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An Extensive Comparisons of 50 Univariate Goodness-of-fit Tests for Normality

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Abstract

The assumption of normality needs to be checked for many statistical procedures, namely parametric tests, because their validity depends on it. Given the importance of this subject and the widespread development of normality tests, comprehensive descriptions and power comparisons of such tests are of considerable interest. Since recent comparison studies do not include several interesting and more recently developed tests, a further comparison of normality tests is considered to be of foremost interest. This study addresses the performance of 50 normality tests available in literature, from 1900 until 2018. Because a theoretical comparison is not possible, Monte Carlo simulation were used from various symmetric and asymmetric distributions for different sample sizes ranging from 10 to 100. The simulations results show that for symmetric distributions with support on $(-\infty, \infty)$ the tests Robust Jarque–Bera and Gel–Miao–Gastwirth have generally the most power. For asymmetric distributions with support on $(-\infty, \infty)$ the tests 1st Cabana-Cabana and 2nd Zhang-Wu have the most power. For distributions with support on $(0, \infty)$, and distributions with support on $(0, 1)$ the test 2nd Zhang-Wu has generally the most power.

Keywords: assumption of normality, normality tests, Monte Carlo simulation, power of test, normal distribution, goodness-of-fit test.

1. Introduction

The problem of testing for normality is fundamental in both theoretical and empirical statistical research. Parametric statistical methods assume that the data has a known and specific distribution, often a normal distribution. Therefore, testing normality is one of the most studied goodness-of-fit problems. There are many tests that can be used to check if your data sample deviates from a normal distribution.

Given the importance of this subject and the widespread development of normality tests over the years, comprehensive descriptions and power comparisons of such tests have also been the focus of attention, thus helping the analyst in the choice of suitable tests for his/her particular needs. An extensive simulation study is presented herein to estimate the power of 50 tests for normality, from 1900 until 2018, for several alternative distributions: Beta, Gamma, Gumbel, Laplace, Skew-Normal, Student's t, Uniform, and Weibull. Since a theoretical comparison is not possible, Monte Carlo simulation were used from these alternative distributions for different sample sizes ranging from 10 to 100.

A short description of the 50 normality tests are given in Section 2. The alternative statistical distributions used for the tests are explained in Section 3. The Monte Carlo simulation is explained in Section 4. Results and Recommendations of the power comparisons of the 50 normality tests are discussed in Section 5.

Comparison of the normality tests has received attention in the literature. The goodness-of-fit tests have been discussed by many authors including Shapiro, Wilk, and Mrs Chen (1968), Farrell and Rogers-Stewart (2006), Yazici and Yolacan (2007), Xavier, Raimundo, and Aníbal (2010) Yap and Sim (2011), Noughabi and Arghami (2011), and Torabi, Montazeri, and Grané (2016). Since recent comparison studies do not include several interesting and more recently developed tests, a further comparison of normality tests is considered to be of foremost interest.

2. Tests for normality

Tests for Normality can be classified into tests based on Chi-square, a test of goodness of fit establishes whether an observed frequency distribution differs from a theoretical distribution (Pearson's chi-square test), Empirical distribution function, these tests are based on a comparison of the empirical and hypothetical distribution functions (Cramer-von Mises, Kolmogorov-Smirnov, Lilliefors, Anderson-Darling, HegazyGreen-1, HegazyGreen-2, Frosini, Glen-Leemis-Barr, 1st Zhang-Wu, 2nd Zhang-Wu), Measures of the moments, these tests are derived from the recognition that the departure of normality may be detected based on the sample moments (Geary, Kurtosis test, Skewness test, D'Agostino-skewness, Spiegelhalter, Martinez-Iglewicz, Anscombe-Glynn, Jarque–Bera, 1st Hosking, 2nd Hosking, 3rd Hosking, 4th Hosking, 1st Cabana-Cabana, 2nd Cabana-Cabana, Adjusted Jarque–Bera, Bonett and Seier, Brys–Hubert–Struyf MC–LR, Brys-Hubert-Struyf-Bonett-Seier, 1st Bontemps and Meddahi, 2nd Bontemps-Meddahi, Gel-Miao-Gastwirth, Doornik-Hansen, Gel-Gastwirth Robust Jarque-Bera, Desgagne-LafayeDeMicheaux X_{APD}, Desgagne-LafayeDeMicheaux Z_{EPD}), Regression and correlation, these tests are based on the ratio of two weighted least-squares estimates of scale obtained from order statistics (Shapiro-Wilk, Shapiro–Francia, D'Agostino-Pearson, Filliben, Weisberg-Bingham, Chen-Shapiro, Rahman-Govindarajulu, 1st Zhang Q, 2nd Zhang Q, Barrio-Cuesta-Matran-Rodriguez, Coin), Maximum entropy, this test are based on the property that the normal distribution has the highest entropy of any distribution for a given standard deviation (Vasicek-Song), Empirical characteristic function, the test uses the difference between the characteristic functions of the sample and of the normal distribution (Epps and Pulley), Lagrange Multiplier, this test is maximizing the log-likelihood subject to the constraint (Desgagne-LafayeDeMicheaux-Leblanc test).

We give a short description of the 50 methods of testing for univariate normality. The presentation is in chronological order, from 1900 until 2018. It also contains references to definitions of these tests.

1. Pearson's chi-square test (Pearson (1900), see also Moore (1986), Hogg, McKean, and Craig (2018)):

$$P = \frac{(O_i - E_i)^2}{E_i},$$

where O_i is the observed counts and E_i is the number of expected observations (under H_0) in class i. The classes are build in such a way that they are equiprobable under the null hypothesis of normality.

2. Cramer-von Mises test (Cramér (1928) and Mises (1931), see also Thode Jr. (2002)):

$$W = \frac{1}{12n} + \sum\nolimits_{i=1}^{n} \left(p_{(i)} - \frac{2i-1}{2n} \right)^2,$$

where $p_{(i)} = \Phi\left(\left[x_{(i)} - \overline{x}\right]/s\right)$. Here, Φ is the cumulative distribution function of the standard normal distribution, and $\overline{x}$ and s are mean and standard deviation of the data values. The p-value is computed from the modified statistic $Z = W\,(1.0 + 0.5/n)$ according to the Table 4.9 in Stephens (1986).

3. Geary test (Geary (1935)) for normality is based on the ratio of the mean deviation to standard deviation,
$$d = \frac{1}{ns}\sum\nolimits_{i=1}^{n} \left| X_i - \overline{X} \right|,$$
where
$$s^2 = \frac{1}{n}\sum\nolimits_{i=1}^{n} \left(X_i - \overline{X} \right)^2.$$
Geary's test of normality is a simple compact but sensitive test of normality. If the null hypothesis of normality is true, the expected value of d is approximately $\sqrt{2/\pi} \approx 0.7979$. Thus one rejects the null hypothesis for very large ($d > 0.7979$) or very small ($d < 0.7979$) values of d (D'Agostino (1970)).

   Geary's test never gained widespread usage, possibly because $0 < d \leq \sqrt{2/\pi}$ in leptokurtic distributions so that large increases in leptokurtosis have small numerical effects on d (Bonett and Seier (2002)).

4. Kolmogorov-Smirnov test (Kolmogorov (1933), Smirnov (1948), see also Thode Jr. (2002), Hollander, Wolfe, and Chicken (2014)) for a given cumulative distribution function $F(x)$ is
$$D_n = \sup_x \left| F_{1,n}(x) - F_{2,m}(x) \right|,$$
where $F_{1,n}(x)$ is the empirical distribution of the data and $F_{2,m}(x)$ is empirical distribution function of the normal distribution. Kolmogorov–Smirnov test is not very powerful because it is devised to be sensitive against all possible types of differences between two distribution functions.

5. Shapiro-Wilk test (Shapiro *et al.* (1968), see also Thode Jr. (2002)) is
$$W = \frac{\left(\sum\nolimits_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \overline{x})^2}, \tag{1}$$
where $x_{(i)}$ is the ith order statistic and $\overline{x}$ is the sample mean.

   The coefficients a_i are given by:
$$(a_1, \cdots, a_m) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$$
and the vector m,
$$m = (m_1, \ldots, m_n)^T$$
is made of the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution; V is the covariance matrix of those normal order statistics.

6. Lilliefors test (Lilliefors (1967), see also Thode Jr. (2002)) is a modification of the Kolmogorov-Smirnov test for normality when the mean and the variance are unknown, and must be estimated from the data. The test statistic is the maximal absolute difference between empirical and hypothetical cumulative distribution function. It may be computed as

$$D = \max\left\{D^{+}, D^{-}\right\}$$

with $D^{+} = \max\limits_{i=1,\ldots,n}\left\{i/n - p_{(i)}\right\}$, $D^{-} = \max\limits_{i=1,\ldots,n}\left\{p_{(i)} - (i-1)/n\right\}$where $p_{(i)} = \Phi\left(\left[x_{(i)} - \overline{x}\right]/s\right)$.
Here, Φ is the cumulative distribution function of the standard normal distribution, $x_{(i)}$ is the ith order statistic and $\overline{x}$ and s are mean and standard deviation of the data values.

7. Kurtosis test (Shapiro *et al.* (1968), see also Thode Jr. (2002)) for normality is based on the following statistic

$$b_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^4}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2\right)^2}, \tag{2}$$

b_2 is asymptotically normal with mean 3 and variance $24/n$.

8. Skewness test (Shapiro *et al.* (1968), see also Thode Jr. (2002)) for normality is based on the following statistic

$$\sqrt{b_1} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^3}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2\right)^{3/2}}. \tag{3}$$

Under the null hypothesis of normality, $\sqrt{b_1}$ is asymptotically normal with mean 0 and variance $6/n$.

9. D'Agostino-skewness test (D'Agostino (1970), see also Thode Jr. (2002)) based on a transformation of the distribution of Equation (3) to normality which works well for small sample sizes, $n \geq 8$. Let

$$Y = \sqrt{b_1}\left\{\frac{(n+1)(n+3)}{6(n-2)}\right\}^{1/2}$$

and

$$B_2 = \frac{3\left(n^2 + 27n - 70\right)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)},$$
$$W^2 = \sqrt{2(B_2 - 1)} - 1,$$
$$\delta = 1/\sqrt{\log(W)},$$
$$\alpha = \sqrt{2/(W^2 - 1)},$$

then $Z\left(\sqrt{b_1}\right)$ is distributed $N(0,1)$, where

$$Z\left(\sqrt{b_1}\right) = \delta \log\left(Y/\alpha + \sqrt{(Y/\alpha)^2 + 1}\right). \tag{4}$$

10. Shapiro-Francia test (Shapiro and Francia (1972)) is a modification of Shapiro-Wilk test Equation (1). It is defined as

$$W' = \frac{cov\left(x, m\right)}{\sigma_x \sigma_m} = \frac{\sum_{i=1}^{n}\left(x_{(i)} - \overline{x}\right)\left(m_i - \overline{m}\right)}{\sqrt{\left(\sum_{i=1}^{n}\left(x_{(i)} - \overline{x}\right)^2\right)\left(\sum_{i=1}^{n}\left(m_i - \overline{m}\right)^2\right)}}.$$

Under the null hypothesis that the data is drawn from a normal distribution, this correlation will be strong, so W' values will cluster just under 1, with the peak becoming narrower and closer to 1 as n increases. If the data deviate strongly from a normal distribution, W' will be smaller. Monte Carlo simulations have shown that the transformed statistic $\ln\left(1 - W'\right)$ is nearly normally distributed.

