Supplementary Material for "Estimation of order restricted normal means when the variances are unknown and unequal"

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Program 1. This program is ralated to simulate from two univariate normal distributions to obtain the risk difference to compute the risks difference $\hat{\mu}_1$ and $\hat{\mu}_2$ which are given in Table 1.

```
estmu1 = function(x,y,t)
{xbar = mean(x)
ybar = mean(y)
min(xbar,t*xbar+(1-t)*ybar)
}
estmu2 = function(x,y,t)
{xbar = mean(x)
ybar = mean(y)
max(xbar,t*xbar+(1-t)*ybar)
}
like = function(a,b)
{(a-b)^2
}
like <- Vectorize(like)
R = function(a,b)
{
mean(like(a,b))
}
n1 = 10;n2 = 15
s1 = 4;s2 = 5
mu1 = 3;mu2 = 4
N = 10000
t = (n1/s1)/((n2/s2)+(n1/s1))
Ly = Lx = muhat1 = muhat2 = Lm1 = Lm2 = array(0)
for(i in 1:N)
{
x = rnorm(n1,mu1,s1)
```

```
y = rnorm(n2,mu2,s2)
muhat1[i] <- estmu1(x,y,t)
muhat2[i] <- estmu2(x,y,t)
Lm1[i] = like(mu1,muhat1[i])
Lm2[i] = like(mu2,muhat2[i])
Lx[i] = like(mu1, mean(x))
Ly[i] = like(mu2, mean(y))
}
R(mu1,Lx) - R(mu1,muhat1)
```

Table 1: Simulation from two univariate normal distributions: the values of risks difference $\hat{\mu}_1$ and $\hat{\mu}_2$.

	Sample sizes	$N(\mu_{1r}, s_{1r})$	$N(\mu_{2r}, s_{2r})$	$RD_{\bar{X}_1,\hat{\mu}_1}$	$RD_{\bar{X}_2,\hat{\mu}_2}$
$case1(r=a)$	$n_1 = 10$	$\mu_{1a} = 3$	$\mu_{2a} = 4$	1.179	-0.235
	$n_2 = 15$	$s_{1a} = 4$	$s_{2a} = 5$		
$case2(r=b)$	$n_1 = 10$	$\mu_{1b} = 4$	$\mu_{2b} = 4$	0.011	0.127
	$n_2 = 15$	$s_{1b} = 2$	$s_{2b} = 3$		
$case3(r=c)$	$n_1 = 10$	$\mu_{1c} = 3$	$\mu_{2c} = 3$	-0.139	-0.110
	$n_2 = 10$	$s_{1c} = 5$	$s_{2c} = 6$		
$case1(r=a)$	$n_1 = 20$	$\mu_{1a} = 4$	$\mu_{2a} = 4$	0.048	0.013
	$n_2 = 25$	$s_{1a} = 5$	$s_{2a} = 6$		
$case2(r=b)$	$n_1 = 20$	$\mu_{1b} = 5$	$\mu_{2b} = 5$	-0.019	-0.067
	$n_2 = 25$	$s_{1b} = 4$	$s_{2b} = 6$		
$case3(r=c)$	$n_1 = 20$	$\mu_{1c} = 7$	$\mu_{2c} = 7$	-0.037	-0.068
	$n_2 = 20$	$s_{1c} = 6$	$s_{2c} = 7$		

Program 2. This program is related to compute the risk difference $RD_{\bar{X}_1,\hat{\mu}_1} = R(\mu_1, \bar{X}_1) - R(\mu_1, \hat{\mu}_1)$ which are given in Figure 1, where $xx = \mu$ and $yy = RD_{\bar{X}_1,\hat{\mu}_1}$.

```
par(mfrow=c(2,2))

x=c(0:1)
y=c(.0007:.008)
xx=c(.79,1,1.8,0,.95)
yy=c(.0006,.005,.0004,.002,.0003)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(0:1.3)
y=c(.0005:.009)
xx=c(.89,.9,1.25,1)
yy=c(.0006,.005,.0004,.0003)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(1:3.5)
y=c(0:.0004)
xx=c(1.89,1.9,1.25,2.5)
yy=c(.0035,.0022,.0013,.0004)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(2:3.5)
y=c(0:.99)
xx=c(2.25,3.5,3.3,2.4)
yy=c(.03,0,.85,.67)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))

dev.off()
-----------------
par(mfrow=c(2,2))

x=c(0:2.5)
y=c(.0007:.008)
xx=c(.79,1.2,1.8,2.25,2.45)
yy=c(.0007,.0009,.001,.002,.006)
```

```
 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(0:1.5)
y=c(.0005:.009)
xx=c(.89,.9,1.25,1.4)
yy=c(.0006,.0009,.004,.008)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(1:3.5)
y=c(0:.0004)
xx=c(1.89,1.9,2.25,3)
yy=c(.0035,.0022,.0013,.0004)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))
x=c(2:3.5)
y=c(0:.99)
xx=c(2.25,2.5,3.3,3.4)
yy=c(.85,.65,.35,.17)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[1],hat(mu)[1])]),xlab = expression(mu))

dev.off()
```

