6.02E-2
	REV	1.00	7.20E-1	7.30E-1	7.25E-1	1.00	6.75E-1	6.84E-1	6.79E-1	1.00	6.91E-1	7.00E-1	6.96E-1
	var	1.09E-4	1.11E-4	1.10E-4	9.36E-5	9.40E-5	9.39E-5	9.36E-5	9.40E-5	9.39E-5	9.32E-5	9.36E-5	9.35E-5
30	MV	1.09E-4	1.11E-4	1.10E-4	8.99E-5	9.03E-5	9.02E-5	9.00E-5	9.04E-5	9.03E-5	8.97E-5	9.00E-5	8.99E-5
	RBV	-3.96E-3	2.09E-3	2.20E-3	-3.92E-2	-3.95E-2	-3.94E-2	-3.85E-2	-3.89E-2	-3.87E-2	-3.81E-2	-3.85E-2	-3.83E-2
	REV	1.00	7.29E-1	7.37E-1	7.34E-1	1.00	6.89E-1	6.97E-1	6.94E-1	1.00	7.00E-1	7.07E-1	7.05E-1
	var	8.05E-5	8.16E-5	8.11E-5	6.91E-5	6.93E-5	6.93E-5	6.90E-5	6.93E-5	6.92E-5	6.88E-5	6.91E-5	6.90E-5
40	MV	7.96E-5	8.11E-5	8.07E-5	6.65E-5	6.67E-5	6.67E-5	6.65E-5	6.67E-5	6.67E-5	6.63E-5	6.65E-5	6.65E-5
	RBV	-1.04E-2	-5.86E-3	-5.68E-3	-3.72E-2	-3.74E-2	-3.73E-2	-3.68E-2	-3.70E-2	-3.69E-2	-3.64E-2	-3.67E-2	-3.66E-2
	REV	1.00	7.09E-1	7.17E-1	7.15E-1	1.00	6.74E-1	6.81E-1	6.79E-1	1.00	6.82E-1	6.89E-1	6.87E-1
	var	6.33E-5	6.41E-5	6.39E-5	5.44E-5	5.45E-5	5.45E-5	5.43E-5	5.45E-5	5.44E-5	5.41E-5	5.43E-5	5.43E-5
50	MV	6.26E-5	6.36E-5	6.34E-5	5.25E-5	5.27E-5	5.27E-5	5.25E-5	5.27E-5	5.26E-5	5.24E-5	5.26E-5	5.25E-5
	RBV	-1.16E-2	-7.85E-3	-7.73E-3	-3.33E-2	-3.34E-2	-3.34E-2	-3.29E-2	-3.31E-2	-3.30E-2	-3.27E-2	-3.29E-2	-3.28E-2
	REV	1.00	7.32E-1	7.39E-1	7.38E-1	1.00	7.00E-1	7.06E-1	7.04E-1	1.00	7.06E-1	7.12E-1	7.11E-1
	var	5.19E-5	5.25E-5	5.23E-5	4.45E-5	4.47E-5	4.47E-5	4.45E-5	4.46E-5	4.46E-5	4.44E-5	4.45E-5	4.45E-5
20	MV	5.17E-5	5.24E-5	5.23E-5	4.36E-5	4.37E-5	4.37E-5	4.35E-5	4.37E-5	4.37E-5	4.34E-5	4.36E-5	4.36E-5
	RBV	-4.08E-3	-1.15E-3	-1.07E-3	-2.12E-2	-2.14E-2	-2.13E-2	-2.09E-2	-2.10E-2	-2.10E-2	-2.08E-2	-2.09E-2	-2.09E-2
	REV	1.00	7.23E-1	7.30E-1	7.28E-1	1.00	6.94E-1	7.00E-1	6.99E-1	1.00	6.99E-1	7.05E-1	7.04E-1

Table 5: Under srs: Comparisons between the variances for the different estimators. *var* := population variance of the estimator.

n	$\hat{\theta}_{RR}$	$\tilde{\theta}_{.RR}$	$\tilde{\theta}_{.RH}$	$\tilde{\theta}_{.RJ}$	$\hat{\theta}_{HH}$	$\tilde{\theta}_{.HR}$	$\tilde{\theta}_{.HH}$	$\tilde{\theta}_{.HJ}$	$\hat{\theta}_{JJ}$	$\tilde{\theta}_{.JR}$	$\tilde{\theta}_{.JH}$	$\tilde{\theta}_{.JJ}$
20	1.34E-7	2.74E-5	2.50E-5	8.78E-6	3.36E-5	8.59E-6	9.17E-6	8.35E-6	3.15E-5	8.22E-6	8.29E-6	8.23E-6
30	4.5E-6	1.48E-5	1.37E-5	6.46E-6	2.42E-5	6.21E-6	6.72E-6	6.26E-6	2.34E-5	6.27E-6	6.29E-6	6.26E-6
40	4.85E-8	1.25E-5	1.18E-5	2.29E-6	1.41E-5	1.88E-6	2.33E-6	2.03E-6	1.37E-5	2.04E-6	2.04E-6	2.04E-6
50	7.50E-7	9.60E-6	9.16E-6	1.62E-6	9.86E-6	1.21E-6	1.60E-6	1.38E-6	9.68E-6	1.39E-6	1.39E-6	1.39E-6
60	1.86E-6	8.36E-6	8.04E-6	6.22E-7	6.64E-6	2.44E-7	5.90E-7	4.19E-7	6.57E-6	4.34E-7	4.36E-7	4.37E-7

Table 6: Under srs: Numbers in this Table are the absolute differences between EV, Table(1), and MV, Table(5).

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Austrian Journal of Statistics http://www.ajs.or.at/
published by the Austrian Society of Statistics http://www.osg.or.at/
Volume 44 *Submitted:* 2014-02-19
January 2015 *Accepted:* 2014-07-29

Austrian Journal of Statistics
January 2015, Volume 44, 47–57.
http://www.ajs.or.at/

Ein Pakt mit den Bürgern. Ein Interview mit Ewald Kutzenberger

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Abstract

Das Interview mit Ewald Kutzenberger wurde von Werner Holzer und Matthias Templ am 22.10.2014 durchgeführt. Es beleuchtet den Werdegang von Ewald Kutzenberger, berichtet Hintergründe zur Entstehung des Bundesstatistikgesetzes und gibt Einblicke in den Übergang der Offiziellen Statistik in Österreich von der reinen Datensammlung zur methodischen Analyse von Daten und auch der Anknüpfung zu Universitäten. Unter anderem wird die politikberatende Funktion der Statistik in den Blickpunkt gerückt.

