AUSTRIAN JOURNAL OF STATISTICS
Volume 40 (2011), Number 3, 169–176

Double Acceptance Sampling Plans Based on Truncated Life Tests for the Marshall-Olkin Extended Exponential Distribution

G. Srinivasa Rao
Hamelmalo Agricultural College, Eritrea

Abstract: In this paper, double acceptance sampling plans are developed for a truncated life test, when the lifetime of an item follows the Marshall-Olkin extended exponential distribution. The probability of acceptance is calculated for different consumer's confidence levels fixing the producer's risk at 0.05. The probability of acceptance and the producer's risk are explained by means of examples.

Zusammenfassung: In diesem Aufsatz werden doppelte Akzeptanz-Stichprobenpläne für einen gestutzten Lifetest entwickelt wenn die Lebensdauer eines Artikels der Marshall-Olkin erweiterten Exponentialverteilung folgt. Die Wahrscheinlichkeit der Akzeptanz ist für verschiedene Konsumenten-Konfidenzniveaus berechnet, wobei das Produzentenrisiko auf 0.05 fixiert ist. Die Wahrscheinlichkeit der Akzeptanz und das Produzentenrisiko werden mittels Beispiele erläutert.

Keywords: Marshall-Olkin Extended Exponential Distribution, Double Acceptance Sampling Plan, Probability of Acceptance, Consumer's Risk, Producer's Risk, Truncated Life Test.

1 Introduction

Acceptance sampling (AS) is an inspecting procedure applied in statistical quality control. AS is a part of operations management and services quality maintenance. It is important for industrial, but also for business purposes helping the decision-making process for the purpose of quality management. Producers are very careful about the quality of their products so that they do not face any difficulty in the acceptance when the consumer comes to buy them. AS is most likely to be useful in the situations when testing is destructive, or when the cost of 100% inspection is extremely high, or when 100% inspection is not technologically feasible or would require so much calendar time that the production schedule would be seriously impacted. Sampling plans are hypothesis tests regarding the product that has been submitted for an appraisal and subsequent acceptance or rejection. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test.

A single acceptance sampling plan (SASP) is a specified plan that establishes the minimum sample size to be used for testing. In most AS plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. On the basis of information obtained from this first sample, we accept or reject the lot. If a good lot is rejected on the basis of this information, its probability is called the type-I error probability (producer's risk) and it is denoted by α. The probability of accepting the bad lot is known as the type-II error probability (consumer's risk) and it is denoted as β. If the product is electronic components or having failure mechanism we put a random sample on test and accept the entire lot if no more than c (AS number) failures occur during the experiment time. More recently, Aslam (2007) proposed double acceptance sampling plans (DASPs) based on truncated life tests when the lifetime of an item follows the Rayleigh distribution and Srinivasa Rao, Ghitany, and Kantam (2009) developed DASPs based on truncated life tests following a Marshall-Olkin extended Lomax distribution.

SASPs based on truncated life tests for a variety of distributions were discussed by Epstein (1954), Sobel and Tischendorf (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam and Rosaiah (1998), Kantam, Rosaiah, and Srinivasa Rao (2001), Baklizi (2003), Baklizi and El Masri (2004), Rosaiah and Kantam (2005), Rosaiah, Kantam, and Santosh Kumar (2006); Rosaiah, Kantam, and Pratapa Reddy (2007); Rosaiah, Kantam, and Santosh Kumar (2007), Tsai and Wu (2006), Balakrishnan, Leiva, and Lopez (2007), Aslam and Kantam (2008) and Srinivasa Rao et al. (2009).