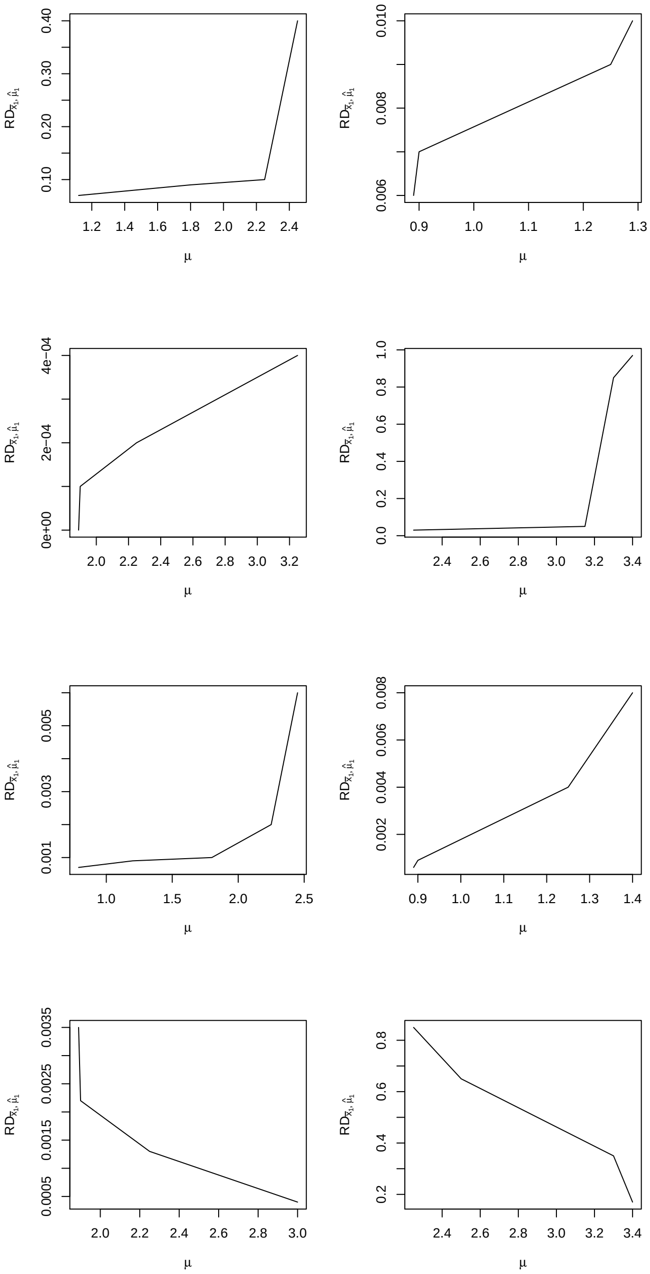

Figure 1: Risk difference $RD_{\bar{X}_1,\hat{\mu}_1} = R(\mu_1, \bar{X}_1) - R(\mu_1, \hat{\mu}_1)$.