Ewald Kutzenberger, geboren 1944 in Steegen (OÖ), absolvierte nach einer kaufmännischen Lehre das Studium der Sozial- und Wirtschaftsstatistik an der Johannes Kepler Universität Linz, wo er auch 1977 promovierte. Er war unter anderem Leiter der OÖ Landesstatistik, Fachstatistischer Generaldirektor der Statistik Austria, Präsident der Österreichischen Statistischen Gesellschaft und Vorsitzender des Statistikrates. Er ist Ehrenmitglied der Österreichischen Statistischen Gesellschaft. In der Pension absolvierte er ein Studium der Fotografie.

Keywords: Interview, Offizielle Statistik, Politikberatung.

Werner Holzer: *Lieber Ewald, vielen Dank, dass du bereit bist, das Interview mit uns zu führen. Was deine berufliche Karriere als einen der bedeutendsten Vertreter der Statistik in Österreich betrifft, und wenn man deinen Lebenslauf liest, so ist dein Weg zur Statistik doch etwas später eingeschlagen worden. Was hat dich bewogen - du warst zuerst Industriekaufmann in der VÖEST (heute: voestalpine) - das Studium der Sozial und Wirtschaftsstatistik zu beginnen?*

Ewald Kutzenberger: Das ist relativ leicht erklärt. Ich habe in der VÖEST Industriekaufmann gelernt und in so einem Großbetrieb ist das eine sehr spezialisierte Geschichte, da man nur einen sehr kleinen Ausschnitt am Gesamten zu bewältigen hat. In einer sehr kleinen Abteilung habe ich dort die Verbuchung der technischen Bürostunden kontrollieren müssen. Das ist natürlich nur mäßig aufregend und ich habe gemerkt, dass dies nicht meins ist. Deshalb habe ich die Matura nachgeholt und hatte das Glück, dass 1966 in Linz die Universität gegründet wurde, an der ich BWL inskribierte. Das war für mich das „technischste“ Studium. Ich habe mich immer für Technik interessiert, besonders für Mathematik. Dann ist Prof. Bruckmann an die Uni Linz gekommen, von 1967 bis 1968, und er hat mich irgendwie fasziniert. Die erste Vorlesung, die ich bei ihm gehört habe, war über Indizes und Indikatoren, das hat mir unheimlich getaugt, und als er dann in Linz das Statistikstudium - Sozial und Wirtschaftsstatistik - eingeführt hat, war ich quasi der erste Student, der dieses Studium inskribiert hat, und schließlich auch der Erste, der es abgeschlossen hat.

Matthias Templ: *Du hast eine lange Verbindung zur Uni Linz wo du bis 2009 Vorlesungen gehalten hast. Gleichzeitig warst du aber auch immer der offiziellen Statistik verbunden. Blättert man deinen Lebenslauf und deine Publikationen durch, fallen besonders die Themen Arbeitsmarktprognosen, Bevölkerungsprognosen, Pendlerprognosen und vor allem auch Mikrozensus auf. Besonders Letzterer hat dich am Anfang deiner Karriere beschäftigt. Welche dieser Themen waren für dich ganz besonders wichtig?*

Ewald Kutzenberger: Ich glaube, ich muss einmal erklären, wie ich zur Amtlichen Statistik gekommen bin. Ich war ein Jahr Assistent an der Uni und habe dort die Aufgabe gehabt, die sehr abstrakten, unheimlich guten Vorlesungen von Prof. Derflinger in den Übungen den Studenten verständlich zu machen. Also den Studenten zu zeigen, was man mit dieser abstrakten, theoretischen Statistik überhaupt machen kann.

Am Institut hat es einen anderen Lehrbeauftragten gegeben, das war Dr. Lackinger, der Leiter der Landesstatistik Oberösterreich. Mit ihm hatte ich natürlich auch Kontakt und er meinte, ich sollte unbedingt zu ihm kommen, weil er seine Abteilung nicht wie einen Beamtenapparat führt, sondern eher wie ein Forschungsinstitut mit Politikberatung, Analysen usw. Da hab ich lange überlegt, für mich als Assistent war das irgendwie imagemäßig ein Abstieg. Zu einer Behörde als Beamter zu gehen, hat mir eigentlich überhaupt nicht zugesagt, aber er hat nicht locker gelassen und so habe ich mich eines Tages entschlossen zum Land zu gehen. Er hat dann auch wirklich das gehalten, was er versprochen hat, es war dort unheimlich interessant. Er war, glaube ich, ein Pionier in der Politikberatung mit Statistik, das hat er wirklich hervorragend gemacht. Er hat eine Nähe zu den Politikern in Oberösterreich gehabt und hat mit der Statistik Akzeptanz gefunden. Daher sind diese Themen, die du angeführt hast, behandelt worden, wie Arbeitsmarktprognosen, Bevölkerungsprognosen, diese ganzen Pendleranalysen, Mikrozensus usw. All dies ist dort als Grundlage für politische Entscheidungen intensivst bearbeitet worden.

Und dann ist der Weg zur Uni zurückgegangen, man hat mich ersucht, ob ich nicht den Studenten aus meiner praktischen Erfahrung ein bisschen was berichten kann. Gerade das Statistikstudium war so abstrakt, dass die meisten noch immer nicht gewusst haben, was man damit praktisch machen kann. Ich habe deshalb Vorlesungen wie Untersuchungsplanung, Amtliche Statistik, Stichprobenplanung und Demographie gehalten. Letzteres hat mich besonders interessiert, so dass ich dann 1975, glaube ich, eine der ersten Bevölkerungsprognosen in Österreich gemacht habe. Diese hat nicht gut ausgeschaut, ihr kennt ja alle die Entwicklung, und das hat soweit geführt, dass mich damals der damalige Landeshauptmann hinausgeschmissen hat, als ich ihm das präsentiert habe und er gemeint hat, „na so ein Blödsinn“.