The purpose of this paper is to propose a DASP based on truncated life tests when the lifetime of a product follows the Marshall-Olkin extended exponential distribution introduced by Ghitany, Al-Awadhi, and Alkhalfan (2007) with known shape parameter and to find the probability of acceptance (PA). The probability density function and the cumulative distribution function (cdf) of the Marshall-Olkin extended exponential distribution, are given by

$$g(t;\nu,\sigma) = \frac{\frac{\nu}{\sigma}\exp(-t/\sigma)}{[1-(1-\nu)\exp(-t/\sigma)]^2}\,, \qquad t>0\,,\nu>0\,,\ \sigma>0\,, \tag{1}$$

$$G(t;\nu,\sigma) = \frac{1-\exp(-t/\sigma)}{1-(1-\nu)\exp(-t/\sigma)}\,, \qquad t>0\,,\nu>0\,,\ \sigma>0\,, \tag{2}$$

where σ is the scale parameter and ν is the index parameter. The mean of this distribution is given by $\mu = 1.3863\sigma$ when $\nu = 2$. Srinivasa Rao et al. (2009) studied SASPs based on the Marshall-Olkin extended exponential distribution. An introduction and some methodology for the proposed DASP for life test is given in Section 2. A description by means of tables and examples is contained in Section 3, and finally some conclusions are given in Section 4.

2 The Double Acceptance Sampling Plan for Life Tests

The DASP is used to minimize the risk of the producer, because it provides another opportunity for acceptance of the product. In the DASP a sample with n_1 items is taken from the lot which is called the first sample. This first sample is put on test. Let c_1 and c_2 be the acceptance numbers for the first and the second sample, respectively. We terminate the experiment if no more than c_1 failures occur during the experiment time t_0, i.e. we reject or accept the lot on the basis of sample 1 if more than c_2 failures occur or the time of experiment has ended (whichever occurs earlier). If (c_1+1) failures occur in the first sample then all possibilities for the second sample are given as:

First Sample	Second Sample
(c_1+1) failures occur in sample 1	$< (c_2-1)$ failures in sample 2 are required to accept
(c_1+2) failures occur in sample 1	$< (c_2-2)$ failures must occur in sample 2 to accept

and so on.

Let μ represent the true average life of a product and μ_0 denote the specified life of an item, under the assumption that the lifetime of an item follows the Marshall-Olkin extended exponential distribution. A product is considered as good and accepted for consumer's use if the sample information supports the hypothesis $H_0 : \mu \geq \mu_0$. Otherwise the lot of the product is rejected. In AS schemes, this hypothesis is tested based on the number of failures from a sample in a pre-fixed time. If the number of failures exceeds the action limit c we reject the lot. We accept the lot if there is enough evidence that $\mu \geq \mu_0$ at a certain level of consumer's risk. Otherwise we reject the lot. In order to determine the parameters of the proposed sampling plan we use the consumer's risk. Often the consumer's risk is expressed by the consumer's confidence level. If the confidence level is p^*, then the consumer's risk is $\beta = 1 - p^*$. In this study we fix the consumer's risk such that

$$\Pr(\text{number of failures} \leq c|p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^*\,, \tag{3}$$

where p is the probability that an item fails before termination time.

Consider a life testing experiment having n_1 items in the first sample put on test, with an acceptance number of $c_1 = 0$ in this first sample, and n_2 items in the second sample put on test and we accept the lots if no more than two failures occur in the second sample, i.e. $c_2 = 2$. If no failure occurs in the first sample of n_1 items put on test, we accept the lot. If the true but unknown lifetime of the product deviates from the specified lifetime of the product it should result in a considerable change in the PA of the lot based on the sampling plan. Hence the PA can be regarded as a function of the deviation of the specified average from the true average. This function is called the operating characteristic (OC) function of the sampling plan. Values of the PA for the first sample using the Marshall-Olkin extended exponential distribution with $\nu = 2$ are given in Table 1. Denote the PAs as $L(p_1)$ and $L(p_2)$ for sampling plans $(n_1, c_1, t/\sigma_0)$ and $(n_2, c_2, t/\sigma_0)$, respectively, then

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n-i}\,, \tag{4}$$

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n-i}\,, \tag{5}$$

where, $p = G_T(t;\sigma) = G_T((t/\sigma_0)\cdot(\sigma_0/\sigma))$ is given in (2).