11. D'Agostino-Pearson test (D'Agostino and Pearson (1973)) proposed the test statistic K^2 that combines normalizing transformations of skewness and kurtosis, $Z\left(\sqrt{b_1}\right)$ and $Z\left(b_2\right)$, respectively. The test statistic K^2 is given by

$$K = \left[Z\left(\sqrt{b_1}\right)\right]^2 + \left[Z\left(b_2\right)\right]^2$$

in which the transformed skewness $Z\left(\sqrt{b_1}\right)$ is obtained by Equation (4) and the transformed kurtosis

$$Z\left(b_2\right) = \left[\left(1 - \frac{2}{9A}\right) - \sqrt[3]{\frac{1 - 2/A}{1 + y\sqrt{2/\left(A - 4\right)}}}\right]\sqrt{\frac{9A}{2}} \tag{5}$$

with A and y obtained by

$$A = 6 + \frac{8}{\gamma_1}\left(\frac{2}{\gamma_1} + \sqrt{1 + \frac{4}{\gamma_1^2}}\right),$$

$$\gamma_1 = \frac{6\left(n^2 - 5n + 2\right)}{\left(n + 7\right)\left(n + 9\right)}\sqrt{\frac{6\left(n + 3\right)\left(n + 5\right)}{n\left(n - 2\right)\left(n - 3\right)}},$$

$$y = \frac{b_2 - 3\left(n - 1\right)/\left(n + 1\right)}{24n\left(n - 2\right)\left(n - 3\right)/\left[\left(n + 1\right)^2\left(n + 3\right)\left(n + 5\right)\right]}.$$

The normality hypothesis of the data is rejected for large values of the test statistic K^2 . The test statistic K^2 is approximately chi-squared distributed with two degrees of freedom.

12. Filliben test (Filliben (1975)) use the correlation between the sample order statistics and estimated median values of the theoretical order statistics. For a sample of size n, Filliben used

$$m_{(i)} = \begin{cases} 1 - 0.5^{(1/n)} & i = 1 \\ \left(i - 0.3175\right)/\left(n + 0.365\right) & 1 < i < n \\ 0.5^{(1/n)} & i = n, \end{cases}$$

where the $m_{(i)}$ were estimated order statistic medians from a uniform distribution. He then used the transformation $M_{(i)} = \Phi^{-1}\left(m_{(i)}\right)$ to obtain an estimate of the median value of the ith normal order statistic. The correlation coefficient r is then defined as

$$r = \frac{\sum_{i=1}^{n} x_{(i)} \cdot M_{(i)}}{\sqrt{\sum_{i=1}^{n} M_{(i)}^2} \cdot \sqrt{\left(n - 1\right) \cdot s^2}}$$

leading to the rejection of the normality hypothesis of the data for small values of r.

13. The Hegazy–Green-1 (Hegazy and Green (1975)) test for normality is based on the following statistic:

$$T_1 = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i - \Phi^{-1}\left(\frac{i}{n+1}\right)\right|,$$

where $Y_i = \frac{X_{(i)}-\overline{X}}{s}$, $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$.

14. The Hegazy–Green-2 (Hegazy and Green (1975)) test for normality is based on the following statistic:

$$T_2 = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \Phi^{-1}\left(\frac{i}{n+1}\right)\right)^2,$$

where $Y_i = \frac{X_{(i)}-\overline{X}}{s}$, $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$.

15. The Weisberg–Bingham (Weisberg and Bingham (1975)) test for normality is based on the following statistic:

$$WB = \frac{\left(\sum_{i=1}^{n} m_i X_{(i)}\right)^2 / \left(\sum_{i=1}^{n} m_i^2\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2},$$

where

$$m_i = \Phi^{-1}\left(\frac{i-3/8}{n+1/4}\right).$$

16. Vasicek-Song (Vasicek (1976), Kai-Sheng Song (2002)) developed a test for normality using an estimate of the sample entropy for $n > 3$. The entropy of a density $f(x)$ is

$$H(f) = -\int_{-\infty}^{\infty} f(x)\log\left(f(x)\right)dx.$$

An estimate of $H(f)$ can be calculated as

$$H_{m,n} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{n}{2m}\left(x_{(i+m)} - x_{(i-m)}\right)\right),$$

where m is a positive integer, $m < n/2$ and $x_{(k)} = x_{(1)}$ for $k < 1$ and $x_{(k)} = x_{(n)}$ for $k > n$. Among all densities with a given variance σ^2, $H(f)$ is maximized by the normal density, with entropy

$$H(f) = \log\left\{\sigma\sqrt{2\pi e}\right\}$$

so that $\exp\left[H(f)\right]/\sigma \leq \sqrt{2\pi e}$ for all $f(x)$, equality being attained under normality. Therefore, an omnibus test for a sample of size n is defined by rejecting the null hypothesis if

$$K_{mn} \leq K^*,$$

where K^* is the appropriate critical value for the test and

$$K_{mn} = \frac{n}{2m\widehat{\sigma}}\left\{\prod_{i=1}^{n}\left(x_{(i+m)} - x_{(i-m)}\right)\right\}^{1/n},$$

where $\widehat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}$.

17. The Spiegelhalter (Spiegelhalter (1977)) test for normality is based on the following statistic:
$$T = \left((c_n u)^{-(n-1)} + g^{-(n-1)}\right)^{1/(n-1)},$$
where
$$u = \frac{X_{(n)}-X_{(1)}}{s}, \quad g = \frac{\sum_{i=1}^{n}|X_i-\overline{X}|}{s\sqrt{n(n-1)}},$$
$$c_n = \frac{(n!)^{1/(n-1)}}{2n}, \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2.$$

18. Martinez and Iglewicz (Martinez and Iglewicz (1981)) have proposed a normality test based on the ratio of two estimators of variance, where one of the estimators is the robust biweight scale estimator S_b^2
$$S_b^2 = \frac{n \cdot \sum_{|\widetilde{z}_i|<1} (x_i - M)^2 \left(1 - \widetilde{z}_i^2\right)^4}{\left[\sum_{|\widetilde{z}_i|<1} \left(1 - \widetilde{z}_i^2\right)\left(1 - 5\widetilde{z}_i^2\right)\right]^2},$$
where M is the sample median, $\widetilde{z}_i = (x_i - M)/(9A)$, with A being the median of $|x_i - M|$, and when $|\widetilde{z}_i| > 1$, $\widetilde{z}_i$ is set to 0. The Martinez and Iglewicz test statistic I_n is then given by
$$I_n = \frac{\sum_{i=1}^{n} (x_i - M)^2}{(n-1) \cdot S_b^2}$$
for which the normality hypothesis of the data is rejected for large values of I_n. Martinez and Iglewicz (1981) have shown that this test is very powerful for heavy-tailed symmetric distributions.

19. Epps and Pulley (Epps and Pulley (1983)) test statistic T_{EP} is based on the following weighted integral
$$T_{EP} = \int_{-\infty}^{\infty} |\varphi_n(t) - \widehat{\varphi}_0(t)|^2 \, dG(t),$$
where $\varphi_n(t)$ is the empirical characteristic function given by $n^{-1}\sum_{j=1}^{n} \exp(i\,t\,x_j)$, $\widehat{\varphi}_0(t)$ is the sample estimate of the characteristic function of the normal distribution given by $\exp\left(i\,t\,\overline{x} - 0.5 m_2 t^2\right)$ and $G(t)$ is an adequate function chosen according to several considerations Epps and Pulley (1983). By setting $dG(t) = g(t)\,dt$ and selecting $g(t) = \sqrt{m_2/2\pi} \cdot \exp\left(-0.5 m_2 t^2\right)$ the following statistic can be obtained by
$$T_{EP} = 1 + \frac{n}{\sqrt{3}} + \frac{2}{n}\sum_{k=2}^{n}\sum_{j=1}^{k-1} \exp\left(\frac{-(x_j - x_k)^2}{2m_2}\right) - \sqrt{2}\sum_{j=1}^{n} \exp\left(\frac{-(x_j - \overline{x})^2}{4m_2}\right)$$
for which the normality hypothesis of the data is rejected when large values of TEP are obtained.

20. Anscombe-Glynn (Anscombe and Glynn (1983))test of kurtosis for normal samples is based on the transformed kurtosis Equation (5):
$$Z(b_2) = \left[\left(1 - \frac{2}{9A}\right) - \sqrt[3]{\frac{1 - 2/A}{1 + y\sqrt{2/(A-4)}}}\right]\sqrt{\frac{9A}{2}}.$$
Under the hypothesis of normality, data should have kurtosis equal to 3.

21. Anderson-Darling test (Stephens (1986)) is an empirical distribution function omnibus test for the composite hypothesis of normality. The test statistic is

$$A = -n - \frac{1}{n}\sum\nolimits_{i=1}^{n} \left[2i-1\right] \left[\ln\left(p_{(i)}\right) + \ln\left(1 - p_{(n-i+1)}\right)\right],$$

where $p_{(i)} = \Phi\left(\left[x_{(i)} - \overline{x}\right]/s\right)$. Here, Φ is the cumulative distribution function of the standard normal distribution, and $\overline{x}$ and s are mean and standard deviation of the data values.

22. Frosini test (Frosini (1987)) for normality is based on the following statistic:

$$B_n = \frac{1}{\sqrt{n}}\sum\nolimits_{i=1}^{n} \left|\Phi\left(Y_i\right) - \frac{i-0.5}{n}\right|,$$

where $Y_i = \frac{X_{(i)} - \overline{X}}{s}$, $\qquad s^2 = \frac{1}{n}\sum\nolimits_{i=1}^{n}\left(X_i - \overline{X}\right)^2$.

23. Jarque–Bera (Jarque and Bera (1987)) test for normality is based on the following statistic:

$$JB = \frac{n}{6}\left(\left(\sqrt{b_1}\right)^2 + \frac{\left(b_2 - 3\right)^2}{4}\right),$$

where $\sqrt{b_1}$ and b_2 are the sample skewness in equation(3) and sample kurtosis in equation (2), respectively. H_0 is rejected for large values of JB. Jarque–Bera test is a large sample test and may not be appropriate in small samples.

24. The 1st Hosking test (Hosking (1990)). Hosking has shown the rth order sample L-moment can be estimated by

$$l_r = \sum\nolimits_{k=0}^{r-1} p^*_{r-1,k} \cdot b_k,$$

where

$$p^*_{r-1,k} = (-1)^{r-k}\begin{pmatrix} r \\ k \end{pmatrix}\begin{pmatrix} r+k \\ k \end{pmatrix}$$

and

$$b_k = \frac{1}{n}\sum\nolimits_{i=1}^{n} \frac{(i-1)(i-2)\cdots(i-k)}{(n-1)(n-2)\cdots(n-k)} x_{(i)}.$$

Based on the second, third and fourth sample L-moments, which have similarities with the corresponding central moments, Hosking (Hosking (1990)) also defines new measures of skewness and kurtosis, termed L-skewness τ_3 and L-kurtosis τ_4 as follows

$$\tau_3 = \frac{l_3}{l_2}, \qquad \tau_4 = \frac{l_4}{l_2}.$$

The value of τ_3 is bounded between $-1$and 1 for all distributions and is close to zero for the normal distribution, while the value of τ_4 is ≤ 1 for all distributions and is close to 0.1226 for the normal distribution. Hosking has suggested that normality could be tested based on τ_3 and τ_4 according to the following statistic T_{Lmom}

$$T_{Lmom} = \frac{\tau_3 - \mu_{\tau_3}}{var(\tau_3)} + \frac{\tau_4 - \mu_{\tau_4}}{var(\tau_4)}, \tag{6}$$

where μ_{τ_3} and μ_{τ_4} are the mean of τ_3 and τ_4, and $var(\tau_3)$ and $var(\tau_4)$ are their corresponding variances. Nonetheless, μ_{τ_3} and μ_{τ_4} are expected to be close to 0 and 0.1226. The normality hypothesis of the data is rejected for large values of T_{Lmom}.

