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Stochastic Modeling and Availability Optimization of Nuts Manufacturing Plant Using Markov Process and Particle Swarm Optimization

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Abstract

Availability is the key system effectiveness measure in process industries, manufacturing plants, and treatment plants like sewage, e-waste etc. The nut-bolt manufacturing industry is very prominent in manufacturing sector. The present work is proposed with a motto to develop a stochastic framework for a nut manufacturing plant to derive steady state availability and its optimization. The Markov birth death approach is applied to develop the stochastic model as well as Chapman-Kolmogorov differential difference equations of the system. The availability function derived using Markov approach is treated as objective function of the optimization problem having decision parameters as failure and repair rates. All the decision variables are considered as exponentially distributed which are i.i.d. in nature. The objective function is optimized using particle swarm optimization to predict the optimal availability and estimated parametric values. The most sensitive component of the system is observed through making variation in failure and repair rates. It is revealed that PSO predicts the optimal availability 0.9999 at population size 50 after 50 iterations. The convergence rate of PSO is very fast in prediction of the availability of nut manufacturing plant. These findings are beneficial for system designers and maintenance engineers to propose the maintenance strategies. The proposed methodology can be utilized to predict the availability of other process industries.

Keywords: nuts manufacturing plant, availability, particle swarm optimization, Markov birth-death process.

1. Introduction

Reliability and maintainability play an important role in product's designing, manufacturing, and planning of maintenance strategies (Saini, Kumar, and Barak 2023). In all the manufacturing, automobile, and production industries to fasten the components nuts and bolts are most frequently used. Nuts are used in making a small electric board to large aircraft. Nut is a small metal item having a hole in the middle. The middle hole has grooved holes named as threads. It cannot be utilized without bolts. The stem of bolt and inside portion of nuts are always circular so that they can easily rotate and held together with a frication of threads. Nuts are very important mechanical tools in mechanical sector. With the help of hot and cold forging, dust proof nuts are made. Nut manufacturing process is very complex due to involvement of various components. As well as the failure of any of its component resulted in the failure of the production line and then impact the allied industries also. So, in such situations, it becomes necessary to operate nut manufacturing plant with utmost care having high reliability.

Several studies conducted related to the performance of mechanical procedure of nuts. Cabrera, Tizani, Mahmood, and Shamsudin (2020) investigated the combined failure mode of the extended hollow bolt and the effect of the column thickness on the tensile behaviour of the blind fastener. Here, a three-dimensional finite element model is proposed and concluded that the failure mode is first controlled by the plastic resistance of the component limited by concrete crushing accompanied with hollow section yielding. Elaziz, Elsheikh, Oliva, Abualigah, Lu, and A. Ewees (2021) discussed various advanced metaheuristic techniques used for performance evaluation of mechanical design problems like PSO etc. Five different mechanical design topics have been discussed considering the utilization of metaheuristic algorithms to improve the design accuracy. He, Huang, Wang, Wang, and Guan (2021) developed a new approach based on stochastic response surface for reliability evaluation. Mishra and Satapathy (2021) proposed a multi-criteria decision-making approach for reliability and maintenance optimization of agricultural machineries. Saini, Sinwar, Swarith, and Kumar (2022b) proposed a methodology based on time between failure and time to repair to investigate the reliability of LHD machines. The parameters of best fitted distribution identified using Anderson-Darling test and optimized reliability using GA and PSO. Kumar, Sinwar, Saini, Saini, Kumar, Kaur, Singh, and Lee (2022b) developed an efficient model for reliability evaluation of E-waste management plant using nature inspired algorithms. Zhou, Zhou, Wang, Feng, and Xie (2022) perform the reliability modelling to inspect the degradation status of ball screws. Sauter, Schmitz, Dikici, Baumgartl, and Buettner (2021) proposed a process for defect detection in metal nuts manufactured in small industry using neural networks. Guang, Zhong, and Ding (2004) optimized the timings of the tool changing equipment using genetic algorithm in nut manufacturing unit. Lin, Zhao, Sun, and Chen (2022) evaluated the reliability of anti-loosing performance of bolted joints and suggested an evaluation method. Pérez-Lechuga, Venegas-Martínez, and Martínez-Sánchez (2021) developed a mathematical model using Markov chain approach for manufacturing lines of process industries. Garg (2014) used PSO and fuzzy theory for RAM analysis of industrial systems. Kumar and Tewari (2017) used particle swarm optimization and Markovian approach for performance optimization for CSDGB filling system in beverage plants. Kumar, Saini, Gupta, Sinwar, Singh, Kaur, and Lee (2022a) optimized the availability of cooling tower using GA and PSO. Saini, Goyal, Kumar, and Patil (2022a) explored the applications of GA and PSO in biological and chemical processing unit of sewage treatment plants. Kumar (2020) developed a mathematical model for performance optimization of rice mill and optimize the performance of plant using PSO. Saini *et al.* (2022b) developed a stochastic model for condenser unit of steam turbine power plants and explored the utility of PSO in performance optimization. Sinwar, Saini, Singh, Goyal, and Kumar (2021) developed a Markov model for physical processing unit of STP and used GA and PSO. Saini, Raghav, Kumar, and Chandnani (2021) optimized the availability of a urea decomposition unit of fertilizer plants using metaheuristics. Jagtap, Bewoor, Kumar, Ahmadi, and Lorenzini (2020) studied a coal-fired thermal plant using Markovian methodology and optimize the availability in availability optimization. Aggarwal and Kumar (2021) and Kumar, Kumar, and Modgil (2019) explored the applicability of PSO and soft computing in complex industries like crushing unit, sugar plants. Kumar (2018) optimized the performance of a pre-heater exchanger system under arbitrary failure and repair laws. Garg (2017) discovered some hybrid soft computing techniques for performance evaluation of industrial systems. Kumar, Negi, Pant, Ram, and Dimri (2021) optimized the cost of butter oil processing system using NIAs. Garg, Rani, and Sharma (2017) and Singhal and Sharma (2019) proposed an efficient approach for availability evaluation using fuzzy differential equations and PSO. It is observed that nut manufacturing process and impact of reliability on it not so much explored so far. Therefore, here nuts manufacturing industry is studies, and its performance is optimized.