The PA for a DASP can be obtained by using (4) and (5) and is

$$\begin{aligned}\Pr(A) = {} & \Pr(\text{no failure occurs in sample 1}) \\ & + \Pr(\text{1 failure occurs in sample 1 and 0 or 1 failure occurs in sample 2}) \\ & + \Pr(\text{2 failures occur in sample 1 and 0 failure occurs in sample 2})\,.\end{aligned}$$

Values of the PA for a DASP are determined at $p^* = 0.75, 0.90, 0.95, 0.99$ and $t/\sigma_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.142, 3.927, 4.712$ with $\nu = 2$ and are given in Table 2.

It is important to note that in the first sample and in the second sample p is a function of the cdf of the Marshall-Olkin extended exponential distribution. These choices are consistent with Gupta and Groll (1961), Gupta (1962), Kantam et al. (2001), Baklizi and El Masri (2004), Balakrishnan et al. (2007).

3 Description of Tables and Examples

Suppose that the lifetime of a product follows the Marshall-Olkin extended exponential distribution with $\nu = 2$ and an experimenter wants to establish that its true unknown mean life is at least 1000 hours with confidence 0.90. The acceptance numbers for this experiment are $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1 = 12$ and $n_2 = 16$. The lot is accepted if during 628 hours no failure is observed in a sample of 12. The PA for this SASP from Table 1 is 0.13117. The PA for the same setup using a DASP from Table 2 is 0.22724. In a DASP scheme as σ/σ_0 increases the PA also increases. For the above sampling plan, the PA is 0.96961 when the ratio of the unknown average lifetime to the specified average lifetime is 12. When the time of the experiment increases, the PA for a DASP also decreases. From Table 2 it is clear that when the time of the experiment is 4712 hours and the ratio $\sigma/\sigma_0 = 2$, the PA is 0.05633. For the same experiment time, when σ/σ_0 increases the PA also increases. It is important to note that a DASP minimizes the producer's risk, but this scheme also exerts the pressure on the producer to improve the quality level of his product. At 4712 hours and with $\sigma/\sigma_0 = 12$ and $p^* = 0.90$, the PA is 0.64988. The producer's risk for the first sample for $p^* = 0.90$ are given in Table 3. For $\sigma/\sigma_0 = 2$ (if the unknown average lifetime is twice the specified average lifetime) the producer's risks when the times of an experiment are 628 and 4712 hours are 0.77276 and 0.94367, respectively. The producer's risk decreases as the quality level of the product increases with $p^* = 0.90$. Table 3, Figure 1 and Figure 2 illustrate our idea.

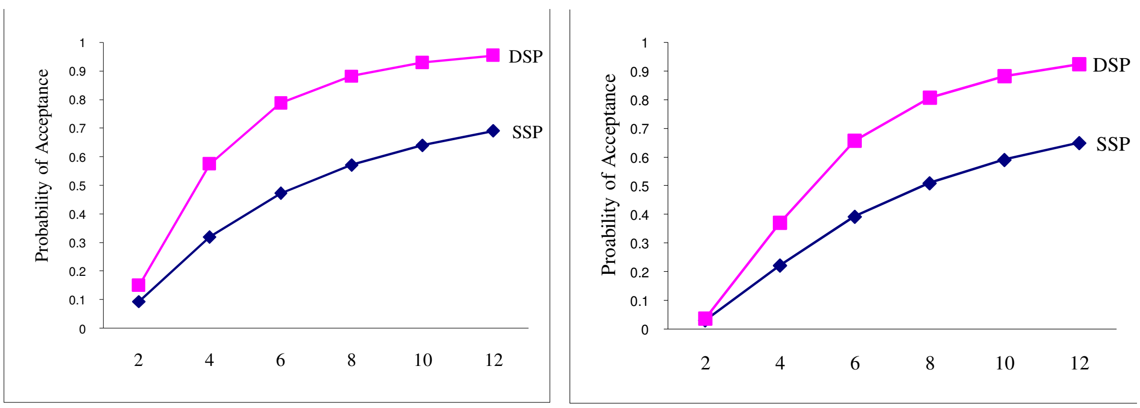

Figure 1: Operating characteristics curve with $p^* = 0.95$ and $t_0 = 628$ hours (left) and with $p^* = 0.95$ and $t_0 = 4712$ hours (right)