25. The 2nd Hosking test (Hosking (1990)). Although L-moments exhibit some robustness towards outliers in the data, as previously referred, they may still be affected by extreme observations (Elamir and Seheult (2003)). A robust generalization of the sample L-moments has, therefore, been formulated by Elamir and Seheult (Elamir and Seheult (2003)) leading to the development of trimmed L-moments. The proposed formulation for the trimmed L-moments allows for both symmetric and asymmetric trimming of the smallest and largest sample observations. For the case of normality testing suggested herein, only symmetric trimming is considered.

    Considering an integer symmetric trimming level t , Elamir and Seheult (Elamir and Seheult (2003)) have shown the rth order sample trimmed L-moment $l_r^{(t)}$ can be estimated by

$$l_r^{(t)} = \frac{1}{r}\sum_{i=t+1}^{n-t} \left\{ \frac{\sum_{k=0}^{r-1}\left[(-1)^k \binom{r-1}{k}\binom{i-1}{r+t-1-k}\binom{n-i}{t+k}\right]}{\binom{n}{r+2t}} \right\} x_{(i)}.$$

    Based on the second, third and fourth sample trimmed L-moments, Elamir and Seheult (Elamir and Seheult (2003)).also define new measures of skewness and kurtosis, termed TL-skewness $\tau_3^{(t)}$ and TL-kurtosis $\tau_4^{(t)}$, given by

$$\tau_3^{(t)} = \frac{l_3^{(t)}}{l_2^{(t)}}, \qquad \tau_4^{(t)} = \frac{l_4^{(t)}}{l_2^{(t)}}.$$

    Based on these new measures, the following test, similiar to that given by Equation(6), is as follows

$$T_{Lmom}^{(t)} = \frac{\tau_3^{(t)} - \mu_{\tau_3}^{(t)}}{var(\tau_3^{(t)})} + \frac{\tau_4^{(t)} - \mu_{\tau_4}^{(t)}}{var(\tau_4^{(t)})},$$

    where for a selected trimming level t , $\mu_{\tau_3}^{(t)}$ and $\mu_{\tau_4}^{(t)}$ are the mean of $\tau_3^{(t)}$ and $\tau_4^{(t)}$, and $var(\tau_3^{(t)})$ and $var(\tau_4^{(t)})$ are their corresponding variances.

    Three versions of this test are considered, which correspond to symmetric trimming levels t of 1, 2 and 3. For each test, the normality hypothesis of the data is rejected for large values of the statistic $T_{Lmom}^{(t)}$.

    The 2nd Hosking test, which correspond to symmetric trimming levels $t = 1$,

$$T_{Lmom}^{(1)} = \frac{\tau_3^{(1)} - \mu_{\tau_3}^{(1)}}{var(\tau_3^{(1)})} + \frac{\tau_4^{(1)} - \mu_{\tau_4}^{(1)}}{var(\tau_4^{(1)})},$$

    the normality hypothesis of the data is rejected for large values of the statistic $T_{Lmom}^{(t)}$.

26. The 3rd Hosking test (Hosking (1990)), which correspond to symmetric trimming levels $t = 2$, is as follows,

$$T_{Lmom}^{(2)} = \frac{\tau_3^{(2)} - \mu_{\tau_3}^{(2)}}{var(\tau_3^{(2)})} + \frac{\tau_4^{(2)} - \mu_{\tau_4}^{(2)}}{var(\tau_4^{(2)})},$$

    the normality hypothesis of the data is rejected for large values of the statistic $T_{Lmom}^{(t)}$.

27. The 4th Hosking test (Hosking (1990)), which correspond to symmetric trimming levels $t = 3$, is as follows,

$$T_{Lmom}^{(3)} = \frac{\tau_3^{(3)} - \mu_{\tau_3}^{(3)}}{var(\tau_3^{(3)})} + \frac{\tau_4^{(3)} - \mu_{\tau_4}^{(3)}}{var(\tau_4^{(3)})},$$

the normality hypothesis of the data is rejected for large values of the statistic $T^{(t)}_{Lmom}$.

28. The 1st Cabana-Cabana test (Cabana and Cabana (1994)). The Cabana-Cabana test statistics are based on the definition of approximate transformed estimated empirical processes (ATEEP) sensitive to changes in skewness or kurtosis. The proposed ATEEP sensitive to changes in skewness is defined as:

$$W_{S,L}(x) = \Phi(x) \cdot \overline{H}_3 - \phi(x) \sum\nolimits_{j=1}^{L} \frac{1}{\sqrt{j}} H_{j-1}(x) \cdot \overline{H}_{j+3},$$

where L is a dimensionality parameter, $\phi(x)$ is the probability density function of the standard normal distribution, $H_j(\cdot)$ represents the j th order normalized Hermite polynomial given by

$$\forall i > 1, \quad H_i(u) = \frac{1}{\sqrt{i}} \left[u \cdot H_{i-1}(u) - \sqrt{i-1} \cdot H_{i-2}(u) \right], \quad H_0(u) = 1, \quad H_1(u) = u, \tag{7}$$

and $\overline{H}_j$ is the jth order normalized mean of the Hermite polynomial defined as

$$\overline{H}_j = \frac{1}{\sqrt{n}} \sum\nolimits_{i=1}^{n} H_j(x_i).$$

The proposed ATEEP sensitive to changes in kurtosis is defined as

$$w_{K,L}(x) = -\phi(x) \cdot \overline{H}_3 + [\Phi(x) - x \cdot \phi(x)] \cdot \overline{H}_4 - \phi(x) \sum\nolimits_{j=2}^{L} \left(\sqrt{\frac{j}{j-1}} H_{j-2}(x) \cdot H_j(x) \right) \cdot \overline{H}_{j+3}.$$

According to (Cabana and Cabana (1994)), the dimensionality parameter L ensures that the test is consistent against alternative distributions differing from the normal distribution having the same mean and variance in at least one moment of order not greater than $L+3$. The Kolmogorov–Smirnov type test statistics sensitive to changes in skewness and in kurtosis, $T_{S,L}$ and $T_{K,L}$ respectively, are defined as

$$T_{S,L} = \max |w_{S,L}(x)| \tag{8}$$

and

$$T_{K,L} = \max |w_{K,L}(x)|. \tag{9}$$

Based on results presented in (Cabana and Cabana (1994)), parameter L was considered to be five.

The 1st Cabana-Cabana test for normality is based on equaion (8):

$$T_{S,L} = \max |w_{S,L}(x)|.$$

The normality hypothesis of the data is rejected for large values of the test statistic.

29. The 2nd Cabana-Cabana (Cabana and Cabana (1994)) is based on equaion (9):

$$T_{K,L} = \max |w_{K,L}(x)|.$$

The normality hypothesis of the data is rejected for large values of the test statistic.

30. Chen-Shapiro test (Chen and Shapiro (1995)) is based on normalized spacings and defined as

$$CS = \frac{1}{(n-1) \cdot s} \sum\nolimits_{i=1}^{n-1} \frac{X_{(i+1)} - X_{(i)}}{M_{(i+1)} - M_{(i)}}$$

in which M_i is the ith quantile of a standard normal distribution obtained by $\Phi^{-1}[(i-0.375)/(n+0.25$
The normality hypothesis of the data is rejected for small values of CS.

31. Adjusted Jarque–Bera test (Urzua (1996)) for normality is based on the following statistic:

$$AJB = \frac{\left(\sqrt{b_1}\right)^2}{Var\left(\sqrt{b_1}\right)} + \frac{\left(b_2 - E\left(b_2\right)\right)^2}{Var\left(b_2\right)},$$

where $\sqrt{b_1}$ and b_2 are the sample skewness in equation(3) and sample kurtosis in equation (2), respectively.

$$Var\left(\sqrt{b_1}\right) = \frac{6\left(n-2\right)}{\left(n+1\right)\left(n+3\right)}, \quad E\left(b_2\right) = \frac{3\left(n-1\right)}{n+1}, \quad Var\left(b_2\right) = \frac{24n\left(n-2\right)\left(n-3\right)}{\left(n+1\right)^2\left(n+3\right)\left(n+5\right)}.$$

32. Rahman-Govindarajulu (Rahman and Govindarajulu (1997)) have proposed a modification to the Shapiro-Wilk test, hereon termed W_{RG}, which relies on a new definition of the weights a as follows,

$$a_i = -\left(n+1\right)\left(n+2\right)\phi\left(m_i\right)\left[m_{i-1}\phi\left(m_{i-1}\right) - 2m_i\phi\left(m_i\right) + m_{i+1}\phi\left(m_{i+1}\right)\right],$$

where it is assumed that $m_0\phi\left(m_0\right) = m_{n+1}\phi\left(m_{n+1}\right) = 0$. With this modification, the new test statistic W_{RG} assigns larger weights to the extreme order statistics than the original W test, which has been seen to result in higher power against short tailed alternative distributions. As for the original W test, the normality hypothesis of the data is rejected for small values of W_{RG}.

33. The 1st Zhang Q test (Zhang (1999)) is based on the ratio of two unbiased estimators of standard deviation, q_1 and q_2, and given by

$$Q = ln(q_1/q_2).$$

The estimators q_1 and q_2 are obtained by

$$q_1 = \sum\nolimits_{i=1}^{n} a_i x_{(i)} \tag{10}$$

and

$$q_2 = \sum\nolimits_{i=1}^{n} b_i x_{(i)}, \tag{11}$$

where the ith order linear coefficients a_i and b_i result from

$$a_i = \left[\left(u_i - u_1\right)\left(n-1\right)\right]^{-1}, \quad \text{for } i \neq 1; \quad a_1 = \sum\nolimits_{i=2}^{n} a_i;$$

$$b_i = \begin{cases} -b_{n-i+1} = \left[\left(u_i - u_{i+4}\right)\left(n-4\right)\right]^{-1} & i = 1, \ldots, 4 \\ \left(n-4\right)^{-1} \cdot \left[\left(u_i - u_{i+4}\right)^{-1} - \left(u_{i-4} - u_i\right)^{-1}\right] & i = 5, \ldots, n-4 \end{cases}.$$

The ith expected value of the order statistics of a standard normal distribution, u_i , is defined by $\Phi^{-1}[(i-0.375)/(n+0.25)]$. According to Zhang (Zhang (1999)), Q is less powerful against negatively skewed distributions. Q approximately follows normal distribution.

Based on the definition of Q, the normality hypothesis of the data is rejected for both small and large values of the statistic using a two-sided test.