Werner Holzer: *Hat es ihm vom Ergebnis her nicht gefallen?*

Ewald Kutzenberger: Es hat ihm überhaupt nicht gefallen. Ihr müsst euch vorstellen, 1975 das war die Zeit des Riesenaufschwungs, die Industrie hat geboomt und es hat nur ein „mehr und mehr“ gegeben und dann kommt so ein kleiner Statistiker daher, der sagt, „hey, passt's auf so geht es nicht weiter, da kommen einmal große Probleme auf uns zu“. Dies hat überhaupt nicht in das Konzept hineingepasst. Aber ich bin irgendwie an dem Thema drangeblieben und wie ich dann 1985 die Abteilung übernommen habe, als Dr. Lackinger in Pension gegangen ist, bin ich zu einer Art Vorstellungsvortrag von der

Wirtschaftskammer eingeladen worden, wo ein großer Kreis von Wirtschaftsleuten aus der Industrie usw. und auch von Behörden anwesend war. Dort habe ich diese Prognosen noch einmal präsentiert und alle waren so beeindruckt, dass das irgendwie fast zu einer Art Markenzeichen von mir geworden ist, und ich überall Bevölkerungsprognosen präsentieren konnte. Auch in der Regierungssitzung bin ich zweimal eingeladen gewesen und im Parlament. Das war schon eine tolle Geschichte. Man konnte ein bisschen zeigen, was man eigentlich mit Statistik machen kann. So bin ich von der Praxis her wieder mit der Uni verbunden gewesen in dem ich versucht habe dieses Wissen, diese Ideen, den Studenten und Studentinnen beizubringen, und ihnen zu zeigen, was man mit Statistik machen kann.

Werner Holzer: *Um auf die Rolle des Chefstatistikers in Oberösterreich zur Politikberatung zurück zu kommen: Wie kann man sich diese Rolle vorstellen, in welche Projekte wart ihr dort eingebunden? Waren das Themen, die ad hoc von der Politik gekommen sind, oder hattet ihr bestimmte Themen von vornherein besetzt?*

Ewald Kutzenberger: Es war beides. Durch die Nähe, die Dr. Lackinger zur Politik gehabt hat, gab es Themen aus der Kenntnis der Arbeit der Politiker. Wir haben uns gesagt, hier könnten wir Grundlagen liefern, hier könnten wir genauer analysieren. Damals war die Raumplanung das große Thema und da hat man natürlich sehr, sehr viel an Material von der Statistik her liefern können. Was wir damals auch in der Amtlichen Statistik gemacht haben, war die Einführung des Computers. Das war auch ein Grund, warum Dr. Lackinger mich geholt hat, weil er gesehen hat, da gibt es jetzt ein Instrument, mit dem man viel mehr machen kann, und er brauchte einen Jungen, der das beherrscht und so bin ich dort hineingekommen.

Das ganz Große und Wichtige war, dass es uns gelungen ist, die Daten des damaligen Statistischen Zentralamtes wirklich anzuzapfen. Wir hatten ja damals auch diese berühmte 15a-Vereinbarung mit dem Bund, diesen Staatsvertrag über die Nutzung der Daten der Amtlichen Statistik gemacht, so nach dem Motto: da liegen derartig viele Daten und die wissen gar nicht, was man im Land spezifisch an Themen hat. Wir wissen aber, was man braucht, um die Politiker beraten zu können. Wir haben vom damaligen Statistischen Zentralamt die Daten bekommen, um sie dann landesspezifisch analysieren zu können. Und das war, glaube ich, die große Errungenschaft damals und das ist auch heute noch so. Das versteht das Bundeskanzleramt noch immer nicht, weil von dort kommen immer wieder Ideen, die Landesstatistiker einzusparen. Aber diese Politiknähe und die Nähe zur Verwaltung, die man in den Ländern hat, die hat man im Bund nicht und das ist eigentlich schade.

Werner Holzer: *Der Vorteil ist ja, dass ihr nach dieser 15a-Vereinbarung mit Einzeldaten arbeiten konntet, das ist ja der große Fortschritt, glaube ich, dass man damit wirklich regional tiefe Analysen machen kann.*

Ewald Kutzenberger: Man kann wirklich ganz, ganz tiefe Analysen durchführen und hochqualitative Beratung anbieten. Das war wirklich eine ganz große Errungenschaft und hat zu einer hohen Qualität der Angewandten Amtlichen Statistik in den Ländern geführt.

Werner Holzer: *Ich springe jetzt in die 90er Jahre. Ich kenne dich seit 1994. Da hatte ich angefangen in der Statistik, damals in der Wanderungsstatistik und ich habe dich als aktivstes Mitglied oder eines der aktivsten Mitglieder in der Statistischen Zentralkommission und in den Fachbeiräten erlebt. In diesen Rollen hast du sicher viel beigetragen zur Bundesstatistik und du bist dann - und jetzt geh ich in die Jahre 1998/1999 - einer der Väter des modernen Bundesstatistikgesetzes 2000 geworden und hast eine ganz wichtige Rolle übernommen, nämlich an der Vorbereitung der Ausgliederung der Bundesstatistik aus dem Bereich des Bundeskanzleramtes mitzuarbeiten als Grundlage der Modernisierung des damaligen Österreichischen Statistischen Zentralamtes. Kannst du uns über diese Zeit etwas berichten?*

Ewald Kutzenberger: Zuerst einmal zur Zeit der beratenden Gremien Zentralkommission und Fachbeiräte: Das war natürlich eine fantastische Geschichte, dass man da als Landesstatistiker in diesen Gremien drinnen war - und ich sag jetzt - ein bisschen mitgestalten konnte, was die Amtliche Bundesstatistik macht, und dass man zu den Daten oder zu den Informationen kommt, die man als Land braucht. Ich nenne nur ein Beispiel: Das erste Mikrozensusmodell hatte keine Bundesländerdaten vorgesehen und da haben wir uns als Vertreter der Länder in diesen beratenden Gremien wirklich auf die Füße gestellt und gesagt: das kann nicht sein. Wir sind bereit, am Mikrozensus mitzuarbeiten, aber wir wollen dafür ein Modell haben, welches es ermöglicht, dass auch Bundesländerdaten erhoben werden. Das Modell hatte sogar ich damals gemacht und es war ein Teil meiner Dienstprüfung für Statistik in Wien. Die Länder haben sich dafür bereit erklärt, den ganzen Interviewerstab in den Ländern zu organisieren und zu betreuen. Das war so ein typisches Geschäft, wo man sagt, es war jedem geholfen und diese Rückkoppelung war halt nur möglich, weil man in diesen Gremien auch drinnen war. Von daher hatte ich natürlich schon einen sehr, sehr langjährigen Einblick in die Amtliche Statistik in Wien.