By keeping above facts and figures in mind, present work is proposed with a motto to develop a stochastic framework for a nut manufacturing plant to derive steady state availability and its optimization. The Markov birth death approach is applied to develop the stochastic model as well as Chapman-Kolmogorov differential difference equations of the system. The availability function derived using Markov approach is treated as objective function of the optimization problem having decision parameters as failure and repair rates. All the decision variables are considered as exponentially distributed which are i.i.d. in nature. The objective function is optimized using particle swarm optimization to predict the optimal availability and estimated parametric values as PSO is very efficient to optimize the non-linear and complex problems. The most sensitive component of the system is observed through making variation in failure and repair rates. It is revealed that PSO predicts the optimal availability 0.9999 at population size 50 after 50 iterations. The convergence rate of PSO is very fast in prediction of the availability of nut manufacturing plant. These findings are beneficial for system designers and maintenance engineers to propose the maintenance strategies. The proposed methodology can be utilized to predict the availability of other process industries.

The whole manuscript is divided into five sections including the present introduction section. Section 2 incorporates material and methods while section 3 presented the steady state availability analysis of nuts manufacturing plant. Section 4 appended results and discussion whereas section 5 presented the conclusion of study.

2. Material and methods

2.1. Notations

The following notations used for development of transition and mathematical model for nut manufacturing industry.

Table 1: Nomenclature

S. No.	Sub-system	Code		Failure rate/hr. $\boldsymbol{\alpha_i}$	Repair rate/hr. $\boldsymbol{\beta_i}$
		Operative Mode	Failed Mode		
1	Receipt of raw material & storage	A	a	α_1	β_1
2	Raw material inspection	B	b	α_2	β_2
3	Blank cutting	C	c	α_3	β_3
4	Heat treatment	D	d	α_4	β_4
5	CNC machine	E	e	$2\alpha_5$	β_5
		E'	e	α_5	β_5
6	Hex milling	F	f	α_6	β_6
		F'	f	α	β

S. No.	Sub-system	Code		Failure rate/hr. $\boldsymbol{\alpha_i}$	Repair rate/hr. $\boldsymbol{\beta_i}$
		Operative Mode	Failed Mode		
7	Surface protection	G	G	α_7	β_7
8	Final inspection	H	H	α_8	β_8
9	Packing & Dispatch	I	i	α_9	β_9
	$P_0'(t)$: Derivative of the $P_0(t)$				
	$P_0(t)$: At time t, the system at 0 state				
	○ : Operative state				
	◇ : Partially failed state				
	□ : Failed state				

2.2. System description

In this section, configuration of the nut manufacturing industry is discussed and presented. It is a very complex system having nine units namely receipt of raw material & storage, raw material inspection, blank cutting, CNC machine, hex milling, surface protection, final inspection, and packing & dispatch. The detailed description is as follows:

Receipt of raw material & storage (RRMS)

It is the first step in the nuts manufacturing process. Here we check the receipt and quantity of the raw material (RRMS) and store it to a safe place to maintain their surface. RRMS undergoes phosphating and wire drawing process. In proposed system a single RRMS unit available and its failure resulted complete system failure. It's failure and repair rates follow exponential distribution.

Raw material inspection (RMI)

After completing the first process, manufacturing process proceed to the 2nd step i.e., raw material inspection. In this process, all the major factor like gate entry, quality of RM and unload it to the allocated location are checked by inspection team. Failure cause may be cracks/scratches, chemical composition, pitting marks and rough surface. In proposed system RMI is considered as a single unit and its failure causes complete shutdown. The failure and repair rate follow exponential distribution.

Blank cutting (BC)

It is the 3rd stage of nut manufacturing process. In blank cutting, the coil wire is cut into a particular size. It can be done in transformative and including subtractive method. In proposed system BC is considered as a single unit and its failure causes complete shutdown. The failure and repairs rate follow exponential distribution.

Heat treatment (HT)

It is the process of heating the metal, holding it at that temperature, convert to the required shape with the help of die and then cooling it back i.e. this is the process that metal part will undergo change in its mechanical properties. Here, we check the hardness of metal, components life, parts will break or not and heating time. In proposed system HT is considered as a single unit and its failure causes complete shutdown. The failure and repair rates follow exponential distribution.

Computer numerical control (CNC) machine

It is the process of creating thread as per the length required on nuts. In this all the functions like proper machine maintenance, component life, fitment problem, setting of die as per the required size and right offset is checked. In proposed system, two CNC machines work in parallel redundancy. The failure and repair rates of both machines are same and exponentially distributed.

Hex milling (HM)

It is the 6th stage of manufacturing nuts. Hex milling is the process where nuts are inserted in the shape of hex. It comprises two non-identical components having distinct failure and repair rates. The failure and repair rates are exponentially distributed.

Surface protection (SP)

In this stage, surface protection is done to protect it from rust and make it look good. Here plating like zinc and trivalent yellow passivation is done. In proposed system SP is considered as a single unit and its failure causes complete shutdown. The failure and repairs rate follows exponentially distribution.

Final inspection (FI)

After manufacturing of nuts final inspection is done. In this stage, the product is tested as per the customer's recruitments. Here the cause of failure may be lack of awareness, components mismatch, loss of production and customer line stoppage. In proposed system FI is considered as a single unit and its failure causes complete shutdown. The failure and repairs rate follows exponential distribution.

Packing & dispatch (PD)

In last packing & dispatch is done. After checking mixing, quantity, washing, quality nuts are packed into a box and ready for dispatch. In proposed system PD is considered as a single unit and its failure causes complete shutdown. The failure and repair rates follow exponential distribution.

The failure and repair rates range given in Table 2.