34. The 2nd Zhang Q test (Zhang (1999)) is based on the the alternative statistic Q^* by switching the ith order statistics $x_{(i)}$ in Equation (10) and Equation (11) by $x^*_{(i)} = -x_{(n-i+1)}$, and obtain

$$Q^* = ln(q_1^*/q_2^*).$$

Q^* approximately follows normal distribution.

35. The Barrio-Cuesta-Matran-Rodriguez test (del Barrio, Cuesta-Albertos, Matrán, and Rodríguez-Rodríguez (1999)) for normality is based on the L_2-Wasserstein distance between a sample distribution and the set of normal distributions as a measure of nonnormality,

$$BCMR = \frac{m_2 - \left[\sum_{i=1}^{n} x_{(i)} \cdot \int_{(i-1)/n}^{i/n} \Phi^{-1}(t)\, dt\right]^2}{m_2},$$

where the numerator represents the squared L_2-Wasserstein distance, m_2 is the sample standardized second moment. The normality hypothesis of the data is rejected for large values of the test statistic.

36. Glen-Leemis-Barr test (Glen, Leemis, and Barr (2001))for normality is based on the quantiles of the order statistics. Given the relation between the order statistics and the empirical distribution function. The Glen-Leemis-Barr test statistic is given by

$$P_S = -n - \frac{1}{n}\sum_{i=1}^{n} \left[(2n+1-2i)\ln\left(p_{(i)}\right) + (2i-1)\ln\left(1-p_{(i)}\right)\right],$$

where $p_{(i)}$ are the elements of the vector p containing the quantiles of the order statistics sorted in ascending order. The elements of p can be obtained by defining vector u, with elements sorted in ascending order and given by $u_{(i)} = \Phi(z_{(i)})$.Considering that $u_{(1)}, u_{(2)}, \ldots, u_{(n)}$ represent the order statistics of a sample taken from a uniform distribution $U(0,1)$, their quantiles, which correspond to the elements of p, can be determined knowing that $u_{(i)}$ follows a Beta distribution $B(i; n-i+1)$. The normality hypothesis of the data is rejected for large values of the test statistic.

37. Bonett and Seier (Bonett and Seier (2002)) test is based on a modification of Geary's measure of kurtosis (Geary (1935)), Z_w,

$$Z_w = \frac{\sqrt{n+2}\,(\widehat{w}-3)}{3.54},$$

where $\widehat{w} = 13.29\left(\ln\sqrt{m_2} - \log\left(n^{-1}\sum_{i=1}^{n}|x_i - \overline{x}|\right)\right)$. Z_w approximately follows a standard normal distribution. The normality hypothesis H_0 is rejected for both small and large values of Z_w using a two-sided test.

38. The Brys–Hubert–Struyf MC–LR test (Brys, Hubert, and Struyf (2008)) is based on robust measures of skewness and tail weight. The considered robust measure of skewness is the medcouple MC defined as

$$MC = \underset{x_{(i)} \le m_F \le x_{(j)}}{med} h\left(x_{(i)}, x_{(j)}\right),$$

where med stands for median, m_F is the sample median and the kernel function h is given by

$$h\left(x_{(i)}, x_{(j)}\right) = \frac{\left(x_{(j)} - m_F\right) - \left(m_F - x_{(i)}\right)}{x_{(i)} - x_{(j)}}.$$

For the case where $x_{(i)} = x_{(j)} = m_F$, h is then set by

$$h\left(x_{(i)}, x_{(j)}\right) = \begin{cases} 1 & i > j \\ 0 & i = j \\ -1 & i < j. \end{cases}$$

The left medcouple (LMC) and the right medcouple (RMC) are the considered robust measures of left and right tail weight, respectively, and are defined by

$$LMC = -MC\,(x < m_F)\,, \quad RMC = MC\,(x > m_F)\,.$$

The test statistic T_{MC-LR} is then defined by

$$T_{MC-LR} = n\,(w-\omega)^t \cdot V^{-1} \cdot (w-\omega)$$

in which w is set as $[MC, LMC, RMC]^t$, and ω and V are obtained based on the influence function of the estimators in w. For the case of a normal distribution, ω and V are defined as

$$\omega = \begin{bmatrix} 0, & 0.199 & 0.199 \end{bmatrix}^t, \quad V = \begin{bmatrix} 1.25 & 0.323 & -0.323 \\ 0.323 & 2.62 & -0.0123 \\ -0.323 & -0.0123 & 2.62 \end{bmatrix}.$$

The normality hypothesis of the data is rejected for large values of T_{MC-LR}. Note that T_{MC-LR} approximately follows the chi-square distribution with three degrees of freedom.

39. The Brys–Hubert–Struyf–Bonett–Seier joint test. The Brys–Hubert–Struyf MC–LR test (Brys *et al.* (2008)) is a skewness associated test and that the Bonett and Seier test (Bonett and Seier (2002)) is a kurtosis based test, a joint test, termed $T_{MC-LR} - Z_w$, considering both these measures is proposed herein for testing normality. This joint test attempts to make use of the two referred focused tests in order to increase the power to detect different kinds of departure from normality. This joint test is proposed herein based on the assumption that the individual tests can be considered independent. The normality hypothesis of the data is rejected for the joint test when rejection is obtained for either one of the two individual tests for a significance level of $\alpha/2$.

40. The 1st Zhang-Wu test (Zhang and Wu (2005)) for normality is

$$Z_C = \sum\nolimits_{i=1}^{n} \left[\ln \frac{\left(1/\Phi\left(z_{(i)}\right)-1\right)}{(n-0.5)\,/\,(i-0.75)-1}\right]^2,$$

the normality hypothesis of the data is rejected for large values of the test statistic.

41. The 2nd Zhang-Wu test (Zhang and Wu (2005)) for normality is

$$Z_A = -\sum\nolimits_{i=1}^{n} \left[\frac{\ln \Phi\left(z_{(i)}\right)}{n-i+0.5} + \frac{\ln\left[1-\Phi\left(z_{(i)}\right)\right]}{i-0.5}\right],$$

the normality hypothesis of the data is rejected for large values of the test statistic.

42. The 1st Bontemps and Meddahi (Bontemps and Meddahi (2005)) proposed a family of normality tests based on moment conditions known as Stein equations and their relation with Hermite polynomials.The test statistics are developed using the generalized method of moments approach associated with Hermite polynomials, which leads to test statistics that are robust against parameter uncertainty. The general expression of the test family is thus given by

$$BM_{3-p} = \sum\nolimits_{k=3}^{p} \left(\frac{1}{\sqrt{n}} \sum\nolimits_{i=1}^{n} H_k\left(z_i\right)\right)^2, \qquad (12)$$

where $z_i = (x_i - \overline{x})/s$ and $H_k(\cdot)$ represents the kth order normalized Hermite polynomial having the general expression given by Equation (7). The general BM_{3-p} family of tests asymptotically follows the chi-square distribution with $p-2$ degrees of freedom

It can be seen from Equation (12) that a number of different tests can be obtained by assigning different values to p, which represents the maximum order of the considered normalized Hermite polynomials. Two different tests are considered in Bontemps-Meddahi (Bontemps and Meddahi (2005)), these tests are termed BM_{3-4} and BM_{3-6}. Thus, The 1st Bontemps-Meddahi test for normality (Bontemps and Meddahi (2005)) is given by

$$BM_{3-4} = \sum\nolimits_{k=3}^{4} \left(\frac{1}{\sqrt{n}} \sum\nolimits_{i=1}^{n} H_k(z_i) \right)^2,$$

the normality hypothesis of the data is rejected for large values of the test statistic.

43. The 2nd Bontemps-Meddahi test for normality (Bontemps and Meddahi (2005)) is given by

$$BM_{3-6} = \sum\nolimits_{k=3}^{6} \left(\frac{1}{\sqrt{n}} \sum\nolimits_{i=1}^{n} H_k(z_i) \right)^2,$$

the normality hypothesis of the data is rejected for large values of the test statistic.

44. The Gel-Miao-Gastwirth test for normality (Gel, Miao, and Gastwirth (2007)) is based on the ratio of the standard deviation s and on the robust measure of dispersion J_n,

$$J_n = \frac{\sqrt{\pi/2}}{n} \sum\nolimits_{i=1}^{n} |x_i - M| \tag{13}$$

in which M is the sample median. The normality test statistic R_{sJ} is therefore given by

$$R_{sJ} = s/Jn$$

which should tend to one under a normal distribution. The normality hypothesis of the data is rejected for large values of R_{sJ} and the statistic $\sqrt{n}(R_{sJ} - 1)$ asymptotically follows the normal distribution $N(0, \pi/2 - 1.5)$. The Gel-Miao-Gastwirth test has higher power against heavy-tailed observations.

45. The Doornik-Hansen test for normality (Doornik and Hansen (2008)) is based on the transformed skewness in Equation (4) and the use of a transformed kurtosis z_2. The statistic

of the Doornik–Hansen test DH is thus given by

$$DH = \left[Z\left(\sqrt{b_1}\right) \right]^2 + [z_2]^2$$

and the transformed kurtosis z_2 is given by

$$z_2 = \left[\left(\frac{\xi}{2a} \right)^{1/3} - 1 + \frac{1}{9a} \right] \sqrt{9a},$$

and

$$\xi = (b_2 - 1 - b_1)\, 2k,$$

$$k = \frac{(n+5)(n+7)\left(n^3 + 37n^2 + 11n - 313\right)}{12(n-3)(n+1)(n^2 + 15n - 4)},$$

$$a = \frac{(n+5)(n+7)\left((n-2)\left(n^2+27n-70\right)+b_1(n-7)\left(n^2+2n-5\right)\right)}{6(n-3)(n+1)\left(n^2+15n-4\right)}.$$

The null hypothesis H_0 is rejected for large values of DH.

46. Coin test (Coin (2008)) for normality is based on a polynomial regression focused on detecting symmetric non-normal alternative distributions. According to Coin (Coin (2008)), the analysis of standard normal $Q-Q$ plots of different symmetric non-normal distributions suggests that fitting a model of the type

$$z_{(i)} = \beta_1 \cdot \alpha_i + \beta_3 \cdot \alpha_i^3,$$

where β_1 and β_3 are fitting parameters and α_i represent the expected values of standard normal order statistics, leads to values β_3 different from zero when in presence of symmetric non-normal distributions. Therefore, Coin (Coin (2008)) suggests the use of β_3^2 as a statistic for testing normality, thus rejecting the normality hypothesis of the data for large values of β_3^2. The values of α_i are obtained using the approximations provided in Royston (1982).

47. The Gel-Gastwirth Robust Jarque-Bera test. Gel and Gastwirth (Gel and Gastwirth (2008)) proposed a modification of Jarque-Bera test that uses a robust estimate of the dispersion in the skewness and kurtosis instead of the second order central moment m_2. The selected robust dispersion measure is the average absolute deviation from the median and leads to the following statistic of the Robust Jarque–Bera test RJB given by

$$RJB = \frac{n}{6}\left(\frac{m_3}{J_n^3}\right)^2 + \frac{n}{64}\left(\frac{m_4}{J_n^4}\right)^2$$

with J_n obtained from Equation (13). The normality hypothesis of the data is rejected for large values of the test statistic. RJB asymptotically follows the chi-square distribution with two degrees of freedom. RJB test is more powerful in detecting moderately heavy-tailed departures from normality, especially in small and moderate samples.