Program 3. This program is related to compute the risk difference $RD_{\bar{X}_2,\hat{\mu}_2} = R(\mu_2, \bar{X}_2) - R(\mu_2, \hat{\mu}_2)$ which are given in Figure 2, where $xx = \mu$ and $yy = RD_{\bar{X}_2,\hat{\mu}_2}$.

```
par(mfrow=c(2,2))

x=c(0:3)
y=c(0,05)
xx=c(0,1.5,2.6,2.7,3)
yy=c(0,.02,.04,.04,.05)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(1:3)
y=c(0:.8)
xx=c(1.25,2.5,2.39,2.45,2.9,3)
yy=c(.3,.4,.6,.65,7,.8)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(0:5)
y=c(.0004:.00035)
xx=c(1.75,3,3.5,4.45,4.5,5)
yy=c(.0005,.0007,.00010,.00020,.00025,.00034)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(0:4)
y=c(.2:.8)
xx=c(0,2.5,2.75,3,3.5,4)
yy=c(.2,.4,.5,.6,.7,.8)



 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))

dev.off()
-------------
par(mfrow=c(2,2))

x=c(0:1)
y=c(0:.05)
```

```
xx=c(.65,.75,.95,1)
yy=c(.05,.03,.02,0)

 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(0:1)
y=c(0:.004)
xx=c(0,.1,.2,.5,.75)
yy=c(.004,.0035,.0023,.001,0)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(0:1.2)
y=c(0:.25)
xx=c(0,.6,1.15,1.2)
yy=c(.25,.15,.10,.05)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))
x=c(1:3)
y=c(0:.008)
xx=c(1.12,1.4,1.5,2,2.7)
yy=c(.007,.005,.004,.003,.007)


 plot(xx,yy,type="l",ylab=expression
 (RD[list(bar(x)[2],hat(mu)[2])]),xlab = expression(mu))

dev.off()
```

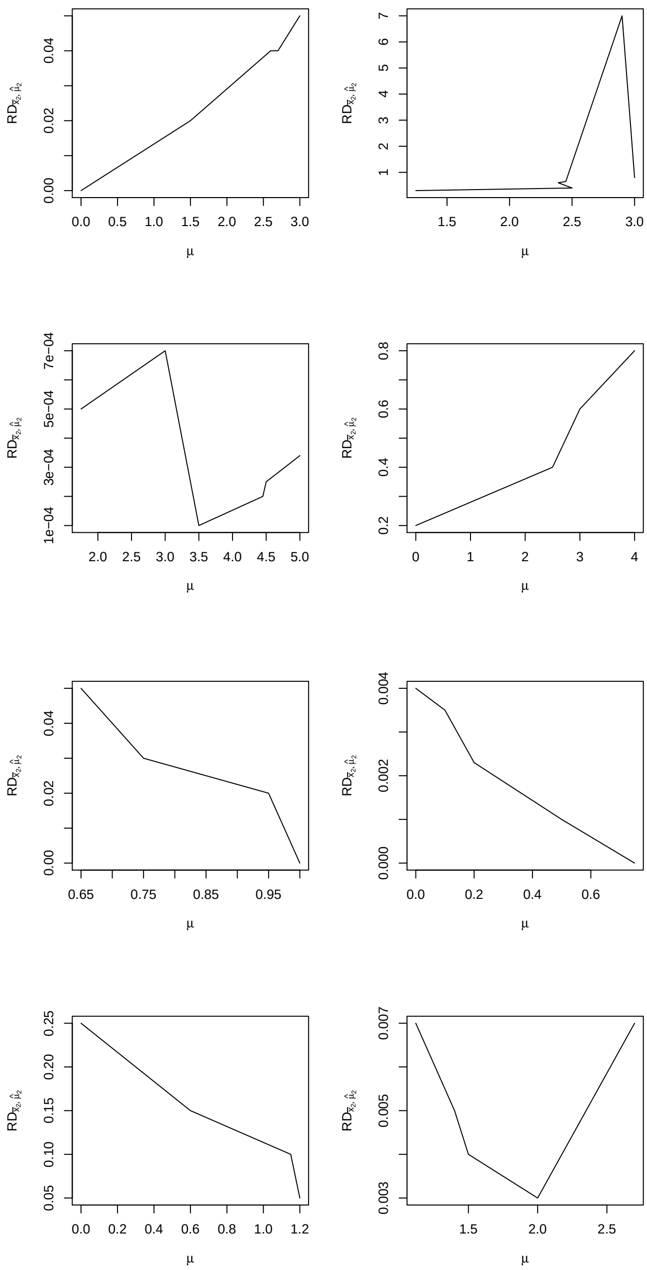

Figure 2: Risk difference $RD_{\bar{X}_2,\hat{\mu}_2} = R(\mu_2, \bar{X}_2) - R(\mu_2, \hat{\mu}_2)$.