Table 1: Operating characteristics for the first sample of the sampling plan $(n_1, c_1, t/\sigma_0)$ when $c_1 = 0$ for the Marshall-Olkin extended exponential distribution with $\nu = 2$

p^*	n_1	t/σ_0	σ/σ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.25816	0.52068	0.65076	0.72603	0.77481	0.80891
	6	0.942	0.20642	0.47331	0.61295	0.69514	0.74882	0.78653
	4	1.257	0.23426	0.50780	0.64345	0.72139	0.77160	0.80656
	4	1.571	0.15383	0.42226	0.57244	0.66232	0.72143	0.76308
	3	2.356	0.10437	0.36358	0.52390	0.62241	0.68786	0.73423
	2	3.142	0.11844	0.39222	0.55352	0.64982	0.71264	0.75660
	2	3.927	0.06060	0.29712	0.46779	0.57665	0.64985	0.70194
	2	4.712	0.02999	0.22168	0.39231	0.50941	0.59085	0.64988
0.90	12	0.628	0.13117	0.37571	0.52496	0.61863	0.68201	0.72752
	8	0.942	0.12199	0.36886	0.52068	0.61579	0.68000	0.72603
	6	1.257	0.11338	0.36186	0.51615	0.61271	0.67777	0.72435
	5	1.571	0.09634	0.34039	0.49792	0.59750	0.66487	0.71320
	3	2.356	0.10437	0.36358	0.52390	0.62241	0.68786	0.73423
	3	3.142	0.04076	0.24563	0.41182	0.52382	0.60160	0.65811
	2	3.927	0.06060	0.29712	0.46779	0.57665	0.64985	0.70194
	2	4.712	0.02999	0.22168	0.39231	0.50941	0.59085	0.64988
0.95	14	0.628	0.09350	0.31915	0.47150	0.57105	0.63986	0.68996
	9	0.942	0.09378	0.32563	0.47989	0.57957	0.64799	0.69755
	7	1.257	0.07888	0.30546	0.46228	0.56467	0.63523	0.68645
	6	1.571	0.06034	0.27439	0.43311	0.53902	0.61275	0.66658
	4	2.356	0.04914	0.25950	0.42234	0.53142	0.60720	0.66238
	3	3.142	0.04076	0.24563	0.41182	0.52382	0.60160	0.65811
	3	3.927	0.01492	0.16196	0.31995	0.43789	0.52387	0.58810
	2	4.712	0.02999	0.22168	0.39231	0.50941	0.59085	0.64988
0.99	19	0.628	0.04011	0.21225	0.36047	0.46749	0.54555	0.60431
	13	0.942	0.03276	0.19777	0.34628	0.45480	0.53435	0.59437
	9	1.257	0.03818	0.21767	0.37082	0.47960	0.55799	0.61649
	7	1.571	0.03779	0.22119	0.37673	0.48627	0.56472	0.62301
	5	2.356	0.02314	0.18521	0.34047	0.45373	0.53601	0.59757
	4	3.142	0.01403	0.15383	0.30639	0.42226	0.50786	0.57244
	3	3.927	0.01492	0.16196	0.31995	0.43789	0.52387	0.58810
	3	4.712	0.00519	0.10437	0.24572	0.36358	0.45416	0.52390

4 Conclusion

We find the AS plans for various values of σ/σ_0 and different experiment times assuming that the life test follows the Marshall-Olkin extended exponential distribution. This distribution provides the high probability for $\sigma/\sigma_0 > 6$.