Table 2: Failure and repair rates of various subsystems

Subsystems	RAMD and Markov Analysis Base values		Search Space	
	Failure Rates	Repair rates	Failure Rates	Repair rates
Receipt of raw material & storage unit	$\alpha_1 = 0.00312$	$\beta_1 = 0.256$	[0.00001-0.91]	[0.00001-1.781]
Raw material inspection unit	$\alpha_2 = 0.00412$	$\beta_2 = 0.314$	[0.00001-0.81]	[0.00001-2.845]
Blank cutting unit	$\alpha_3 = 0.00916$	$\beta_3 = 0.518$	[0.00001-0.95]	[0.00001-1.765]
Heat treatment	$\alpha_4 = 0.00301$	$\beta_4 = 0.413$	[0.00001-0.97]	[0.00001-1.854]
CNC machine	$\alpha_5 = 0.00817$	$\beta_5 = 0.167$	[0.00001-0.96]	[0.00001-2.913]
Hex milling	$\alpha_6 = 0.00768$	$\beta_6 = 0.378$	[0.00001-0.94]	[0.00001-1.748]
	$\alpha = 0.0129$	$\beta = 0.267$	[0.00001-0.87]	[0.00001-2.981]
Surface protection unit	$\alpha_7 = 0.00767$	$\beta_7 = 0.299$	[0.00001-0.91]	[0.00001-2.852]
Final inspection	$\alpha_8 = 0.00563$	$\beta_8 = 0.319$	[0.00001-0.15]	[0.00001-3.962]
Packing & Dispatch	$\alpha_9 = 0.00997$	$\beta_9 = 0.407$	[0.00001-0.257]	[0.00001-1.853]

2.3. Particle swarm optimization (PSO)

The development of optimization models to solve complex engineering problems is quite challenging. However, the advent of state-of-the-art meta-heuristic algorithms enables the task of optimizing various engineering problems. Meta-heuristics are generic methods that may be used to solve complex and challenging combinatorial search problems. These algorithms are not only approximate but also provide stochastic solutions. Meta-heuristic-based approaches can be classified into two main approaches viz. single-solution-based approaches and population-based approaches. As compared to single-solution approaches, the population-based approaches are more popular due to their numerous advantages such as generating multiple candidate solutions, avoiding local optima, and greater exploration capabilities. Keeping in view of such advantages, the well-known particle swarm optimization (PSO) is utilized here to optimize the availability of nut manufacturing industry. Inspired from the social behaviour of bird flocking, the PSO is developed by Kennedy and Eberhart (1995). Here the individuals of problem-space are termed as 'particles' that coordinate themselves in achieving the optimal solution. The population of particles is referred as 'swarm'. In the search of optimal solution, the particles flown in search space and adjusts their positions according to their own experiences (personal best) as well as from the global experiences (global best). Personal best indicates the best position of a particle whereas the global best indicates the global best position based on the cumulative experiences of all particles. Let $x_i(t)$ represents the position of a particle i in the search space at time step t. The new position is obtained by adding the velocity $v_i(t)$ to the current position of x_i as given in eq. (1) as follows:

$$x_i(t+1) = x_i(t) + v_i(t+1)\,. \tag{1}$$

To update the position of particles, the velocity equation is characterized by $v_i(t+1)$ as given in eq. (2) as follows:

$$v_i(t+1) = w \cdot v_i(t) + c_1(p_i(t) - x_i(t)) + c_2(g(t) - x_i(t)). \tag{2}$$

Here, w represents the inertia coefficient, $v_i(t)$ the initial velocity, c_1 and c_2 are acceleration coefficients, $p_i(t)$ indicates the personal best and $g(t)$ indicates the global best. The coefficients help achieving efficiency followed by an optimal solution. The detailed working of PSO is shown in Figure 1.

2.4. Assumptions

The study is performed under the following assumptions:

- The failure and repair rates are exponentially distributed and i.i.d in nature as well as no concurrent failures reported (Kumar, Singh, and Singh 1988).
- Switch devices and repairs are perfect (Sandler 1963)
- Sufficient repair facilities available and systems works as new after repair (Srinath 1991).

3. Steady state availability analysis of nut manufacturing plant

The most important metric of system effectiveness is availability that is not only affected by the rate of malfunctions and restorations but also by the amount of time that passes. Hence, before coming to any conclusions concerning the system's performance, it is essential to look through its steady state availability. The assessment of availability of various systems is done by various techniques among which Markov birth-death process is recommended when failure and repair rates are exponentially distributed. So, here a mathematical model is developed for nut manufacturing plant using Markov approach. Figure 2 shows a state transition diagram constructed utilising abstractions and assumptions outlined in material and methods.

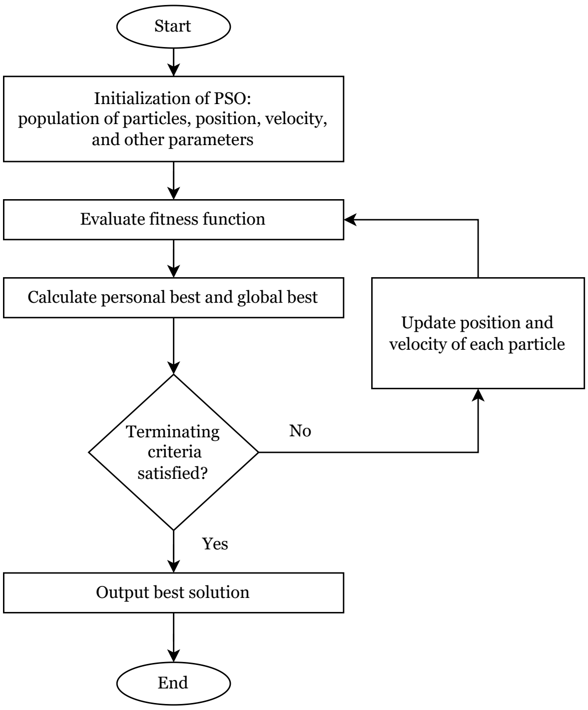

Figure 1: Flowchart of particle swarm optimization

The system description is also presented in Section 2.2. Each state change corresponded with a given rate parameter. The mathematical model is created with the use of a changeover diagram as well as some elementary probability arguments. The differential equations described as follows:

$$\begin{aligned}
P_0\left(t+\delta t\right) &= (1-(\alpha_1+\alpha_2+\alpha_3+\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8+\alpha_9)\delta t)P_0(t)\\
&\quad+\boldsymbol{\beta}_1P_1(t)\delta t+\boldsymbol{\beta}_2P_2(t)\delta t+\boldsymbol{\beta}_3P_3(t)\delta t+\boldsymbol{\beta}_4P_4(t)\delta t+\boldsymbol{\beta}_5P_5(t)\delta t\\
&\quad+\boldsymbol{\beta}_6P_6(t)\delta t+\boldsymbol{\beta}_7P_7(t)\delta t+\boldsymbol{\beta}_8P_8(t)\delta t+\boldsymbol{\beta}_9P_9(t)\delta t,\\
\frac{P_0\left(t+\delta t\right)-P_0(t)}{\delta t} &= (-\alpha_1-\alpha_2-\alpha_3-\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)P_0(t)\\
&\quad+\boldsymbol{\beta}_1P_1(t)+\boldsymbol{\beta}_2P_2(t)+\boldsymbol{\beta}_3P_3(t)+\boldsymbol{\beta}_4P_4(t)+\boldsymbol{\beta}_5P_5(t)+\boldsymbol{\beta}_6P_6(t)\\
&\quad+\boldsymbol{\beta}_7P_7(t)+\boldsymbol{\beta}_8P_8(t)+\boldsymbol{\beta}_9P_9(t).
\end{aligned}$$

Taking limit $\delta t\to 0$, we get

$$\begin{aligned}
P_0'(t) &= (-\alpha_1-\alpha_2-\alpha_3-\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)P_0(t)+\boldsymbol{\beta}_1P_1(t)+\boldsymbol{\beta}_2P_2(t)\\
&\quad+\boldsymbol{\beta}_3P_3(t)+\boldsymbol{\beta}_4P_4(t)+\boldsymbol{\beta}_5P_5(t)+\boldsymbol{\beta}_6P_6(t)+\boldsymbol{\beta}_7P_7(t)+\boldsymbol{\beta}_8P_8(t)+\boldsymbol{\beta}_9P_9(t).
\end{aligned} \tag{3}$$