Auch durch die Vorlesungen, die ich an der Uni gehalten habe, hatte ich mich immer mit der Amtlichen Statistik auseinandergesetzt, z.B. wie in modernen Institutionen diese organisiert sein sollte, und hinterfragt, was die Aufgabe einer modernen Statistikorganisation ist, und wie diese ablaufen sollte.

1998 hatte mich dann der damalige Präsident Bader angerufen und gesagt, dass er einen Entwurf vom Bundeskanzleramt über ein neues Bundesstatistikgesetz hat, weil das alte Gesetz nicht mehr den EU-Anforderungen entspricht. Er meinte, dass ich mir das einmal anschauen sollte. Und ich habe mir den Entwurf angesehen und es war halt überhaupt nicht das, was man sich unter einer modernen Amtlichen Statistik vorgestellt hat. Es war beispielsweise über die Unabhängigkeit der Statistik gar nichts zu finden. Diesbezüglich hatte ich auch mit dem damaligen Landesstatistiker von Vorarlberg, Dr. Feurstein, der auch Abgeordneter zum Nationalrat war, gesprochen und er hat gemeint, da sollte man etwas unternehmen. Er ist mit mir zum damaligen Sektionschef Mayer – so hat er glaub ich, geheißen - ins Bundeskanzleramt gegangen und er hat uns - um es klipp und klar zu sagen - hinausgeschmissen. Er sagte uns: „Da steht alles, was wollt ihr? Ihr werdet doch nicht gescheiter sein als unsere Legisten“ , und das war’s! Aber Dr. Feurstein hat nicht losgelassen und hat einen Termin mit dem damals zuständigen Staatssekretär vereinbart. Es war auch Dr. Schittengruber dabei, der diesen Gesetzesentwurf erstellt hat, und ich habe ihnen meine Ideen und Bedenken und meine Vorstellungen erklärt. Daraufhin hat der Staatssekretär gemeint „Na wenn Sie glauben, dass Sie es besser können, dann setzen sie sich mit Dr. Schittengruber zusammen und machen Sie einen neuen Entwurf“. Das war klipp und klar. Das alles war vor Weihnachten und ich kann mich noch erinnern, dass wir über Weihnachten, Neujahr bis zu den Heiligen Drei Königen immer wieder zusammen gesessen sind. Am Anfang hatte ich einen ziemlichen Bammel gehabt, muss ich sagen, weil ich als kleiner Landesstatistiker da zu dem großen Legisten in das Bundeskanzleramt kam und die Befürchtung hatte, dass er mir gar nicht zuhören wird. Aber ich muss sagen, Dr. Schittengruber war wirklich super. Er hat sich alles genau angehört und hat gesagt, dass das Hand und Fuß hat. Aber er hat auch gemeint, dass wir das Gesetz nicht neu machen können, und wir schauen müssen, dass wir meine Anregungen irgendwo einbauen. Durch dieses Einbauen ist das Gesetz so unleserlich geworden. Da war immer wieder irgendwo ein Verweis und ein neuer Aspekt und wieder nur Rückverweise, es war nicht möglich, den Entwurf völlig neu zu schreiben. Trotzdem hatte er, glaube ich, wirklich fast alles, was an meinen Ideen da war, reingebracht. Und dann ist plötzlich von politischer Seite die Aufforderung gekommen „Naja, wenn ihr das jetzt ohnehin neu macht’s mit Unabhängigkeit usw., dann lagern wir es gleich aus“. Das war, glaube ich, zu Pfingsten während einer Regierungsklausur, in welcher schließlich tatsächlich beschlossen worden ist, die Statistik auszulagern. Ich habe zu Dr. Schittengruber gesagt, bei diesem Anstaltsrecht kenne ich mich überhaupt nicht aus, da sollte man noch jemand anderen beiziehen, jemanden, der von Statistik und vom Unternehmensrecht eine Ahnung hat und ich habe dann Dr. Richter vorgeschlagen. Dies ist auch akzeptiert worden und dann haben eben wir drei, Dr. Schittengruber, Dr. Richter und ich den neuen Entwurf mit dieser Ausgliederung erstellt.

Werner Holzer: *Du bist dann im Jahr 2000 zum Fachstatistischen Generaldirektor der neu errichteten bzw. ausgegliederten Bundesanstalt Statistik Österreich ernannt worden. Das war, glaube ich, eine sehr spannende Zeit damals. Also einerseits hast du den Wechsel vom Statistiknutzer zum Chefstatistikproduzenten vollzogen, wenn man das jetzt so salopp sagen kann, und auf der anderen Seite bzw. gleichzeitig hast du dich der großen Aufgabe gestellt, gemeinsam mit der kaufmännischen Generaldirektorin das Haus von Grund auf zu modernisieren und neue Projekte aufzusetzen. Wie war die Zeit damals, also diese ersten Jahre der Ausgliederung während deiner Funktionsperiode als Fachstatistischer Generaldirektor?*