48. The Desgagne-LafayeDeMicheaux-Leblanc (Desgagné, de Micheaux, and Leblanc (2013)) test for normality is tailored to detect departures from normality in the tails of the distribution. The proposed test for normality is given by

$$R_n = n\mathbf{r}_n\left(\overline{X}_n, S_n\right)^T \left(\mathbf{J}_0 - \frac{1}{2}\boldsymbol{\nu}_0\boldsymbol{\nu}_0^T\right)^{-1} \mathbf{r}_n\left(\overline{X}_n, S_n\right)$$

when $X \sim N\left(\mu, \sigma^2\right)$,

$$\mathbf{r}_n\left(\overline{X}_n, S_n\right) = \mathbf{r}_n(\mu, \sigma) - \frac{1}{2}(1-T_n)\boldsymbol{\nu}_0 + o_P\left(n^{-\frac{1}{2}}\right)\mathbf{1}_3,$$

$$T_n = \frac{1}{n}\sum\nolimits_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2,$$

and where $\mathbf{1}_3 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ and $\mathbf{J}_0$ is

$$\mathbf{J}_0 = \begin{pmatrix} 042423099 & 013285410 & 005401580 \\ 013285410 & 010080860 & 004056528 \\ 005401580 & 004056528 & 001632673 \end{pmatrix},$$

and the vector $\boldsymbol{\nu}_0$ is

$$\boldsymbol{\nu}_0 = \begin{pmatrix} 086481850 & 038512925 & 015594892 \end{pmatrix}^T,$$

$$\mathbf{J}_0 - \frac{1}{2}\boldsymbol{\nu}_0\boldsymbol{\nu}_0^T = \begin{pmatrix} 00502754623 & -00336793487 & -00134179540 \\ -00336793487 & 00266463308 & 00105350321 \\ -00134179540 & 00105350321 & 000416669944 \end{pmatrix}.$$

Under the null hypothesis R_n approximately follows the chi-square distribution with three degrees of freedom.

49. The Desgagne-LafayeDeMicheaux X_{APD} test for normality (Desgagné and de Micheaux (2018)) for finite sample sizes $n \geq 10$ is given by

$$X_{APD} = \frac{nB_2^2}{(3-8/\pi)(1-1.9/n)} + \frac{n\left[\left(K_2 - B_2^2\right)^{1/3} - ((2-\log 2-\gamma)/2)^{1/3}(1-1.026/n)\right]^2}{72^{-1}((2-\log 2-\gamma)/2)^{-4/3}(3\pi^2-28)(1-2.25/n^{0.8})},$$

the '2nd-power skewness' and '2nd-power kurtosis' are, respectively, denoted by B_2 and K_2, and defined by (Desgagné and de Micheaux (2018)) as

$$B_2 = \frac{1}{n}\sum\nolimits_{i=1}^{n} Z_i^2 \operatorname{sign}(Z_i),$$

$$K_2 = \frac{1}{n}\sum\nolimits_{i=1}^{n} Z_i^2 \log|Z_i|, \tag{14}$$

$$Z_i = S_n^{-1}\left(X_i - \overline{X}_n\right),$$

$$\overline{X}_n = \frac{1}{n}\sum X_i,$$

$$S_n = \left[\frac{1}{n}\sum\left(X_i - \overline{X}_n\right)^2\right]^{1/2},$$

$$\gamma = 0.577215665..., \tag{15}$$

γ is the Euler–Mascheroni constant. X_{APD} approximately follows the chi-square distribution with two degrees of freedom for all $n \geq 10$.

50. The Desgagne-LafayeDeMicheaux Z_{EPD} test for normality (Desgagné and de Micheaux (2018)) for finite sample sizes $n \geq 10$, is given by

$$Z_{EPD} = \frac{n^{1/2}\left[\left((2K_2)^{\alpha_n}-1\right)/\alpha_n + \left((2-\log 2-\gamma)^{-0.06}-1\right)/0.06 + 1.32/n^{0.95}\right]}{\left[(2-\log 2-\gamma)^{-2.12}(3\pi^2-28)/2 - 3.78/n^{0.733}\right]^{1/2}},$$

where K_2 is given in Equation (14) and γ in Equation (15) and

$$\alpha_n = -0.06 + 2.1/n^{0.67}.$$

Z_{EPD} approximately follows the standard normal distribution $N(0,1)$ for all $n \geq 10$.

3. Statistical distributions used in the simulation study

The simulation study uses a number of alternative statistical distributions over which the performance of the presented normality tests is to be assessed. The selected alternative distributions were chosen in order to be a representative set exhibiting different values of important properties such as skewness and kurtosis. Following Esteban, Castellanos, Morales, and Vajda (2001), these alternative distributions are categorized into four groups, depending on the support and shape of their densities as follows:

1. Group I: Symmetric distributions with support on $(-\infty, \infty)$:
   - Student's t-distribution with 3 degrees of freedom.
   - Laplace distribution with parameters location = 0 and scale = 1.
   - Logistic distribution with parameters location = 0 and scale = 4.
2. Group II: Asymmetric distributions with support on $(-\infty, \infty)$:
   - Gumbel distribution with parameters location = 0 and scale = 3.
   - Skew-Normal with parameters location = 0, scale = 1 and slant = 7.
3. Group III: Distributions with support on $(0, \infty)$:
   - Gamma distribution with parameters shape = 2 and scale = 1.
   - Weibull distribution with parameters shape = 1.5 and scale = 1.
4. Group IV: Distributions with support on $(0, 1)$:
   - Beta distribution with parameters shape1 = 2 and shape2 = 5.
   - Uniform distribution with parameters (0,1).

4. Simulation study

Since a theoretical comparison is not possible, power comparisons of tests for normality are made by using Monte Carlo simulation. To compare the power of the tests we generate samples of sizes $n = 10$, 30 , 50 , 70 and 100 from the alternative distributions in Section 3. The number of simulation is 10000 and the level of significance $\alpha = 0.05$. We compute the power of the test as the proportion of times we correctly reject the null hypothesis in 10000 replications at $\alpha = 0.05$ level of significance. For doing the simulation and computing the estimated powers of the tests for normality, R language (R Core Team (2021)) and the R packages are used. The followings R packages are used to test the normality: DescTools (Signorell (2020)), evd (Stephenson (2002)), fBasics (Wuertz, Setz, and Chalabi (2020)), lawstat (Gastwirth, Gel, Hui, Lyubchich, Miao, and Noguchi (2020)), moments (Komsta and Novomestky (2015)), normtest (Gavrilov and Pusev (2014)), nortest (Gross and Ligges (2015)), PoweR (Lafaye de Micheaux and Tran (2016)), rmutil (Swihart and Lindsey (2020)), sn (Azzalini (2021)), and vsgoftest (Lequesne and Regnault (2020)).

Table 1 through 9 respectively report the estimates of the power of the 50 tests for normality, in order of increasing power, under the alternative distributions in Section 3.

The difference of the power of the tests becomes more apparent when the comparison is carried out graphically. Figures 1 through 17 respectively present the simulated power curves for 50 normality tests under the alternative distributions in Section 3 for sample $n = 10$, 30 , 50 , 70 and 100 based on the results of Table 1 through 9 respectively. The vertical axis of the figures measure the simulated power of the tests for normality and the horizontal axis represents the sample sizes n.

Table 1: Power comparisons of normality tests under alternative Student's t-distribution with 3 degrees of freedom for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Geary	0.0230	0.0042	0.0010	0.0002	0.0000
2	Brys-Hubert-Struyf	0.0024	0.0332	0.0478	0.0612	0.0694
3	Kolmogorov-Smirnov	0.0084	0.0500	0.1024	0.1624	0.2366
4	4th Hosking	0.0536	0.0840	0.0984	0.1220	0.1448
5	3rd Hosking	0.0572	0.1112	0.1444	0.1898	0.2230
6	Vasicek-Song	0.0396	0.1612	0.3066	0.4342	0.7196
7	2nd Hosking	0.0878	0.2034	0.2728	0.3814	0.4792
8	Pearson Chi-Square	0.1510	0.2330	0.3300	0.4216	0.5206
9	1st Zhang	0.1462	0.2932	0.3886	0.4702	0.5534
10	Rahman-Govindarajulu	0.1540	0.2968	0.4336	0.5502	0.6854
11	2nd Zhang	0.1480	0.2988	0.3870	0.4676	0.5520
12	Barrio-Cuesta-Matran-Rodriguez	0.1452	0.3008	0.3866	0.4690	0.5496
13	Lilliefors	0.1628	0.3348	0.4722	0.5974	0.7314
14	Frosini	0.1712	0.3896	0.5498	0.6904	0.8188
15	Cramer-von Mises	0.1818	0.4056	0.5634	0.6988	0.8204
16	Skewness	0.2168	0.4186	0.5200	0.5856	0.6376
17	D'Agostino-skewness	0.2172	0.4196	0.5198	0.5852	0.6394
18	Anderson-Darling	0.1898	0.4356	0.5948	0.7252	0.8480
19	Hegazy–Green 1	0.1820	0.4360	0.6030	0.7430	0.8674
20	1st Bontemps-Meddahi	0.0666	0.4378	0.6384	0.7860	0.8936
21	Chen-Shapiro	0.1930	0.4396	0.6048	0.7358	0.8552
22	Glen-Leemis-Barr	0.1956	0.4408	0.5996	0.7312	0.8536
23	Epps-Pulley	0.2032	0.4454	0.5984	0.7364	0.8576
24	1st Cabana-Cabana	0.2164	0.4474	0.5744	0.6766	0.7692
25	1st Zhang-Wu	0.1882	0.4490	0.6100	0.7398	0.8520
26	2nd Zhang-Wu	0.2024	0.4524	0.5968	0.7206	0.8350
27	Shapiro-Wilk	0.1914	0.4548	0.6256	0.7628	0.8740
28	Anscombe-Glynn	0.1758	0.4592	0.6322	0.7784	0.8866
29	Bonett-Seier	0.1426	0.4610	0.6508	0.7956	0.9026
30	Desgagné-LafayeDeMicheaux-Leblanc	0.1296	0.4614	0.6494	0.7912	0.8942
31	Spiegelhalter	0.1580	0.4772	0.6926	0.8382	0.9282
32	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.1778	0.4820	0.6778	0.8186	0.9170
33	Brys-Hubert-Struyf & Bonett-Seier	0.2388	0.4842	0.6652	0.7974	0.8970
34	Coin	0.1786	0.4918	0.6852	0.8136	0.9170
35	1st Hosking	0.2184	0.4932	0.6438	0.7800	0.8802
36	D'Agostino-Pearson	0.2318	0.5018	0.6518	0.7772	0.8746
37	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1976	0.5108	0.6838	0.8202	0.9114
38	2nd Cabana-Cabana	0.1498	0.5120	0.6978	0.8306	0.9168
39	Weisberg–Bingham	0.2092	0.5124	0.6816	0.8098	0.9038
40	Doornik-Hansen	0.2054	0.5158	0.6858	0.8142	0.9036
41	Jarque–Bera	0.2192	0.5168	0.6812	0.8136	0.9052
42	Shapiro-Francia	0.2166	0.5180	0.6866	0.8140	0.9064
43	Filliben	0.2140	0.5198	0.6912	0.8174	0.9094
44	Kurtosis	0.1612	0.5200	0.6978	0.8286	0.9174
45	Adjusted Jarque–Bera	0.2196	0.5262	0.6946	0.8210	0.9100
46	2nd Bontemps-Meddahi	0.1422	0.5304	0.7276	0.8576	0.9380
47	Hegazy–Green 2	0.2184	0.5330	0.7054	0.8326	0.9196
48	Martinez-Iglewicz	0.2178	0.5618	0.7416	0.8602	0.9422
49	Gel–Miao–Gastwirth	0.2576	0.5676	0.7310	0.8544	0.9356
50	Robust Jarque–Bera	0.2376	0.5726	0.7468	0.8516	0.9346