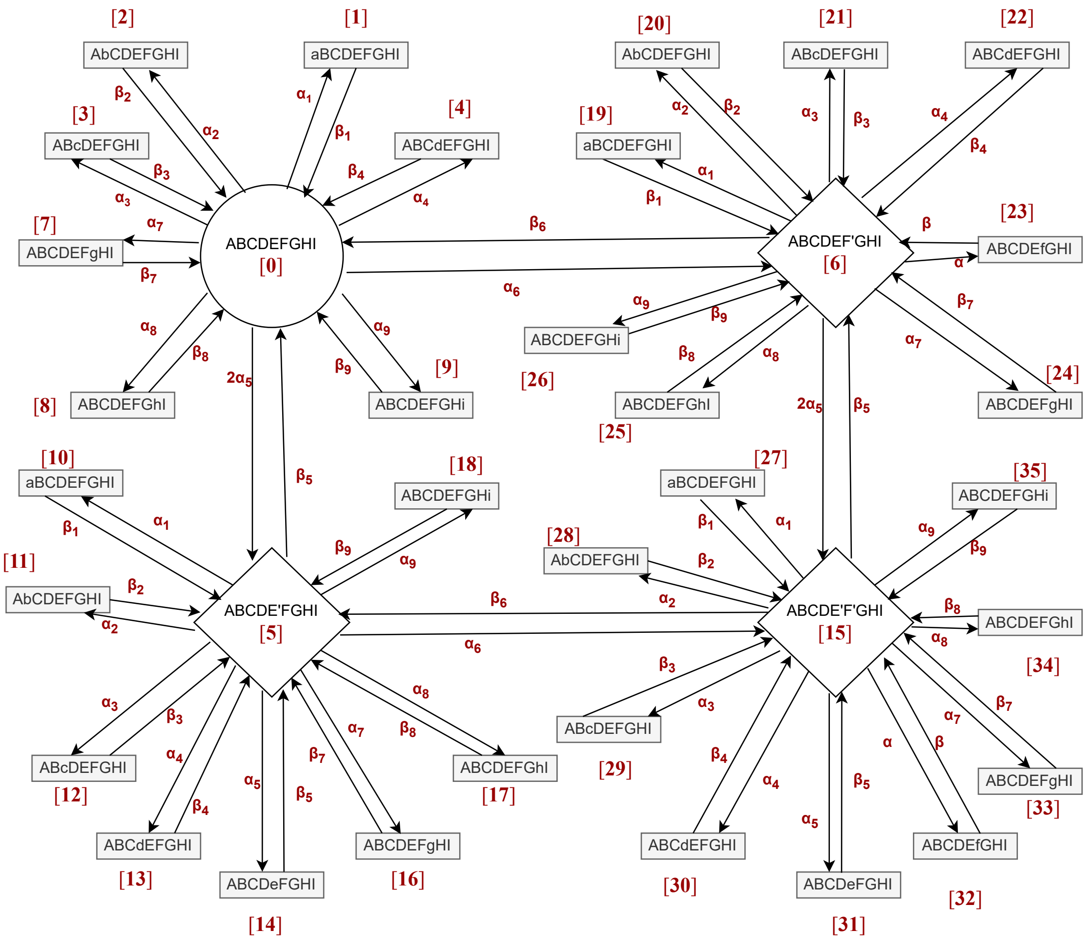

Figure 2: State transition diagram of nuts manufacturing plant

Similarly,

$$\begin{aligned}
P_1'(t) &= -\boldsymbol{\beta}_1 P_1(t) + \boldsymbol{\alpha}_1 P_0(t), && (4)\\
P_2'(t) &= -\boldsymbol{\beta}_2 P_2(t) + \boldsymbol{\alpha}_2 P_0(t), && (5)\\
P_3'(t) &= -\boldsymbol{\beta}_3 P_3(t) + \boldsymbol{\alpha}_3 P_0(t), && (6)\\
P_4'(t) &= -\boldsymbol{\beta}_4 P_4(t) + \boldsymbol{\alpha}_4 P_0(t), && (7)\\
P_5'(t) &= (-\boldsymbol{\beta}_5 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 - \alpha_6 - \alpha_7 - \alpha_8 - \alpha_9) P_5(t)\\
&\quad + \mathbf{2}\alpha_5 P_0(t) + \boldsymbol{\beta}_1 P_{10}(t) + \boldsymbol{\beta}_2 P_{11}(t) + \boldsymbol{\beta}_3 P_{12}(t) + \boldsymbol{\beta}_4 P_{13}(t)\\
&\quad + \boldsymbol{\beta}_5 P_{14}(t) + \boldsymbol{\beta}_6 P_{15}(t) + \boldsymbol{\beta}_7 P_{16}(t) + \boldsymbol{\beta}_8 P_{17}(t) + \boldsymbol{\beta}_9 P_{18}(t), && (8)\\
P_6'(t) &= (\boldsymbol{\beta}_6 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha - 2\alpha_5 - \alpha_7 - \alpha_8 - \alpha_9) P_6(t) + \alpha_6 P_0(t)\\
&\quad + \boldsymbol{\beta}_1 P_{19}(t) + \boldsymbol{\beta}_2 P_{20}(t) + \boldsymbol{\beta}_3 P_{21}(t) + \boldsymbol{\beta}_4 P_{22}(t) + \boldsymbol{\beta} P_{23}(t) + \boldsymbol{\beta}_7 P_{24}(t)\\
&\quad + \boldsymbol{\beta}_5 P_{15}(t) + \boldsymbol{\beta}_8 P_{25}(t) + \boldsymbol{\beta}_9 P_{26}(t), && (9)\\
P_7'(t) &= -\boldsymbol{\beta}_7 P_7(t) + \boldsymbol{\alpha}_7 P_0(t), && (10)\\
P_8'(t) &= -\boldsymbol{\beta}_8 P_8(t) + \boldsymbol{\alpha}_8 P_0(t), && (11)\\
P_9'(t) &= -\boldsymbol{\beta}_9 P_9(t) + \boldsymbol{\alpha}_9 P_0(t), && (12)\\
P_{10}'(t) &= -\boldsymbol{\beta}_1 P_{10}(t) + \boldsymbol{\alpha}_1 P_5(t), && (13)\\
P_{11}'(t) &= -\boldsymbol{\beta}_2 P_{11}(t) + \boldsymbol{\alpha}_2 P_5(t), && (14)\\
P_{12}'(t) &= -\boldsymbol{\beta}_3 P_{12}(t) + \boldsymbol{\alpha}_3 P_5(t), && (15)\\
P_{13}'(t) &= -\boldsymbol{\beta}_4 P_{13}(t) + \boldsymbol{\alpha}_4 P_5(t), && (16)\\
P_{14}'(t) &= -\boldsymbol{\beta}_5 P_{14}(t) + \boldsymbol{\alpha}_5 P_5(t), && (17)\\
P_{15}'(t) &= (-\boldsymbol{\beta}_5 - \boldsymbol{\beta}_6 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha - \alpha_5 - \alpha_7 - \alpha_8 - \alpha_9) P_{15}(t) + 2\alpha_5 P_6(t)\\
&\quad + \boldsymbol{\beta}_1 P_{27}(t) + \boldsymbol{\beta}_2 P_{28}(t) + \alpha_6 P_5(t) + \boldsymbol{\beta}_3 P_{29}(t) + \boldsymbol{\beta}_4 P_{30}(t)\\
&\quad + \boldsymbol{\beta} P_{32}(t) + \boldsymbol{\beta}_7 P_{33}(t) + \boldsymbol{\beta}_5 P_{31}(t) + \boldsymbol{\beta}_8 P_{34}(t) + \boldsymbol{\beta}_9 P_{35}(t), && (18)\\
P_{16}'(t) &= -\boldsymbol{\beta}_7 P_{16}(t) + \boldsymbol{\alpha}_7 P_5(t), && (19)\\
P_{17}'(t) &= -\boldsymbol{\beta}_8 P_{17}(t) + \boldsymbol{\alpha}_8 P_5(t), && (20)\\
P_{18}'(t) &= -\boldsymbol{\beta}_9 P_{18}(t) + \boldsymbol{\alpha}_9 P_5(t), && (21)\\
P_{19}'(t) &= -\boldsymbol{\beta}_1 P_{19}(t) + \boldsymbol{\alpha}_1 P_6(t), && (22)\\
P_{20}'(t) &= -\boldsymbol{\beta}_2 P_{20}(t) + \boldsymbol{\alpha}_2 P_6(t), && (23)\\
P_{21}'(t) &= -\boldsymbol{\beta}_3 P_{21}(t) + \boldsymbol{\alpha}_3 P_6(t), && (24)\\
P_{22}'(t) &= -\boldsymbol{\beta}_4 P_{22}(t) + \boldsymbol{\alpha}_4 P_6(t), && (25)\\
P_{23}'(t) &= -\boldsymbol{\beta} P_{23}(t) + \boldsymbol{\alpha} P_6(t), && (26)\\
P_{24}'(t) &= -\boldsymbol{\beta}_7 P_{24}(t) + \boldsymbol{\alpha}_7 P_6(t), && (27)\\
P_{25}'(t) &= -\boldsymbol{\beta}_8 P_{25}(t) + \boldsymbol{\alpha}_8 P_6(t), && (28)\\
P_{26}'(t) &= -\boldsymbol{\beta}_9 P_{26}(t) + \boldsymbol{\alpha}_9 P_6(t), && (29)\\
P_{27}'(t) &= -\boldsymbol{\beta}_1 P_{27}(t) + \boldsymbol{\alpha}_1 P_{15}(t), && (30)\\
P_{28}'(t) &= -\boldsymbol{\beta}_2 P_{28}(t) + \boldsymbol{\alpha}_2 P_{15}(t), && (31)\\
P_{29}'(t) &= -\boldsymbol{\beta}_3 P_{29}(t) + \boldsymbol{\alpha}_3 P_{15}(t), && (32)\\
P_{30}'(t) &= -\boldsymbol{\beta}_4 P_{30}(t) + \boldsymbol{\alpha}_4 P_{15}(t), && (33)\\
P_{31}'(t) &= -\boldsymbol{\beta}_5 P_{31}(t) + \boldsymbol{\alpha}_5 P_{15}(t), && (34)\\
P_{32}'(t) &= -\boldsymbol{\beta} P_{32}(t) + \boldsymbol{\alpha} P_{15}(t), && (35)\\
P_{33}'(t) &= -\boldsymbol{\beta}_7 P_{33}(t) + \boldsymbol{\alpha}_7 P_{15}(t), && (36)\\
P_{34}'(t) &= -\boldsymbol{\beta}_8 P_{35}(t) + \boldsymbol{\alpha}_8 P_{15}(t), && (37)\\
P_{35}'(t) &= -\boldsymbol{\beta}_9 P_{35}(t) + \boldsymbol{\alpha}_9 P_{15}(t), && (38)
\end{aligned}$$