Ewald Kutzenberger: Vielleicht einmal kurz erklärt, wie bin ich überhaupt Generaldirektor geworden? Ich habe das nie angestrebt und ich sage immer, wenn ich das vorher gewusst hätte, hätte ich im Gesetz so einiges ein bisschen anders vorgeschlagen. Dr. Schittengruber hat mich damals angerufen und gesagt, so, wir haben jetzt die Ausschreibung gemacht und Sie melden sich gefälligst. Zuerst hatte ich gemeint, dass dies überhaupt nicht in Frage kommen würde. Ich war so etabliert in Oberösterreich, dass es für mich einfach undenkbar war wegzugehen. Ich hatte es mir wirklich nicht vorstellen können. Ausschlaggebend war dann ein Gespräch mit Dr. Findl, den ich gut kannte, weil er von der Demographie kam und wir in der demographischen Gesellschaft sehr viel Kontakt zueinander gehabt hatten. Er hat mir damals ins Gewissen geredet und gemeint „Pass auf, wenn du nicht kommst, wer weiß, wer sonst kommt. Bei dir wissen wir, dass du dich auskennst, du kennst das Amt, du kennst das Haus, du kennst sehr viele Leute hier, wir kennen dich. Du musst dich bewerben, weil ansonsten kommt irgendwer daher und dreht wieder alles total um und macht es wieder ganz anders als du es dir vorgestellt hast“. Das Gespräch hat mich dann bewogen, dass ich gesagt habe, ok, da hat er irgendwie Recht. Ich habe mich dann beworben und bin zum Fachstatistischen Generaldirektor ernannt worden. Wie war das am Anfang? Ich glaube, ich war mit Dr. Petrovic ein sehr, sehr gutes Team. Sie hat sich in Statistik nicht ausgekannt, ich hab mich im Personalrecht nicht ausgekannt. Wir haben uns gegenseitig sehr, sehr gut ergänzt. Sie hatte darauf geschaut, dass die Finanzen in Ordnung sind und ich habe versucht, in der ganzen statistischen Reorganisation die Ideen des Gesetzes umzusetzen. Da hast du mir (Anm.: Werner Holzer) sehr viel geholfen bei der Reorganisation. Das war eine ganz wichtige Geschichte, diese vielen Abteilungen, die damals existiert haben, in wenige Direktionen umzuwandeln und innerhalb der Direktionen eine Struktur hineinzubringen. Ich bin heute noch begeistert, wie alle am selben Strang gezogen haben. Das war wirklich hervorragend, die alten Abteilungsleiter, wenn ich so sagen darf, die haben genau gewusst, wenn wir jetzt von 8 Abteilungen auf 4 Direktionen zusammenlegen, werden 4 Abteilungsleiter nicht mehr gebraucht und trotzdem haben alle mitgezogen und wir haben das neue Konzept wirklich miteinander umgesetzt. Das war eine ganz tolle Geschichte und ich glaube, dass diese Reorganisation ganz gut gelungen ist. Sie steht heute noch so, wie es damals gemacht wurde, und ist, soweit ich weiß, für so manche andere statistische Ämter sogar Beispiel gewesen für Neuorganisationen.

Matthias Templ: *Dieses Alleinstellungsmerkmal von zwei Generaldirektoren ist sehr ungewöhnlich. Ist dies auch auf deinen Vorschlag zurückzuführen?*

Ewald Kutzenberger: Das ist eigentlich vom GmbH-Recht übernommen worden und das Vier-Augen-Prinzip ist üblich in der Leitung von so einem Unternehmen. Die finanzielle Situation war damals im Haus katastrophal und vor allem auch der Umgang mit dem

Geld war katastrophal. Es wurde die Idee geboren, eine kaufmännische Leitung zu bestellen, die sich wirklich darum kümmert, dass das in Ordnung kommt. Eine gegenseitige Kontrolle war aus der damaligen Sicht, glaube ich, eine ganz gescheite Entscheidung.

Werner Holzer: *Dieses Prinzip der umfassenden Modernisierung hat sich aus meiner Sicht auch bei deinem TQM-, Total Quality Management-Ansatz gezeigt. Ich kann mich erinnern, dass ich sehr beeindruckt war, wie du gekommen bist - ganz am Anfang - und gesagt hast, das ist mein Managementansatz, mein strategisches Management, und wie es dann sukzessive in der Statistik Austria umgesetzt wurde.*

Ewald Kutzenberger: Ich habe damals nach etwas gesucht, wie man allen Mitarbeitern und auch externen Leuten und den Beamten in den Ministerien, im Bundeskanzleramt, im Wirtschaftsrat und dem Statistikrat erklären kann, was wir machen und warum wir das machen. Dabei ist mir dieser TQM-Ansatz eingefallen und das ist etwas, was man, glaube ich, sehr gut erklären kann und wo kaum einer etwas dagegen haben kann. Mit der entsprechenden Erklärung hat sich eigentlich immer jeder abgefunden. Das hatte „Hand und Fuß“, man hatte ein Ziel und hat dadurch eine sehr positive Mitwirkung aller Beteiligten erzielt. Du (Anm.: Werner Holzer) hast dabei auch sehr mitgeholfen diesen TQM-Gedanken zu tragen und in die Direktionen hinauszubringen und den Mitarbeitern zu erklären. Dies hatte immer sehr gut funktioniert, weil für jeden einleuchtend war, was man da will und was das Ziel ist.

Matthias Templ: *Vielleicht darf ich noch einmal über das Bundesstatistikgesetz zu sprechen kommen. Dadurch war die Statistik Austria praktisch Vorreiter in Europa bezüglich Unabhängigkeit der Statistik von der Politik. Wir erleben auch heute noch, dass Länder direkter politischer Einflussnahme ausgesetzt sind. Wie hast du das erlebt? Ich denke es gab viele Fälle bei denen die Politik Einfluss nehmen wollte. Soweit ich dies beurteilen kann warst du damals sehr robust gegen Einflussnahmen.*

Ewald Kutzenberger: Es war meine Grundidee bei dem neuen Bundesstatistikgesetz, dass Statistik politisch völlig unabhängig agieren muss, weil sie sonst unglaubwürdig ist. Es ist gelungen, einige wichtige Aspekte in das Gesetz aufzunehmen, z.B. die Weisungsungebundenheit des Fachstatistischen Generaldirektors bei fachlich-methodischen Fragen. Dies steht auch dezidiert so im Bundesstatistikgesetz, d.h. niemand kann einem Fachstatistischen Generaldirektor sagen, wie er einen Verbraucherpreisindex rechnen soll oder wie er dieses oder jenes berechnen soll. Es war eigentlich kein Problem, dies in das Gesetz aufzunehmen, da wir auf die skandinavischen Länder verweisen konnten, wo das damals schon üblich war, im Gegensatz zu Mitteleuropa. Und in Südeuropa hat man sich das überhaupt nicht vorstellen können. Die haben gesagt, also das ist bei uns undenkbar. Dies hat sich heutzutage auch schon geändert.