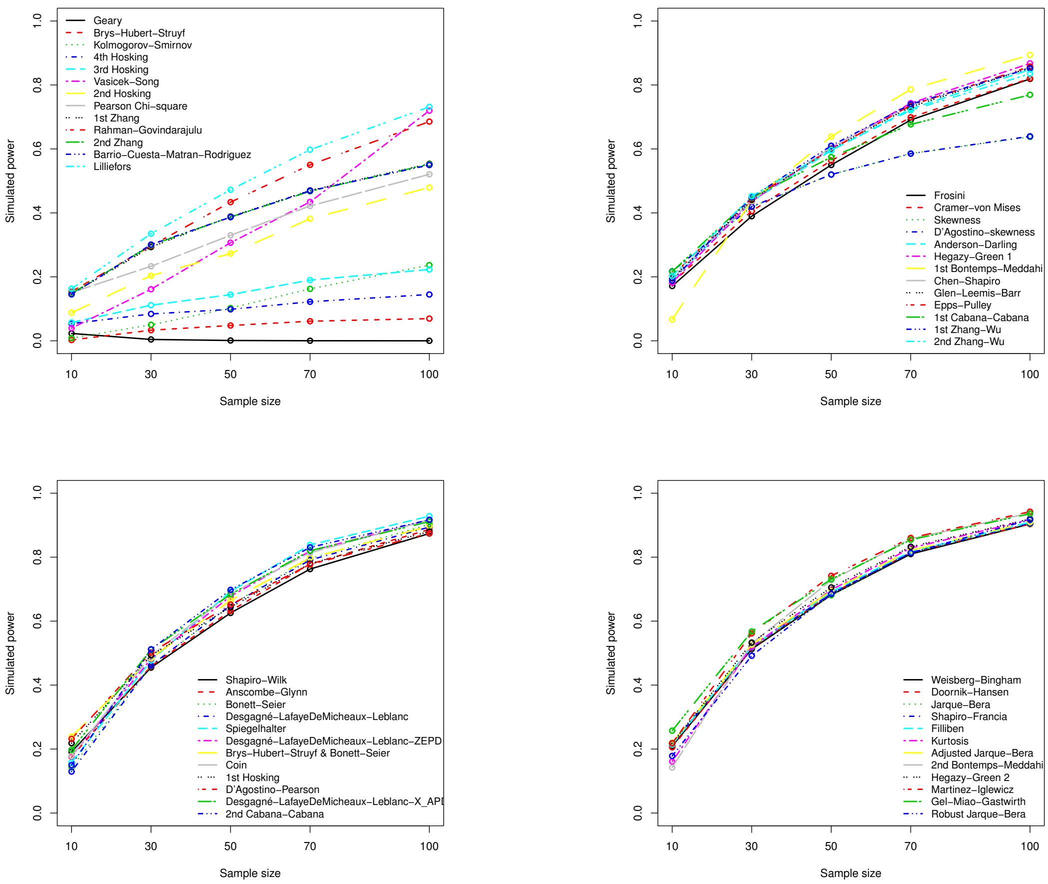

Figure 1: Simulated power curves for 50 normality tests under alternative Student's t-distribution with 3 degrees of freedom for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

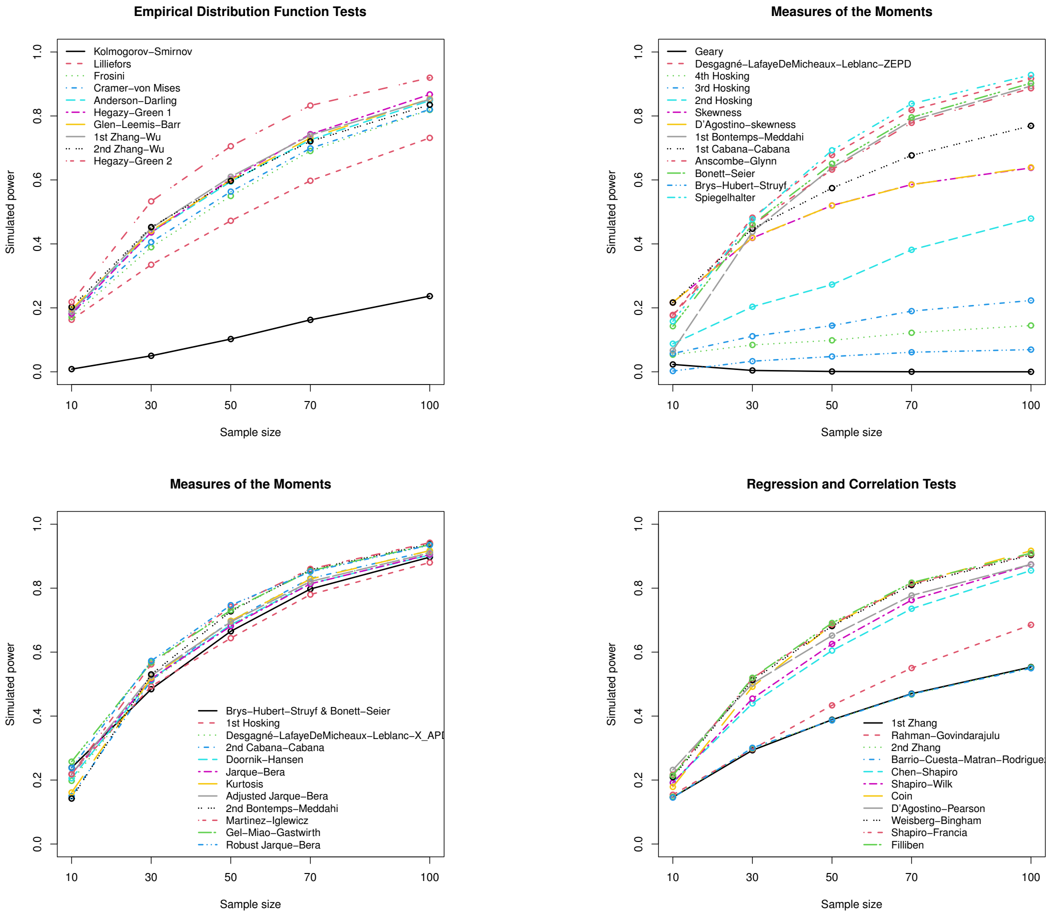

Figure 2: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Student's t-distribution with 3 degrees of freedom for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 2: Power comparisons of normality tests under alternative Laplace distribution $(0, 1)$ for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Geary	0.0204	0.0022	0.0004	0.0002	0.0000
2	Kolmogorov-Smirnov	0.0002	0.0120	0.0278	0.0470	0.0840
3	Brys-Hubert-Struyf	0.0008	0.0322	0.0512	0.0782	0.0978
4	Vasicek-Song	0.0170	0.0854	0.1880	0.2734	0.6148
5	4th Hosking	0.0488	0.1434	0.2240	0.3094	0.4058
6	Rahman-Govindarajulu	0.1184	0.1852	0.2732	0.3564	0.4864
7	Pearson Chi-Square	0.1282	0.1932	0.2704	0.3604	0.4710
8	3rd Hosking	0.0650	0.1942	0.2954	0.4056	0.5248
9	1st Zhang	0.1204	0.2216	0.2648	0.3100	0.3766
10	Barrio-Cuesta-Matran-Rodriguez	0.1196	0.2220	0.2696	0.3146	0.3728
11	2nd Zhang	0.1214	0.2246	0.2740	0.3190	0.3722
12	2nd Hosking	0.1082	0.2810	0.4308	0.5610	0.7080
13	Lilliefors	0.1356	0.2856	0.4350	0.5486	0.7072
14	D'Agostino-skewness	0.1726	0.3048	0.3596	0.3886	0.4164
15	Skewness	0.1706	0.3054	0.3618	0.3874	0.4150
16	1st Bontemps-Meddahi	0.0316	0.3060	0.4940	0.6292	0.7758
17	1st Zhang-Wu	0.1494	0.3234	0.4598	0.5658	0.7000
18	1st Cabana-Cabana	0.1742	0.3270	0.4266	0.4844	0.5708
19	2nd Zhang-Wu	0.1644	0.3298	0.4566	0.5590	0.6874
20	Chen-Shapiro	0.1510	0.3316	0.4902	0.6078	0.7626
21	Anscombe-Glynn	0.1366	0.3340	0.4870	0.6262	0.7694
22	Frosini	0.1412	0.3356	0.5220	0.6650	0.8176
23	Epps-Pulley	0.1628	0.3506	0.5258	0.6538	0.8070
24	Shapiro-Wilk	0.1496	0.3538	0.5260	0.6524	0.7932
25	Cramer-von Mises	0.1520	0.3632	0.5436	0.6794	0.8236
26	Anderson-Darling	0.1556	0.3640	0.5518	0.6908	0.8276
27	Hegazy–Green 1	0.1454	0.3706	0.5586	0.7032	0.8402
28	Glen-Leemis-Barr	0.1574	0.3724	0.5588	0.6968	0.8298
29	D'Agostino-Pearson	0.1926	0.3862	0.5104	0.6164	0.7378
30	Desgagné-LafayeDeMicheaux-Leblanc	0.1086	0.3862	0.5722	0.7182	0.8542
31	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.1434	0.3972	0.6058	0.7518	0.8882
32	Doornik-Hansen	0.1720	0.4084	0.5558	0.6840	0.8030
33	Jarque–Bera	0.1774	0.4090	0.5580	0.6734	0.8020
34	2nd Cabana-Cabana	0.1126	0.4106	0.5922	0.7234	0.8468
35	Kurtosis	0.1258	0.4130	0.5820	0.7176	0.8388
36	Bonett-Seier	0.1132	0.4156	0.6336	0.7790	0.9098
37	Coin	0.1580	0.4194	0.5976	0.7416	0.8662
38	Adjusted Jarque–Bera	0.1806	0.4224	0.5756	0.6920	0.8112
39	Weisberg–Bingham	0.1730	0.4226	0.5962	0.7210	0.8454
40	Shapiro-Francia	0.1822	0.4276	0.6014	0.7230	0.8492
41	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1642	0.4278	0.6138	0.7476	0.8766
42	Filliben	0.1772	0.4310	0.6038	0.7250	0.8484
43	2nd Bontemps-Meddahi	0.1010	0.4354	0.6390	0.7772	0.8944
44	Spiegelhalter	0.1300	0.4366	0.6870	0.8342	0.9420
45	1st Hosking	0.1882	0.4378	0.6168	0.7500	0.8738
46	Brys-Hubert-Struyf & Bonett-Seier	0.2200	0.4404	0.6316	0.7736	0.9046
47	Hegazy–Green 2	0.1788	0.4500	0.6260	0.7466	0.8664
48	Martinez-Iglewicz	0.1862	0.4934	0.6802	0.8010	0.9078
49	Robust Jarque–Bera	0.1990	0.5018	0.6786	0.7982	0.8978
50	Gel–Miao–Gastwirth	0.2488	0.5528	0.7474	0.8610	0.9500