where initial conditions are given as:

$$P_k(t=0) = \begin{cases} 1, & \text{if } k = 0, \\ 0, & \text{if } k \neq 0 \end{cases} \tag{39}$$

Applying limit $t \to \infty$ on above equations (38), the system convetes into a system of linear equation along with initil condition (39). As sum of transition probabilities is one, i.e., $\sum_{k=0}^{35} P_i = 1$, we have

$$\begin{aligned} P_0 = \Bigg\{ & 1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + 2\frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} + \frac{\alpha_7}{\beta_7} + \frac{\alpha_8}{\beta_8} + \frac{\alpha_9}{\beta_9} + 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right) \\ & + 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_5}{\beta_5}\right) \\ & + 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_1}{\beta_1}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_2}{\beta_2}\right) \\ & + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_3}{\beta_3}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_4}{\beta_4}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha}{\beta}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_7}{\beta_7}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_8}{\beta_8}\right) \\ & + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_9}{\beta_9}\right) + 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \\ & + 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha}{\beta}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \\ & + 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \Bigg\}^{-1} \end{aligned} \tag{40}$$

and probabilities at other states is derived mathematically as:

$$\begin{aligned} & P_1 = \frac{\alpha_1}{\beta_1} P_0\,;\; P_2 = \frac{\alpha_2}{\beta_2} P_0\,;\; P_3 = \frac{\alpha_3}{\beta_3} P_0\,;\; P_4 = \frac{\alpha_4}{\beta_4} P_0\,;\; P_5 = \frac{2\alpha_5}{\beta_5} P_0\,;\; P_6 = \frac{\alpha_6}{\beta_6} P_0\,; \\ & P_7 = \frac{\alpha_7}{\beta_7} P_0\,; P_8 = \frac{\alpha_8}{\beta_8} P_0\,; P_9 = \frac{\alpha_9}{\beta_9} P_0\,; P_{10} = 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{11} = 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,; \\ & P_{12} = 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{13} = 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{14} = 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,; \\ & P_{15} = 2\left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{16} = 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{17} = 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,; \\ & P_{18} = 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right) P_0\,;\; P_{19} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_1}{\beta_1}\right) P_0\,;\; P_{20} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_2}{\beta_2}\right) P_0\,; \\ & P_{21} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_3}{\beta_3}\right) P_0\,;\; P_{22} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_4}{\beta_4}\right) P_0\,;\; P_{23} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha}{\beta}\right) P_0\,; \\ & P_{24} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_7}{\beta_7}\right) P_0\,;\; P_{25} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_8}{\beta_8}\right) P_0\,;\; P_{26} = \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_9}{\beta_9}\right) P_0\,; \\ & P_{27} = 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,;\; P_{28} = 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,; \\ & P_{29} = 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,;\; P_{30} = 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,; \\ & P_{31} = 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,;\; P_{32} = 2\left(\frac{\alpha}{\beta}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,; \\ & P_{33} = 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,;\; P_{34} = 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,; \\ & P_{35} = 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) P_0\,. \end{aligned}$$

The steady state availability of the nut manufacturing plant is defined as the sum of probabilities of all operative states and shown in equation (41).

$$
\begin{aligned}
SSA &= P_0 + P_5 + P_6 + P_{15} \\
&= \left(1 + 2\frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} + 2\left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_5}{\beta_5}\right)\right)\Bigg\{1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + 2\frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} + \frac{\alpha_7}{\beta_7} \\
&\quad + \frac{\alpha_8}{\beta_8} + \frac{\alpha_9}{\beta_9} + 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right) \\
&\quad + 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right) + 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right) \\
&\quad + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_1}{\beta_1}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_2}{\beta_2}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_3}{\beta_3}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_4}{\beta_4}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha}{\beta}\right) \\
&\quad + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_7}{\beta_7}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_8}{\beta_8}\right) + \left(\frac{\alpha_6}{\beta_6}\right)\left(\frac{\alpha_9}{\beta_9}\right) + 2\left(\frac{\alpha_1}{\beta_1}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \\
&\quad + 2\left(\frac{\alpha_2}{\beta_2}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_3}{\beta_3}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_4}{\beta_4}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \\
&\quad + 2\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha}{\beta}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_7}{\beta_7}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) \\
&\quad + 2\left(\frac{\alpha_8}{\beta_8}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right) + 2\left(\frac{\alpha_9}{\beta_9}\right)\left(\frac{\alpha_5}{\beta_5}\right)\left(\frac{\alpha_6}{\beta_6}\right)\Bigg\}^{-1}.
\end{aligned}
\tag{41}
$$

The SSA of nut manufacturing plant is derived for a particular case by considering the rate parameters as given in Table 2 and numerical results appended in Table 4-5. It is a local solution and to predict the optimum availability of nut manufacturing plant nature inspired algorithm namely particle swarm optimization is utilized. The optimal solution is predicted in the search space given in Table 2. The PSO algorithm is applied under the set of constraints given in Table 3.