Der zweite wichtige Aspekt war das Veröffentlichen der Ergebnisse. Also nicht wie die Vorgehensweise von früher, dass die Ergebnisse einmal dem Minister vorgelegt wurden, der dann darüber befunden hat, wann das veröffentlicht wird und was man darüber sagt, sondern wir haben gesagt, das geht so nicht. Die Statistik hat laut Gesetz eine bestimmte Statistik zu erstellen und wenn diese fertig ist, hat die Statistik diese zu veröffentlichen und gleichzeitig bekommt der Minister die Ergebnisse, welcher dann ohnehin dazu Stellung nehmen kann. Dass wir das ins Statistikgesetz hineingebracht haben, darüber bin ich heute noch etwas verwundert. Sagen wir so, ich glaube das hatten sie nicht gut gelesen. Das hat tatsächlich danach am meisten Probleme gemacht. Man wollte natürlich immer noch wie früher arbeiten: so nach dem Motto, „Jetzt habt ihr das fertig, können wir das haben und dann machen wir halt einmal eine Präsentation“. Wir haben gesagt, nein, das geht nicht, und da waren sie einmal ganz verwundert wieso und weshalb das unmöglich ist; worauf ich gesagt habe: „Lesen sie das Gesetz“. Es hat viele Diskussionen gegeben, aber es hat sich dann doch eingespielt und man hat begriffen, dass das nun anders ist und schon Sinn macht. Ein weiterer, völlig neuer Aspekt war, dass alle

Hauptergebnisse der Öffentlichkeit im Internet zur Verfügung gestellt werden müssen. Mein Motto war, dass die Amtliche Statistik quasi einen Pakt mit den Bürgern und den Unternehmen schließt: Sie geben uns Daten und wir geben dafür Informationen zurück. Auch EUROSTAT hat sich, allerdings erst viel später, dieser Meinung angeschlossen.

Matthias Templ: *Damit kann man auch sehr offen zu Universitäten sein, praktisch auch Wissenschaftlichkeit in der Statistik Austria fördern. Ich kann mir vorstellen, dass in den frühen 1980er Jahren die Amtliche Statistik eher mit dem negativen Ausdruck „Erbsenzählen“ behaftet war. Ich glaube dabei hat sich einiges geändert. Du warst einer der Pioniere, der die Statistik modernisiert hat; und dass die Nähe und Verbindung zu den Wissenschaften und zu den Universitäten gesucht wurde.*

Ewald Kutzenberger: Ich hab das so gesehen, dass die Bundesstatistik - die Amtliche Statistik - die Entscheidungsgrundlagen für die Regierung, für die Politik, für die Wirtschaft, für Medien und für die öffentlichkeit liefern muss. Diese Entscheidungsgrundlagen sind die Publikationen, die wir zu den einzelnen Bereichen erstellen. Diese sollen schon so verständlich wie möglich sein, so dass der Nutzer auch versteht, was drinnen steht. Man muss aufpassen, dass man in diesen Publikationen nicht zu viel „Wissenschaftlichkeit“ hineinbringt. Man kann parallel einmal eine Publikation erstellen, wo man irgendwelche Methoden erklärt, aber die Hauptpublikationen sollen so klar, einfach und verständlich wie möglich sein. Dass die Wissenschaft, die statistische Theorie, im Erarbeiten dieser Publikationen - also dieser Ergebnisse - eine ganz wichtige Rolle spielt, war mir immer ein Anliegen. Ich habe immer die Idee gehabt, dass wir im Haus so eine Art Analysegruppe etablieren, was am Anfang nicht auf viel Gegenliebe gestoßen ist. Eigentlich wollte ich so eine zentrale Analyseinstitution haben. Das hat sich damals überhaupt nicht durchsetzen lassen, aber wir haben es daraufhin innerhalb der Direktionen versucht und haben dort Gruppen geschaffen, die mit qualitativen und mit wissenschaftlichen Methoden und Analysen sich der Daten angenommen haben.

Ich bin immer noch der Meinung, dass die Amtliche Statistik, auf einem - ich bin dafür immer ausgelacht worden, weil ich das gesagt habe - Datenschatz sitzt, der bei weitem nicht gehoben wird. Wir haben immer noch dieses *Stove-Pipe*-Prinzip, dass man eine bestimmte Statistik zu machen hat und danach die Ergebnisse präsentiert werden. Aber dass man aktiv die Vernetzungen der unterschiedlichen Bereiche vorantreibt, hier ist glaube ich noch unheimlich viel zu machen. Wenn unabhängige wissenschaftliche Institute zu diesen Daten unter strengsten Datenschutzregeln zugreifen könnten und mit den Daten arbeiten könnten wie sie wollten, da wäre einiges drinnen. Aber so sind die Ressourcen beschränkt, man hat nicht genug Leute, nicht genug Geld, um sich sozusagen auftragsfrei solcher Daten anzunehmen. Ideal wäre, dass ich mir als Mitarbeiter sage, „da haben wir etwas und das muss ich mir einmal anschauen und analysieren, vielleicht ist da mehr drinnen“. Das geht leider aus Kapazitätsgründen nicht.

Werner Holzer: *Ich komme jetzt auf eine andere ganz wichtige Funktion zu sprechen, die du im System der Amtlichen Statistik inne hattest. Du bist nach deiner Funktionsperiode als Fachstatistischer Generaldirektor zum Vorsitzenden des Statistikrates ernannt worden und hast sozusagen damals die Seiten gewechselt. Also vom Chefstatistiker zum Vorsitzenden dieses wichtigen Beratungsgremiums. Wie war das für dich und wie hast du diesen Wechsel erlebt?*