Table 2: Operating characteristics for the second sample of the sampling plan $(n_2, c_2, t/\sigma_0)$ when $c_1 = 0$ and $c_2 = 2$ for the Marshall-Olkin extended exponential distribution with $\nu = 2$

p^*	n_1	n_2	t/σ_0	σ/σ_0					
				2	4	6	8	10	12
0.75	8	12	0.628	0.45096	0.82895	0.93048	0.96560	0.98062	0.98804
	6	8	0.942	0.39539	0.80628	0.92098	0.96089	0.97797	0.98641
	4	6	1.257	0.43317	0.83137	0.93356	0.96775	0.98206	0.98903
	4	5	1.571	0.32643	0.77674	0.90904	0.95517	0.97484	0.98453
	3	4	2.356	0.21884	0.70478	0.87543	0.93778	0.96485	0.97831
	2	3	3.142	0.25957	0.74736	0.89933	0.95140	0.97315	0.98369
	2	3	3.927	0.12549	0.61206	0.82982	0.91416	0.95142	0.97004
	2	3	4.712	0.05633	0.47827	0.74747	0.86655	0.92250	0.95143
0.90	12	16	0.628	0.22724	0.66946	0.84673	0.91868	0.95216	0.96961
	8	11	0.942	0.20922	0.65970	0.84288	0.91698	0.95132	0.96915
	6	8	1.257	0.20470	0.66556	0.84839	0.92083	0.95394	0.97098
	5	7	1.571	0.16554	0.62694	0.82734	0.90900	0.94679	0.96637
	3	5	2.356	0.16742	0.63960	0.83823	0.91651	0.95189	0.96991
	3	4	3.142	0.07518	0.51513	0.76617	0.87537	0.92692	0.95381
	2	4	3.927	0.08075	0.50533	0.75912	0.87145	0.92470	0.95248
	2	3	4.712	0.05633	0.47827	0.74747	0.86655	0.92250	0.95143
0.95	14	19	0.628	0.14872	0.57448	0.78747	0.88255	0.92912	0.95418
	9	12	0.942	0.15666	0.59797	0.80539	0.89450	0.93714	0.95973
	7	9	1.257	0.13649	0.58175	0.79765	0.89057	0.93494	0.95840
	6	8	1.571	0.09748	0.52331	0.76080	0.86828	0.92086	0.94905
	4	5	2.356	0.09159	0.53764	0.77683	0.88036	0.92945	0.95519
	3	4	3.142	0.07518	0.51513	0.76617	0.87537	0.92692	0.95381
	3	4	3.927	0.02313	0.34713	0.64125	0.79562	0.87541	0.91925
	2	4	4.712	0.03578	0.37001	0.65774	0.80684	0.88314	0.92472
0.99	19	25	0.628	0.05495	0.38573	0.64428	0.78598	0.86364	0.90850
	13	16	0.942	0.04727	0.37987	0.64398	0.78743	0.86536	0.91006
	9	12	1.257	0.05363	0.40491	0.66761	0.80508	0.87804	0.91925
	7	10	1.571	0.05062	0.39926	0.66493	0.80405	0.87770	0.91919
	5	7	2.356	0.03055	0.34676	0.62707	0.78021	0.86246	0.90904
	4	5	3.142	0.02094	0.32643	0.61752	0.77674	0.86148	0.90904
	3	4	3.927	0.02313	0.34713	0.64125	0.79562	0.87541	0.91925
	3	4	4.712	0.00686	0.21884	0.51528	0.70478	0.81272	0.87543

Table 3: Producer's risk with respect to time of the experiment for double sampling with $p^* = 0.90$

c_1	c_2	n_1	n_2	t/σ_0	σ/σ_0					
					2	4	6	8	10	12
0	2	12	16	0.628	0.77276	0.33054	0.15327	0.08132	0.04784	0.03039
0	2	2	3	4.712	0.94367	0.52173	0.25253	0.13345	0.07750	0.04857

References

Aslam, M. (2007). Double acceptance sampling based on truncated life tests in Rayleigh distribution. *European Journal of Scientific Research*, *17*, 605-611.