Program 4. This program is related to simulate from two univariate normal distributions to obtain the values of $MPN_{\mu_1} = MPN_{\mu_1}(\hat{\mu}_1(\gamma), \bar{X}_1)$ and $MPN_{\mu_2} = MPN_{\mu_2}(\hat{\mu}_2(\gamma), \bar{X}_2)$ which are given in Table 2, where $t = \gamma$.

```
estmu1 = function(x,y,t)
{xbar = mean(x)
ybar = mean(y)
min(xbar,t*xbar+(1-t)*ybar)
}
estmu2 = function(x,y,t)
{xbar = mean(x)
ybar = mean(y)
max(xbar,t*xbar+(1-t)*ybar)
}
n1 = 10;n2 = 20
s1 = 5;s2 = 7
mu1 = 4;mu2 = 5
N = 10000
t = (n1/s1)/((n2/s2)+(n1/s1))
muhat1 = muhat2 = array(0)
z = 0
for(i in 1:N)
{
x = rnorm(n1,mu1,s1)
y = rnorm(n2,mu2,s2)
muhat1[i] <- estmu1(x,y,t)
muhat2[i] <- estmu2(x,y,t)
if((abs(muhat2[i]-mu2) < abs(mean(y)-mu2)))
z=z+1
}
z/N
```

Table 2: Simulation from two univariate normal distributions: the values of $MPN_{\mu_1} = MPN_{\mu_1}(\hat{\mu}_1(\gamma), \bar{X}_1)$ and $MPN_{\mu_2} = MPN_{\mu_2}(\hat{\mu}_2(\gamma), \bar{X}_2)$.

	Sample sizes	$N(\mu_{1r}, s_{1r})$	$N(\mu_{2r}, s_{2r})$	MPN_{μ_1}	MPN_{μ_2}
$case1(r = a)$	$n_1 = 15$	$\mu_{1a} = 6$	$\mu_{2a} = 7$	0.208	0.568
	$n_2 = 15$	$s_{1a} = 4$	$s_{2a} = 5$		
$case2(r = b)$	$n_1 = 10$	$\mu_{1b} = 4$	$\mu_{2b} = 5$	0.252	0.580
	$n_2 = 20$	$s_{1b} = 5$	$s_{2b} = 7$		
$case3(r = c)$	$n_1 = 15$	$\mu_{1c} = 3$	$\mu_{2c} = 3$	0.303	0.618
	$n_2 = 20$	$s_{1c} = 5$	$s_{2c} = 7$		
$case1(r = a)$	$n_1 = 20$	$\mu_{1a} = 9$	$\mu_{2a} = 9$	0.428	0.379
	$n_2 = 25$	$s_{1a} = 7$	$s_{2a} = 4$		
$case2(r = b)$	$n_1 = 20$	$\mu_{1b} = 6$	$\mu_{2b} = 7$	0.188	0.549
	$n_2 = 20$	$s_{1b} = 4$	$s_{2b} = 5$		
$case3(r = c)$	$n_1 = 15$	$\mu_{1c} = 5$	$\mu_{2c} = 7$	0.074	0.372
	$n_2 = 20$	$s_{1c} = 3$	$s_{2c} = 6$		