Ewald Kutzenberger: Nicht gut. Vielleicht erkläre ich noch einmal kurz, was die Idee mit dem Statistikrat war. Diesbezüglich haben wir lange diskutiert, damals mit Dr. Schittengruber. Unsere Bedenken waren folgende: Wenn die Statistik eine ausgegliederte Dienststelle, eine ausgelagerte Institution wird, gibt es einen Wirtschaftsrat, der die finanzielle Kontrolle bezüglich des Budgets übernimmt. Es könnte die Qualität der Statistik dadurch leiden, dass es dann darum geht, das Ganze immer nur billiger zu machen und mit weniger Geld auszukommen. Was ist dann naheliegend? Dass man die Qualität der Statistik herunterfährt, dass man z.B. sagt: „Na, machen wir halt kleinere und weniger Stichproben“. Somit würde die Qualität sinken. Daher stellte sich die Frage, wie man das kontrollieren kann. Die Idee war, einen Statistikrat einzurichten, parallel zum Wirtschaftsrat, der die Qualität dieser Statistik überprüft, damit ja nicht die Qualität der Statistik durch irgendwelchen finanziellen Druck sinkt. Und man hat dann überlegt, wen man in dieses Gremium hineinnimmt. Wer kann das kontrollieren? Die Grundidee war, dass man Vertreter der wichtigsten Nutzer einbezieht, also die bedeutendsten Ministerien, Wirtschaftskammer, Arbeiterkammer, Länder, Gemeindebund, Städtebund usw.. und man auch die Wissenschaft hineinnimmt, die auch kontrolliert, ob hinsichtlich der verwendeten Methoden alles passt. Im Gesetz heißt es so schön, dass man von den Methoden her immer auf dem neuesten Stand sein muss. Das muss ja irgendwer kontrollieren und da war wieder die Grundidee, dass Vertreter im Statistikrat Fachleute in der Statistik sind, die aber nicht die entsendete Institution repräsentieren, sondern nur das Know-how von dort mitnehmen, aber unabhängig auf die Qualität der Statistik schauen. Das hat nicht ganz funktioniert. Also diese Trennung, ein Abgesandter eines Ministeriums zu sein und im Statistikrat womöglich aus methodischen Gründen oder Qualitätsgründen eine Entscheidung mitzutragen, die gegen das Ministerium ist, das war schwierig. Und deshalb hat das nicht ganz funktioniert. Aber trotzdem war die Rolle wahnsinnig wichtig, und wie ich dann in den Statistikrat hinübergewechselt bin, war ich also in dieser Zwickmühle. Auf der einen Seite habe ich die Forderungen der Nutzer verstanden, aber natürlich im Hintergrund genau gewusst, was man alles nicht machen kann, aus budgetären oder personellen Gründen, usw. Ich war immer so mittendrinnen und habe beide Seiten verstanden, die aber sehr oft extreme Positionen hatten. Ich habe mich überhaupt nicht wohl gefühlt in der Rolle, das muss ich ganz offen gestehen.

Werner Holzer: *Du hast nicht zuletzt auch die Funktion des Präsidenten der Österreichischen Statistischen Gesellschaft inne gehabt. Weil wir vorhin über das Verhältnis der Amtlichen Statistik und der Wissenschaft gesprochen haben: Wie wichtig war und ist für dich die ÖSG als Brückenbauer zwischen akademischer, angewandter und der offiziellen bzw. amtlichen Statistik?*

Ewald Kutzenberger: Die Statistische Gesellschaft ist die einzige Institution wo diese drei großen Bereiche der Statistik, also die Amtliche Statistik, die wissenschaftliche und die angewandte - also auch die Nutzer - vertreten sind. Ich sehe das so, dass die Amtliche Statistik, wie ich schon erwähnt habe, die wissenschaftlichen Methoden sehr dringend braucht, um bei den Analysen am neuesten Stand zu sein, um hochqualitative Analysen machen zu können und die Ergebnisse „herauszukitzeln“ aus den Daten, was vielleicht mit einfacheren Methoden nicht mehr möglich wäre. Dass das nicht so einfach ist, hatte ich auch gemerkt. Es bestehen zwei Lager, welche nicht recht viel miteinander zu tun haben wollen, das ist ganz eigenartig: Für die statistischen Theoretiker von der Uni ist die reine Amtliche Statistik ein bisschen was sehr Einfaches, die Erbsenzähler, „das können sie ohne uns auch“, ja so ungefähr, und für die Statistiker in der Amtlichen Statistik ist das oft so, dass die sagen: „die mit ihren Theorien, die sind so abgehoben, die haben eigentlich mit der Praxis überhaupt nichts mehr zu tun, die wissen ja gar nicht mehr, warum sie alle diese Theorien entwickeln“. Das waren so diese zwei Lager und daher haben wir uns in der ÖSG schon immer bemüht, durch Veranstaltungen der Wissenschaft und der Statistik Austria - die hauptsächlich in der Statistik Austria stattgefunden haben - dagegen zu wirken. Nur es war leider so, dass der gegenseitige Besuch immer relativ marginal war, also wenn ein theoretisches Thema angesetzt war, haben oft zwei, drei Leute von der Statistik Austria daran teilgenommen und umgekehrt, wenn jemand von der Statistik Austria vorgetragen hat, sind von der Wissenschaft - wenn es gutgegangen ist – vielleicht zwei, drei Leute gekommen. Also so wirklich angenommen wurde das nicht. Ich glaube aber trotzdem, dass die Statistische Gesellschaft eine wichtige Funktion hat. Nur wenn ich mir so die Publikationen der letzten Zeit ansehe, wird das Ganze für meine Begriffe sehr, sehr theorielastig, das heißt, die angewandte Statistik geht in meinen Augen völlig unter. Ich weiß nicht, wie die Situation bei den Veranstaltungen der ÖSG derzeit ist, aber was ich so mitbekomme, wäre - glaube ich - eine neue Zielsetzung notwendig, um die Bereiche besser zusammen zu bringen.

Matthias Templ: *Ich denke man kann immer wieder beobachten, dass die Theorie bzw. die Theoretiker mit der Anwendung bzw. den „Angewandten“ - und vice versa - eigentlich wenig zu tun haben wollen. Darum, finde ich, hat die ÖSG hier wirklich eine gute Brückenbauerfunktion.*

Ewald Kutzenberger: Das wäre eine Möglichkeit, um diese zusammenzubringen und ich finde, die Theoretiker müssen ein bisschen von ihrem hohen Ross herunter steigen und die „Angewandten“ müssen für neue Methoden ein bisschen offener werden. Das muss man fast ein wenig erzwingen, weil von selber geht es nicht, das ist meine Beobachtung gewesen.

Werner Holzer: *Mir geht es auch so ähnlich, wenn ich mir die Publikationen der ÖSG ansehe. Aber andererseits finde ich schon, dass bei methodischen Vorträgen - insbesondere in der Statistik Austria - die Amtliche Statistik sehr gut vertreten ist.*

Ewald Kutzenberger: Jetzt ist es besser geworden. Ich war bei den letzten Vorträgen nicht dabei, aber ich weiß aus meiner Zeit schon, dass das immer mühsam war und wir immer getrommelt haben: „Bitte geht hin, das ist ein interessanter Vortrag“.