Table 3: Parameter values for metaheuristics algorithms PSO

Algorithm	Parameters
Particle Swarm Optimization	Population size = 10, 50, 100, 200, 500, 1000, number of iterations = 10, 20, 30, 40, 50, 60; inertia weight = 1; damping ratio = 0.9; global best = 2.8; personal best = 1.8

4. Results and discussion

In present section, steady state availability of a nut manufacturing plant is derived for a particular case. The impact of variation in failure and repair rates is observed on the steady state availability. The impact of variation in failure and repair rates is measured by increasing the failure and repair rates by 20%. It is observed from Table 4 that, after making 20% increment in failure rate of RRMS (α_1) the steady state availability decreases from 0.814997706 to 0.813233191 while availability increases up to 0.816473996. It is observed that variation in failure rate (α_2) of RMI unit decrease the availability of nut manufacturing plant up to 0.813107549 and variation in repair rate resulted the enhancement in availability up to 0.816473996. The failure rate (α_5) of CNC machine shows sharp decline in availability of nut manufacturing plant up to 0.796036393. It is seen as the highly sensitive unit in the nut manufacturing plant.

Table 4: Impact of failure rates on nuts manufacturing plant's availability

$\boldsymbol{\alpha}$	**Base values**	$\boldsymbol{\alpha}_1$+20% of $\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$+20% of $\boldsymbol{\alpha}_2$	$\boldsymbol{\alpha}_3$+20% of $\boldsymbol{\alpha}_3$	$\boldsymbol{\alpha}_4$+20% of $\boldsymbol{\alpha}_4$	$\boldsymbol{\alpha}_5$+20% of $\boldsymbol{\alpha}_5$	$\boldsymbol{\alpha}_6$+20% of $\boldsymbol{\alpha}_6$	$\boldsymbol{\alpha}_7$+20% of $\boldsymbol{\alpha}_7$	$\boldsymbol{\alpha}_8$+20% of $\boldsymbol{\alpha}_8$	$\boldsymbol{\alpha}_9$+20% of $\boldsymbol{\alpha}_9$
0.0129	0.814997706	0.813233191	0.813107549	0.812540207	0.813941606	0.800257369	0.814159541	0.811292628	0.812444962	0.814989086
0.0229	0.814457006	0.812694829	0.812569353	0.812002761	0.813402305	0.799727301	0.814513338	0.810756831	0.811907642	0.814448397
0.0329	0.813917023	0.812157179	0.812031869	0.811466026	0.812863719	0.799197934	0.813868158	0.810221741	0.811371033	0.813908425
0.0429	0.813377755	0.811620239	0.811495095	0.81093	0.812325845	0.798669268	0.813224	0.809687357	0.810835132	0.813369169
0.0529	0.812839201	0.81108401	0.810959031	0.810394682	0.811788683	0.7981413	0.812580861	0.809153678	0.810299939	0.812830626
0.0629	0.81230136	0.810548488	0.810423674	0.80986007	0.811252231	0.797614031	0.811938738	0.808620701	0.809765452	0.812292797
0.0729	0.81176423	0.810013673	0.809889024	0.809326162	0.810716487	0.797087457	0.811297629	0.808088426	0.80923167	0.811755678
0.0829	0.81122781	0.809479564	0.809355079	0.808792959	0.810181451	0.796561578	0.810657531	0.807556852	0.80869859	0.811219269
0.0929	0.810692099	0.808946158	0.808821837	0.808260457	0.80964712	0.796036393	0.810018443	0.807025976	0.808166213	0.810683569

Table 5: Impact of repair rates on nuts manufacturing availability with respect to repair rate β

$\boldsymbol{\beta}$	**Base values**	$\boldsymbol{\beta}_1$+20% of $\boldsymbol{\beta}_1$	$\boldsymbol{\beta}_2$+20% of $\boldsymbol{\beta}_2$	$\boldsymbol{\beta}_3$+20% of $\boldsymbol{\beta}_3$	$\boldsymbol{\beta}_4$+20% of $\boldsymbol{\beta}_4$	$\boldsymbol{\beta}_5$+20% of $\boldsymbol{\beta}_5$	$\boldsymbol{\beta}_6$+20% of $\boldsymbol{\beta}_6$	$\boldsymbol{\beta}_7$+20% of $\boldsymbol{\beta}_7$	$\boldsymbol{\beta}_8$+20% of $\boldsymbol{\beta}_8$	$\boldsymbol{\beta}_9$+20% of $\boldsymbol{\beta}_9$
0.267	0.814997706	0.816473996	0.816587295	0.817141473	0.815879886	0.827967212	0.814861814	0.818111222	0.81713728	0.817958191
0.268	0.815000311	0.816476609	0.816589909	0.817144091	0.815882496	0.827969861	0.814863992	0.818113846	0.817139898	0.817960814
0.269	0.815002896	0.816479204	0.816592504	0.817146689	0.815885087	0.82797249	0.814866153	0.818116451	0.817142497	0.817963418
0.27	0.815005462	0.816481779	0.81659508	0.817149269	0.815887658	0.8279751	0.814868298	0.818119036	0.817145076	0.817966002
0.271	0.815008009	0.816484335	0.816597637	0.817151829	0.815890211	0.82797769	0.814870428	0.818121603	0.817147636	0.817968568
0.272	0.815010537	0.816486873	0.816600175	0.817154371	0.815892745	0.827980261	0.814872542	0.81812415	0.817150178	0.817971115
0.273	0.815013047	0.816489391	0.816602695	0.817156894	0.81589526	0.827982814	0.81487464	0.818126679	0.817152701	0.817973643
0.274	0.815015538	0.816491892	0.816605196	0.817159398	0.815897757	0.827985348	0.814876723	0.81812919	0.817155205	0.817976152
0.275	0.815018011	0.816494374	0.816607679	0.817161885	0.815900235	0.827987864	0.814878791	0.818131682	0.817157692	0.817978644

After the steady state availability analysis, an effort is made to predict the optimal availability of the nut manufacturing plant availability. Here, particle swarm optimization algorithm is applied on the availability function (41) to derive optimal value under the search space given in Table 2. The simulation is conducted in various environments by considering various population and iteration sizes. The optimal availability is attained after the 50 iterations at population size 50, inertia weight = 1, damping ratio = 0.9, global best = 2.8, and personal best = 1.8. Table 7 shows the estimated parameters value at various population sizes while Table 8 shows the estimated values at population size 20 for various iterations.