Program 5. This program is related to compute the values of $MPN_{\mu_1} = MPN_{\mu_1}(\hat{\mu}_1(\gamma), \bar{X}_1)$ which are given in Figure 3, where $xx = \mu$ and $yy = MPN_{\mu_1}$.

```
par(mfrow=c(2,2))

x=c(0:2)
y=c(0:.05)
xx=c(.75,1.2,1.5,2)
yy=c(.04,.03,.02,0)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))


x=c(0:2)
y=c(0:.007)
xx=c(0,1,1.25,1.5,1.75)
yy=c(.006,.005,.004,.002,0)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))


x=c(0:1.2)
y=c(0:.25)
xx=c(0,.5,.65,1.2)
yy=c(.25,.25,.10,.05)
```

```
plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))


x=c(1:5)
y=c(0:.008)
xx=c(1.12,2,3.1,3.5,4.5)
yy=c(.007,.005,.004,.001,0)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))

dev.off()
--------------------
par(mfrow=c(2,2))

x=c(0:2)
y=c(0,05)
xx=c(0,.5,.6,.7,2)
yy=c(0,.02,.04,.04,.05)
plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))

x=c(1:4)
y=c(0:.8)
xx=c(1.25,1.5,2.3,3.4,3.5,4)
yy=c(.8,.6,.5,.4,.2,0)
plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))


x=c(0:6)
y=c(.0004:.00025)
xx=c(.75,1.2,2.5,3.5,4.5,5)
yy=c(.0003,.0002,.0001,.00010,.00015,.00022)
plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))

x=c(0:5)
y=c(.2:.8)
xx=c(0,1.5,1.75,2.4,3.5,4)
yy=c(.7,.6,.5,.3,.25,.2)
plot(xx,yy,type="l", ylab=expression
(MPN[mu[1]]),xlab = expression(mu))
```

```
dev.off()
```

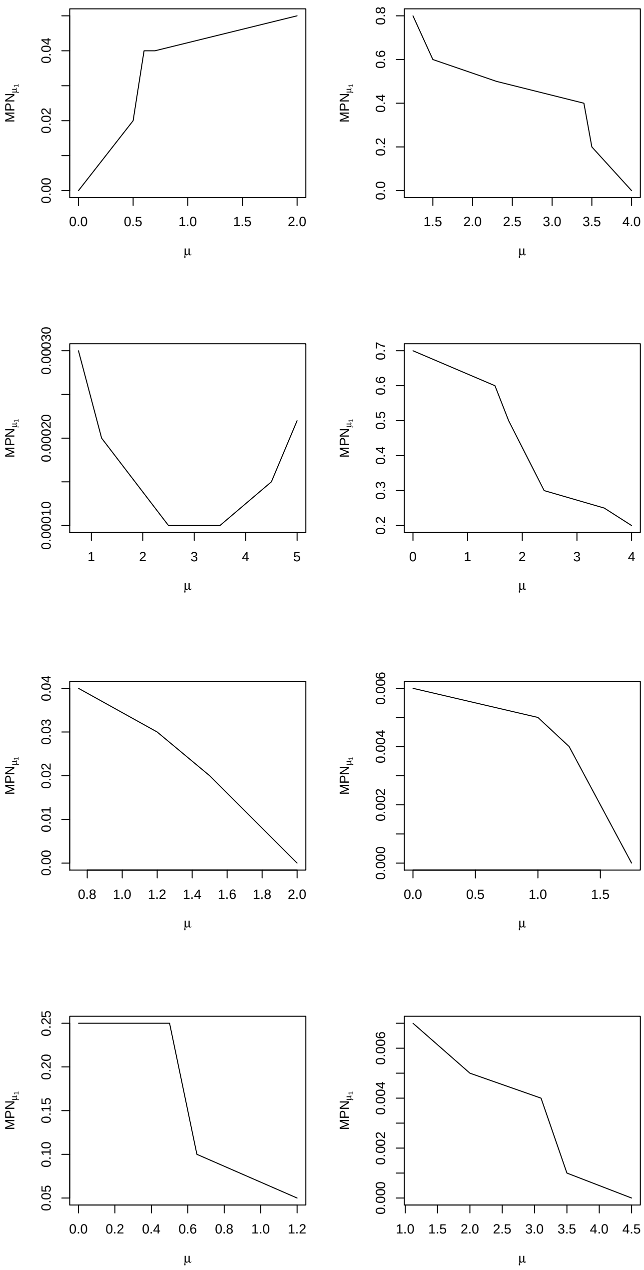

Figure 3: $MPN_{\mu_1} = MPN_{\mu_1}(\hat{\mu}_1(\gamma), \bar{X}_1)$.