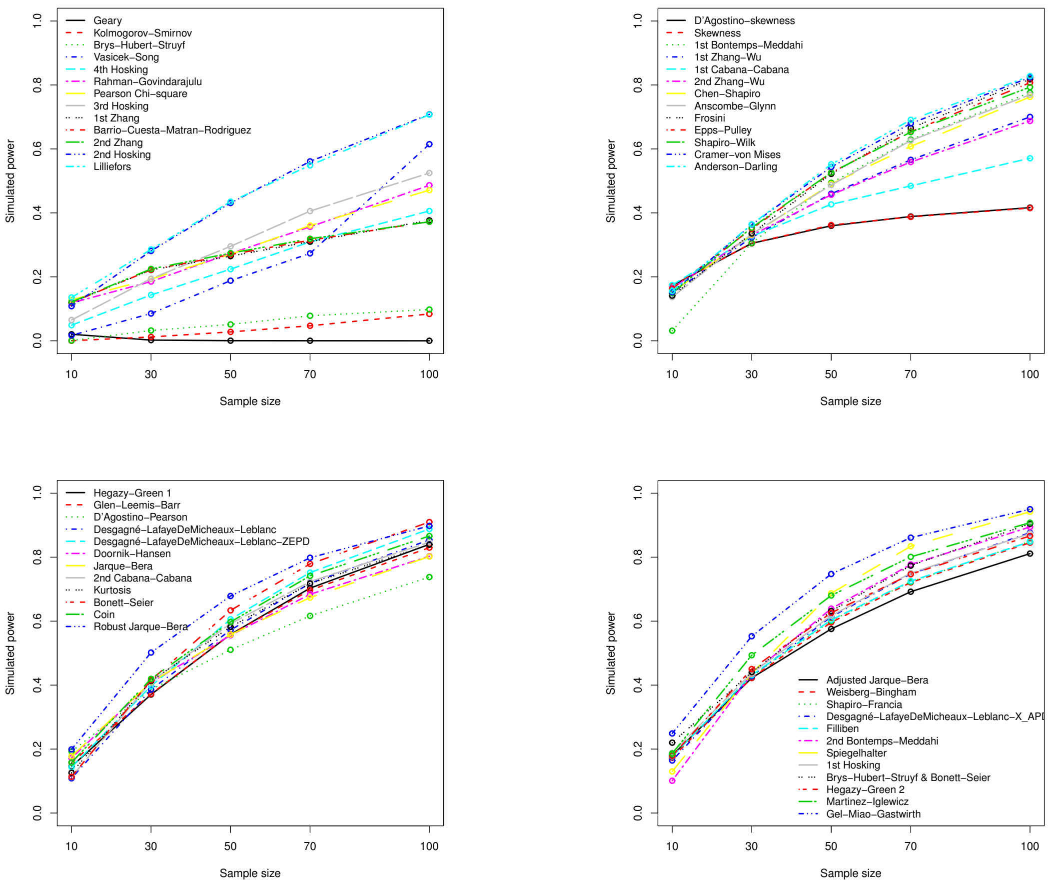

Figure 3: Simulated power curves for 50 normality tests under alternative Laplace distribution $(0, 1)$ for sample sizes $n = 10$, 30, 50, 70 and 100 $(\alpha = 0.05)$

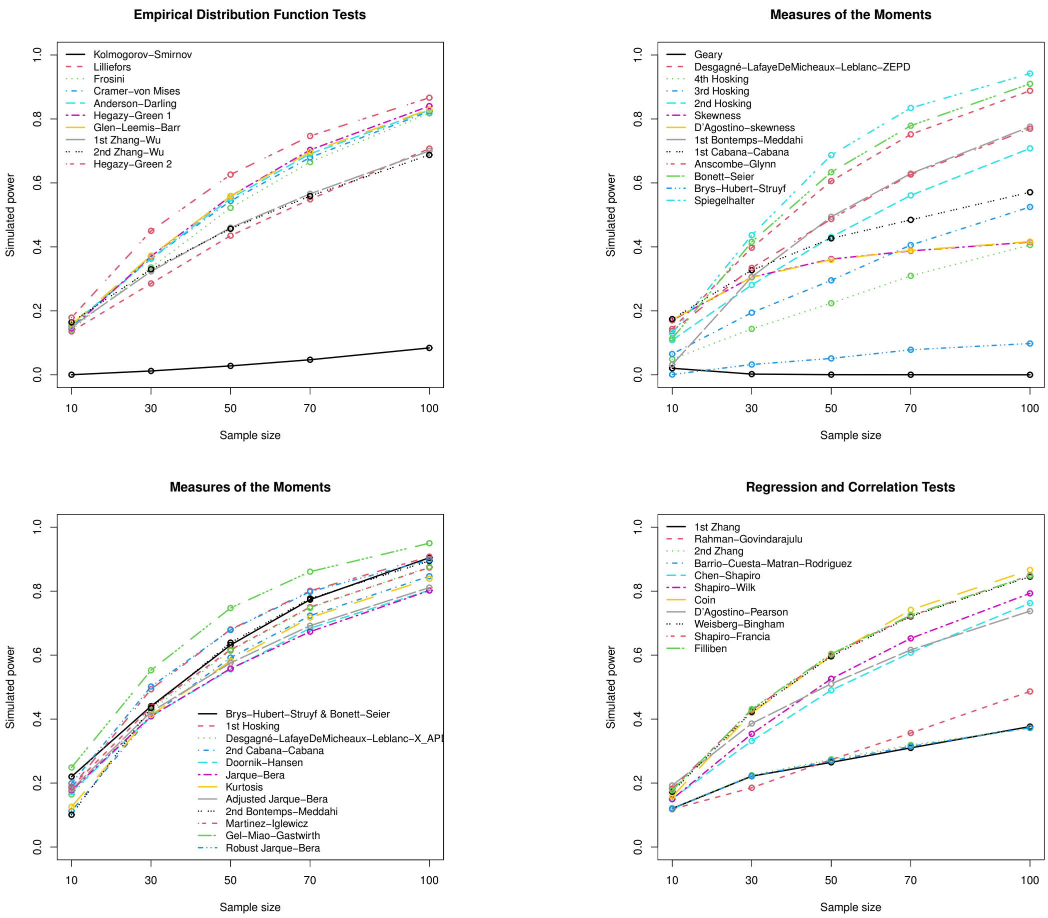

Figure 4: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Laplace distribution $(0, 1)$ for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 3: Power comparisons of normality tests under alternative Logistic distribution ($location = 0,\ scale = 4$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0000	0.0008	0.0014	0.0012	0.0012
2	Geary	0.0402	0.0156	0.0144	0.0062	0.0034
3	Vasicek-Song	0.0136	0.0224	0.0364	0.0460	0.1572
4	Brys-Hubert-Struyf	0.0008	0.0302	0.0396	0.0422	0.0358
5	4th Hosking	0.0492	0.0578	0.0690	0.0730	0.0660
6	3rd Hosking	0.0508	0.0642	0.0768	0.0822	0.0820
7	Pearson Chi-Square	0.0784	0.0668	0.0816	0.0834	0.0858
8	Rahman-Govindarajulu	0.0640	0.0764	0.0926	0.0904	0.1040
9	2nd Hosking	0.0632	0.0832	0.1030	0.1142	0.1216
10	Lilliefors	0.0700	0.0884	0.1146	0.1296	0.1502
11	Frosini	0.0702	0.0990	0.1382	0.1604	0.1972
12	Cramer-von Mises	0.0732	0.1070	0.1464	0.1676	0.2016
13	1st Zhang	0.0764	0.1154	0.1418	0.1598	0.1870
14	2nd Zhang	0.0752	0.1216	0.1420	0.1536	0.1938
15	Barrio-Cuesta-Matran-Rodriguez	0.0738	0.1216	0.1440	0.1556	0.1940
16	Anderson-Darling	0.0776	0.1228	0.1624	0.1846	0.2310
17	Hegazy–Green 1	0.0720	0.1238	0.1746	0.1994	0.2580
18	Epps-Pulley	0.0848	0.1262	0.1750	0.1976	0.2520
19	Glen-Leemis-Barr	0.0798	0.1276	0.1644	0.1904	0.2380
20	1st Bontemps-Meddahi	0.0096	0.1322	0.2164	0.2730	0.3716
21	Chen-Shapiro	0.0820	0.1358	0.1824	0.2148	0.2816
22	Bonett-Seier	0.0600	0.1388	0.1990	0.2436	0.3260
23	Desgagné-LafayeDeMicheaux-Leblanc	0.0574	0.1428	0.2144	0.2602	0.3414
24	Shapiro-Wilk	0.0812	0.1434	0.1988	0.2374	0.3114
25	2nd Zhang-Wu	0.0900	0.1444	0.1862	0.2050	0.2554
26	Anscombe-Glynn	0.0744	0.1452	0.2028	0.2462	0.3416
27	1st Zhang-Wu	0.0856	0.1470	0.2020	0.2368	0.3016
28	Spiegelhalter	0.0720	0.1484	0.2378	0.3096	0.4160
29	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.0796	0.1486	0.2160	0.2698	0.3678
30	1st Hosking	0.0920	0.1522	0.2060	0.2386	0.2926
31	Skewness	0.0980	0.1540	0.1932	0.2068	0.2262
32	D'Agostino-skewness	0.0980	0.1550	0.1936	0.2062	0.2276
33	1st Cabana-Cabana	0.0992	0.1588	0.2048	0.2258	0.2576
34	Coin	0.0776	0.1644	0.2344	0.2904	0.3874
35	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.0822	0.1684	0.2394	0.2864	0.3682
36	Weisberg–Bingham	0.0934	0.1742	0.2422	0.2920	0.3726
37	Shapiro-Francia	0.0986	0.1776	0.2446	0.2976	0.3754
38	Doornik-Hansen	0.0824	0.1780	0.2514	0.2970	0.3880
39	Filliben	0.0958	0.1790	0.2488	0.2976	0.3780
40	2nd Cabana-Cabana	0.0716	0.1828	0.2596	0.3154	0.4198
41	2nd Bontemps-Meddahi	0.0456	0.1828	0.2912	0.3716	0.4778
42	D'Agostino-Pearson	0.1132	0.1842	0.2444	0.2816	0.3510
43	Hegazy–Green 2	0.0966	0.1858	0.2608	0.3146	0.4012
44	Kurtosis	0.0772	0.1886	0.2648	0.3212	0.4176
45	Jarque–Bera	0.1032	0.1908	0.2608	0.3106	0.4014
46	Adjusted Jarque–Bera	0.1030	0.1958	0.2670	0.3188	0.4126
47	Martinez-Iglewicz	0.0940	0.2022	0.2876	0.3602	0.4616
48	Brys-Hubert-Struyf & Bonett-Seier	0.1832	0.2098	0.2586	0.2988	0.3630
49	Gel–Miao–Gastwirth	0.1240	0.2110	0.2832	0.3362	0.4332
50	Robust Jarque–Bera	0.1076	0.2292	0.3086	0.3700	0.4346