Matthias Templ: *Wie ich versucht habe dich zu diesem Interview zu überreden, ist mir nach einigen Telefonaten das Wort Pensionsstress wieder in Erinnerung gerufen worden. Ich glaube du bist nach wie vor sehr engagiert. Ich habe etwas recherchiert, du machst noch Studienauswertungen zum Thema Alter?*

Ewald Kutzenberger: Jetzt nicht mehr, das war eher in den ersten Jahren meiner Pensionierung. Ich muss sagen, ich habe mich von der Statistik total zurückgezogen. Ich bin der Meinung, das sollen jetzt die Jungen machen, man muss auch loslassen können. Lustig war ja, als ich damals als Generaldirektor gesagt habe, dass ich nicht mehr verlängern will, und dem Bundeskanzler einen Brief geschrieben habe, dass ich die Statistik nun den Jungen überlassen möchte. Und dann war mein Nachfolger älter als ich. Das war ein bisschen verblüffend für mich, aber war halt so. Ich glaube schon, dass es wichtig ist, dass junge Energien mit neuen Ideen hereinkommen und es macht keinen Sinn, wenn man hierbei als Alter glaubt, mitreden zu können und seinen Senf dazugeben zu müssen. Das ist nicht gut.

Darum habe ich mich dann wirklich zurückgezogen, habe ganz etwas anderes gemacht. Ich habe in der Pension Fotografie studiert und ich mache das sehr interessiert weiter. Wir sind da eine recht nette Gruppe von Studienkollegen, die immer wieder gemeinsam Projekte machen, das fasziniert mich. Und ich habe fünf Enkelkinder, die halten mich auch auf Trab. Sie kommen jetzt in ein Alter, wo ich mit ihnen etwas anfangen kann. Mit ganz kleinen Kindern kann man ja nicht so viel machen, die älteren Enkel sind jetzt zehn und elf Jahre, da macht es schon Spaß, mit ihnen etwas zu unternehmen und zu sehen, wie sie neugierig werden, was sie alles wissen wollen und was die schon können. Das verblüfft mich immer wieder.

Werner Holzer: *Und glaubst du, ist da ein Statistiker darunter?*

Ewald Kutzenberger: Ich meine, der eine Enkel der jetzt 10 Jahre alt geworden ist, der fragt mich immer wieder, was denn das und das ist, und horcht sehr interessiert zu, der dürfte schon irgendwie ein bisschen in die Richtung Naturwissenschaften gehen. Während die ältere Enkelin, die 11 Jahre ist und meinem Sohn ähnelt, eher sprachlich talentiert ist, die in Kunst und Sprache total aufgeht. Es ist interessant zu sehen, wie die Kinder sich da unterschiedlich entwickeln und zum Teil schon ganz, ganz tolle Ideen haben, das taugt mir.

Matthias Templ: *Wir haben uns einmal beim Wandern getroffen. Jetzt hast du wahrscheinlich bereits einen zweiten Rucksack für den Fotoapparat mit. Du interessierst dich wahrscheinlich auch für die Landschaftsfotografie?*

Ewald Kutzenberger: Ich habe gerade den Fotoapparat gewechselt, weil mir der andere zu schwer war zum Mitnehmen. Diese Profikameras sind ja solche Trümmer. Jetzt habe ich mir wieder einen leichteren zugelegt, den ich zum Fotografieren mitnehmen kann. Ja, ich wandere schon sehr gerne, aber jetzt nicht so fanatisch. Wir fahren auch Rad und haben uns ein E-Bike zugelegt, da die Kondition in dem Alter dann doch nicht mehr so gut ist, wenn viele Berge kommen. Ich lebe ja am Rand vom Mühlviertel, wo man ohne Berge nicht auskommt. So ein E-Bike ist toll, man kann so 70-80 km-Touren ohne Rücksicht auf „Mugln“ machen, das ist eine tolle Geschichte.

Matthias Templ: *Ich kenne Statistiker die alle möglichen Statistiken darüber erstellen.*

Ewald Kutzenberger: Ja, dieses „Runtastic“ und wie das alles heißt. Das habe ich am Anfang auch gemacht. Dann habe ich mir gedacht, na so ein Blödsinn, genieß einfach das Fahren und schau nicht dauernd auf die Zeit und auf die Höhenmeter, die man macht. Irgendwie ist es zwar immer anscheinend menschlich, dass man das nachher anschauen will, aber ich bin draufgekommen, dass es viel gescheiter ist, wenn man sich auf das Wandern als solches, das Radfahren als solches, auf die Umgebung, auf die Landschaft konzentriert und das genießt. Das ist viel gescheiter.

Werner Holzer: *Also ich kann das wirklich gut verstehen, dass du dich nach einem so intensiven Berufsleben als Statistiker in den unterschiedlichsten Bereichen jetzt auf Erholung und auf Familie konzentrierst. Wir wünschen dir noch alles Gute und danken dir vielmals für das Interview.*

Ewald Kutzenberger: Ich danke euch, es war auch für mich interessant wieder einmal auf meine Rolle in der Amtlichen Statistik zurückzublicken!

Die Interviewer bedanken sich herzlich bei Elisabeth Jelinek und Statistik Austria für die Transkription.

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Austrian Journal of Statistics `http://www.ajs.or.at/`
published by the Austrian Society of Statistics `http://www.osg.or.at/`
Volume 44 *Submitted:* 2014-10-22
January 2015 *Accepted:* 2014-11-05

News and Announcements

Regular meetings of the **Vienna R Meetup Group** find place approx. every two months. The meetup group is supported by Revolutions Analytics and data-analysis OG. More information on past and future presentations at the meetup, the orgianisation of the meetup group, members and discussions can be found at http://www.meetup.com/ViennaR/ .

The **Young Statistician Meeting** 2015 will be take place in Vorau, see http://www.stat.tugraz.at/ysm15/ysmhistory.html. It offers our youngsters a warm and cordial first step into the scientific world of the international research community. The main orgainizer (Herwig Friedl) as well as the local organizer (Peter Filzmoser) looking forward for applications from young statisticians. The Austrian Statistical Society supports this event.

The students at the Vienna University of Technology seems to be especially **happy**, look for example here:
http://www.statistik.tuwien.ac.at/public/filz/happystudent/happystudent.html

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ISSN 1026 - 597X

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