Program 6. This program is related to compute the values of $MPN_{\mu_2} = MPN_{\mu_2}(\hat{\mu}_2(\gamma), \bar{X}_2)$ which are given in Figure 4, where $xx = \mu$ and $yy = MPN_{\mu_2}$.

```
par(mfrow=c(2,2))

x=c(.75:2.5)
y=c(.0007:.008)
xx=c(.79,1,1.25,1.5,2.2)
yy=c(.0008,.0009,.0010,.009,.008)


plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))

x=c(0:1.5)
y=c(.0005:.009)
xx=c(.89,.9,1.25,1.5)
yy=c(.0006,.0007,.0009,.002)


plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))


x=c(2:3.5)
y=c(0:.4)
xx=c(2.5,2.9,2.99,3.2)
yy=c(.1,.2,.3,.4)


plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))


x=c(0:1.4)
y=c(0:.7)
xx=c(0,.5,.75,.9,1.3,1.4)
yy=c(.6,.5,.4,.4,.3,.1)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))

dev.off()
```

```
-----------------
par(mfrow=c(2,2))


x=c(0:1.4)
y=c(0:.5)
xx=c(0,.5,.75,1,1.3)
yy=c(.5,.4,.4,.2,.1)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))

x=c(0:1)
y=c(.003:.020)
xx=c(0,.5,.75,1)
yy=c(.005,.007,.008,.010)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))


x=c(0:1.5)
y=c(.003:.015)
xx=c(0,.5,.75,1,1.2,1.4)
yy=c(.004,.007,.009,.010,.014,.015)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))


x=c(0:2.4)
y=c(0:.9)
xx=c(0,.5,.75,1,2.3,2.4)
yy=c(.1,.2,.3,.4,.6,.8)

plot(xx,yy,type="l", ylab=expression
(MPN[mu[2]]),xlab = expression(mu))

dev.off()
```

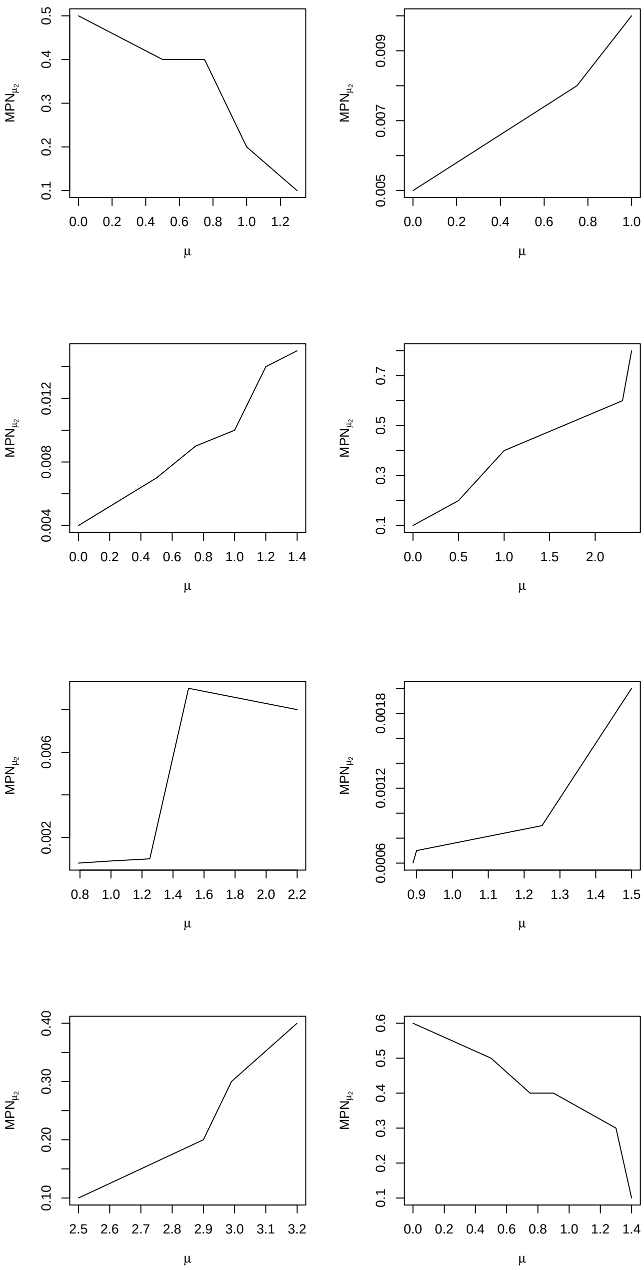

Figure 4: $MPN_{\mu_2} = MPN_{\mu_2}(\hat{\mu}_2(\gamma), \bar{X}_2)$.