Aslam, M., and Kantam, R. R. L. (2008). Economic reliability acceptance sampling based on truncated life tests in the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, *24*, 269-276.

Baklizi, A. (2003). Acceptance sampling based on truncated life tests in the Pareto distribution of the second kind. *Advances and Applications in Statistics*, *3*, 33-48.

Baklizi, A., and El Masri, A. E. K. (2004). Acceptance sampling based on truncated life tests in the Birnbaum-Saunders model. *Risk Analysis*, *24*, 1453-1457.

Balakrishnan, N., Leiva, V., and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution. *Communication in Statistics, Simulation and Computation*, *36*, 643-656.

Epstein, B. (1954). Truncated life tests in the exponential case. *Annals of Mathematical Statistics*, *25*, 555-564.

Ghitany, M. E., Al-Awadhi, F. A., and Alkhalfan, L. A. (2007). Marshal-Olkin extended Lomax distribution and its application to censored data. *Communications in Statistics, Theory and Methods*, *36*, 1855-1866.

Goode, H. P., and Kao, J. H. K. (1961). *Sampling plans based on the Weibull distribution.* Philadelphia, Pennsylvania. (24-40)

Gupta, S. S. (1962). Life test sampling plans for normal and lognormal distribution. *Technometrics*, *4*, 151-175.

Gupta, S. S., and Groll, P. a. (1961). Gamma distribution in acceptance sampling based on life tests. *Journal of the American Statistical Association*, *56*, 942-970.

Kantam, R. R. L., and Rosaiah, K. (1998). Half logistic distribution in acceptance sampling based on life tests. *IAPQR Transactions*, *23*, 117-125.

Kantam, R. R. L., Rosaiah, K., and Srinivasa Rao, G. (2001). Acceptance sampling based on life tests: Log-logistic models. *Journal of Applied Statistics*, *28*, 121-128.

Rosaiah, K., and Kantam, R. R. L. (2005). Acceptance sampling based on the inverse Rayleigh distribution. *Economic Quality Control*, *20*, 277-286.

Rosaiah, K., Kantam, R. R. L., and Pratapa Reddy, J. (2007). Economic reliability test plan with inverse Rayleigh variate. *Pakistan Journal of Statistics*, *24*, 57-65.

Rosaiah, K., Kantam, R. R. L., and Santosh Kumar, C. (2006). Reliability of test plans for exponentiated log-logistic distribution. *Economic Quality Control*, *21*, 165-175.

Rosaiah, K., Kantam, R. R. L., and Santosh Kumar, C. (2007). Exponentiated log-logistic distribution – an economic reliability test plan. *Pakistan Journal of Statistics*, *23*, 147-156.

Sobel, M., and Tischendorf, J. A. (1959). Acceptance sampling with new life test objectives. In *Proceedings of fifth national symposium on reliability and quality control.* Philadelphia, Pennsylvania. (108-118)

Srinivasa Rao, G., Ghitany, M. E., and Kantam, R. R. L. (2009). Reliability test plans for Marshall-Olkin extended exponential distribution. *Applied Mathematical Sciences*, *3*, 2745-2755.

Tsai, T. R., and Wu, S. J. (2006). Acceptance sampling based on truncated life tests for generalized Rayleigh distribution. *Journal of Applied Statistics*, *33*, 595-600.

Author's address:

G. Srinivasa Rao
Department of Basic Sciences
Hamelmalo Agricultural College
PO Box 397
Keren, Eritrea

E-mail: gaddesrao@yahoo.com