Table 6: Availability of the nuts manufacturing plant corresponding to various population sizes at various iterations using particle swarm optimization

	iter\NP	10	50	100	200	500	1000
PSO	10	0.480795566	0.68680626	0.810442759	0.831984501	0.972255963	0.929715512
	20	0.704463894	0.996704583	0.998686263	0.999889988	0.999794883	0.999878965
	30	0.949313045	0.999830875	0.999866482	0.999917489	0.999919745	0.999917963
	40	0.998735856	0.999920988	0.999920982	0.999921045	0.999921047	0.999921047
	50	0.99990059	0.999921048	0.999921048	0.999921048	0.999921048	0.999921048
	60	0.999918615	0.999921048	0.999921048	0.999921048	0.999921048	0.999921048

Table 7: Parameter estimation of various failure and repair rates of the nuts manufacturing plant after 10 iterations using PSO

iter\NP		10	50	100	200	500	1000
PSO	α	0.00001	0.074377082	0.00001	0.00001	0.02817512	0.00001
	α_1	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
10	α_2	0.095009	0.113709764	0.209014171	0.024118812	0.00001	0.002337198
	α_3	0.56109825	0.00001	0.107044412	0.102725866	0.00001	0.021122339
	α_4	0.07903892	0.111148185	0.00001	0.00001	0.028827717	0.00001
	α_5	0.53211261	0.275032904	0.055190441	0.00001	0.048018115	0.412539701
	α_6	0.64107144	0.709452276	0.00001	0.595235448	0.004059011	0.00001
	α_7	0.48047403	0.336560041	0.427278063	0.306312098	0.00001	0.00001
	α_8	0.03605938	0.118142524	0.033759226	0.133872158	0.114380563	0.076096516
	α_9	0.18474713	0.191211683	0.100238989	0.160709481	0.200910639	0.170279691
	β	0.71858601	0.966349219	1.17502008	1.781	1.737424608	1.777058423
	β_1	1.33027682	0.38980712	1.544832551	2.042341686	2.232255627	1.703383822
	β_2	0.68147745	1.094033287	0.297982891	1.30327185	1.405371554	1.387544165
	β_3	0.85538977	1.715554701	0.879398082	1.138945368	1.854	1.761916615
	β_4	1.30247356	2.678143418	1.522905789	2.331521589	1.164759624	2.618964009
	β_5	0.57625584	1.085490084	1.748	1.748	1.264242396	0.863923389
	β_6	1.27991154	2.387129559	1.215893956	1.872743272	2.40628491	1.976226677
	β_7	1.82961297	1.321331311	2.751198762	2.755407172	1.577476461	1.487855494
	β_8	1.52655944	1.944528961	2.227457983	2.337423638	2.856396049	2.595135435
	β_9	1.31519785	0.460989297	1.461099248	0.782362793	0.372889284	0.535609378

Table 8: Parameter estimation of various failure and repair rates of the nuts manufacturing plant after 20 iterations using PSO

iter\NP		10	50	100	200	500	1000
PSO	α	0.006774	0.002737	0.008518	0.003861	0.00001	0.000228
	α_1	0.00001	0.000309	0.00001	0.00001	0.00001	0.00001
20	α_2	0.005723	0.00001	0.001207	0.00001	0.00001	0.00001
	α_3	0.639786	0.005536	0.00001	0.00001	0.00001	0.00001
	α_4	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
	α_5	0.409673	0.001313	0.00001	0.00001	0.00001	0.00001
	α_6	0.397895	0.00001	0.00001	0.00001	0.00001	0.00001
	α_7	0.301072	0.00001	0.00195	0.00001	0.00001	0.00001
	α_8	0.101597	0.059285	0.15	0.122624	0.094297	0.035035
	α_9	0.1489	0.154461	0.238571	0.236328	0.07783	0.177278
	β	0.964693	1.264796	1.238233	1.073105	1.441586	1.101048
	β_1	2.43773	2.395209	1.555335	2.261086	2.438493	2.122592
	β_2	1.034691	0.840715	0.243845	1.612633	1.536053	1.262581
	β_3	1.025494	1.208913	1.434057	1.339085	1.694839	0.803131
	β_4	2.555019	2.040392	2.563745	1.776895	2.071757	0.884208
	β_5	1.220945	1.723002	1.25177	1.194176	1.178397	1.431728
	β_6	1.435182	0.863446	0.088178	1.93531	2.228774	2.200724
	β_7	1.962174	0.102308	1.1167	1.018395	0.832261	2.618695
	β_8	3.32905	2.380993	3.318377	2.29232	2.128231	2.917004
	β_9	0.718178	1.063187	0.073068	0.318165	1.162051	0.920632

5. Conclusion

Once it comes to the design phase as well as the operation phase of the Nuts manufacturing plants, the reliability characteristics, including reliability, availability, and maintainability, play an extremely important role. Because of this circumstance, it is required to evaluate these measurements at any instantaneous time point as well as throughout the course of a long period of time. In order to acquire a global solution, it is required to optimize the availability. As a result, during the first stages of this study, we found that the availability of the system as well as all of its subsystems is higher than 0.9999. On the other hand, in the long run, the availability is drastically reduced, and it eventually reached 0.814997706. The difference in success rates is the root cause of the availability issues that have arisen. Additionally, the fluctuation in the rates of breakdown and repair influences the plant's availability. Because of this circumstance, it is required to evaluate the values of the rates of failure and repair in such a way that the system maintains a high level of availability. Therefore, the metaheuristic approach known as PSO is applied to the availability model, and an optimal value of 0.999921048 is attained for the availability in accordance with the estimated parameter values. The current research focuses on a single Nuts production plant to perform its investigation. It is recommended that the same operation be carried out on the data of numerous plants to achieve more effective outcomes. The estimation of parameters using many additional nature-inspired algorithms will be the focus of subsequent efforts, as will the development of a mathematical model to predict the impact of several failures occurring at the same time.

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