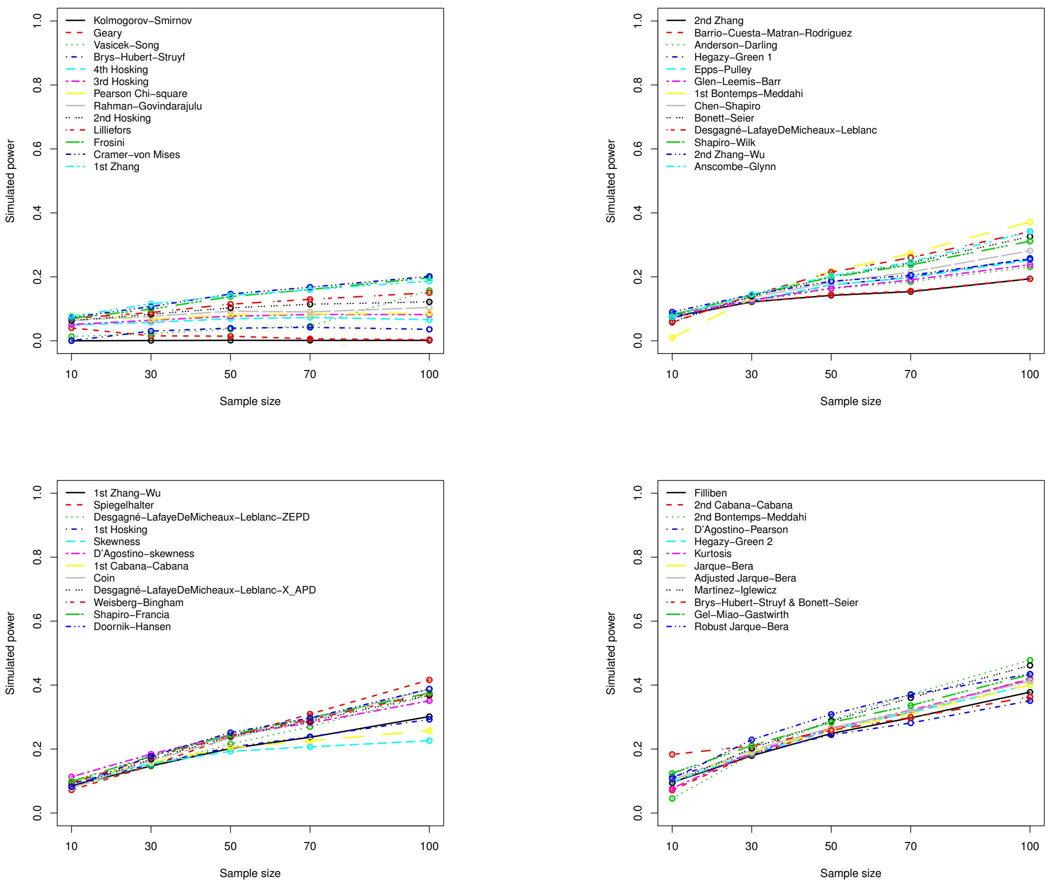

Figure 5: Simulated power curves for 50 normality tests under alternative Logistic distribution ($location = 0,\ scale = 4$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

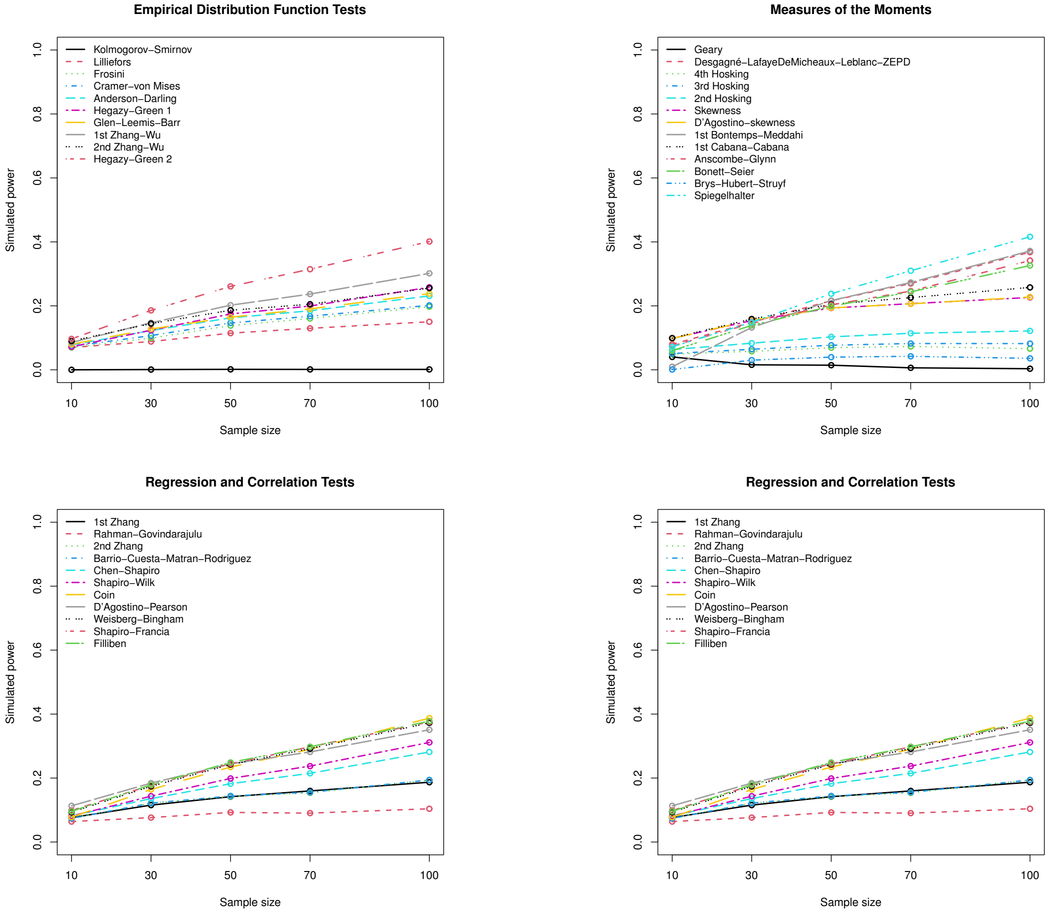

Figure 6: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Logistic distribution ($location = 0,\ scale = 4$) for sample sizes $n = 10,\ 30,\ 50,\ 70$ and 100 ($\alpha = 0.05$)

Table 4: Power comparisons of normality tests under alternative Gumbel distribution $(0,3)$ for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0006	0.0100	0.0294	0.0602	0.1002
2	Geary	0.0500	0.0342	0.0268	0.0178	0.0128
3	Brys-Hubert-Struyf	0.0028	0.0658	0.1160	0.1772	0.2516
4	4th Hosking	0.0548	0.0918	0.1468	0.1992	0.2858
5	3rd Hosking	0.0538	0.1268	0.2184	0.2884	0.4152
6	Coin	0.0706	0.1290	0.1580	0.1906	0.2212
7	Bonett-Seier	0.0650	0.1530	0.2034	0.2438	0.2964
8	Vasicek-Song	0.0342	0.1554	0.2782	0.4006	0.7846
9	Spiegelhalter	0.0780	0.1662	0.2350	0.2956	0.3664
10	Pearson Chi-Square	0.1248	0.1888	0.2740	0.3622	0.4904
11	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.0958	0.2042	0.2810	0.3538	0.4420
12	2nd Hosking	0.0748	0.2080	0.3548	0.4830	0.6494
13	Desgagné-LafayeDeMicheaux-Leblanc	0.0666	0.2234	0.3280	0.4326	0.5504
14	Brys-Hubert-Struyf & Bonett-Seier	0.1724	0.2344	0.3078	0.3770	0.4648
15	Anscombe-Glynn	0.1048	0.2388	0.3292	0.4188	0.5290
16	Gel–Miao–Gastwirth	0.1398	0.2662	0.3530	0.4260	0.5134
17	1st Zhang	0.0896	0.2688	0.4220	0.5594	0.6906
18	2nd Cabana-Cabana	0.0996	0.2704	0.3732	0.4740	0.5852
19	Kurtosis	0.1068	0.2768	0.3842	0.4844	0.5946
20	Lilliefors	0.1136	0.2794	0.4366	0.5784	0.7276
21	2nd Zhang	0.1328	0.2814	0.3766	0.4566	0.5472
22	Barrio-Cuesta-Matran-Rodriguez	0.1322	0.2820	0.3750	0.4552	0.5452
23	1st Bontemps-Meddahi	0.0252	0.3120	0.5332	0.7134	0.8800
24	Martinez-Iglewicz	0.1288	0.3512	0.4934	0.6062	0.7308
25	Cramer-von Mises	0.1294	0.3598	0.5418	0.6998	0.8416
26	Frosini	0.1320	0.3600	0.5484	0.7126	0.8482
27	Adjusted Jarque–Bera	0.1472	0.3874	0.5750	0.7394	0.8902
28	Glen-Leemis-Barr	0.1398	0.3948	0.5866	0.7424	0.8766
29	Anderson-Darling	0.1390	0.3970	0.5948	0.7552	0.8842
30	Doornik-Hansen	0.1198	0.4062	0.6384	0.8072	0.9306
31	1st Hosking	0.1228	0.4064	0.6310	0.7868	0.9168
32	Hegazy–Green 1	0.1326	0.4066	0.6162	0.7814	0.9078
33	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1228	0.4072	0.6362	0.7972	0.9238
34	D'Agostino-Pearson	0.1652	0.4120	0.5940	0.7526	0.8962
35	Robust Jarque–Bera	0.1472	0.4138	0.5948	0.7542	0.8860
36	Jarque–Bera	0.1554	0.4158	0.6108	0.7690	0.9052
37	Rahman-Govindarajulu	0.1350	0.4162	0.6200	0.7876	0.9130
38	Hegazy–Green 2	0.1500	0.4306	0.6282	0.7796	0.9108
39	2nd Bontemps-Meddahi	0.0790	0.4324	0.6674	0.8290	0.9406
40	Epps-Pulley	0.1534	0.4466	0.6462	0.7992	0.9172
41	Filliben	0.1550	0.4496	0.6524	0.8072	0.9222
42	Weisberg–Bingham	0.1510	0.4508	0.6576	0.8108	0.9258
43	Shapiro-Francia	0.1614	0.4584	0.6632	0.8152	0.9276
44	Chen-Shapiro	0.1520	0.4640	0.6778	0.8338	0.9432
45	1st Zhang-Wu	0.1560	0.4664	0.6626	0.8178	0.9318
46	Shapiro-Wilk	0.1516	0.4676	0.6802	0.8314	0.9396
47	Skewness	0.1584	0.4720	0.6930	0.8428	0.9452
48	D'Agostino-skewness	0.1616	0.4736	0.6914	0.8412	0.9474
49	2nd Zhang-Wu	0.1564	0.4822	0.6888	0.8458	0.9458
50	1st Cabana-Cabana	0.1568	0.4908	0.7130	0.8556	0.9532

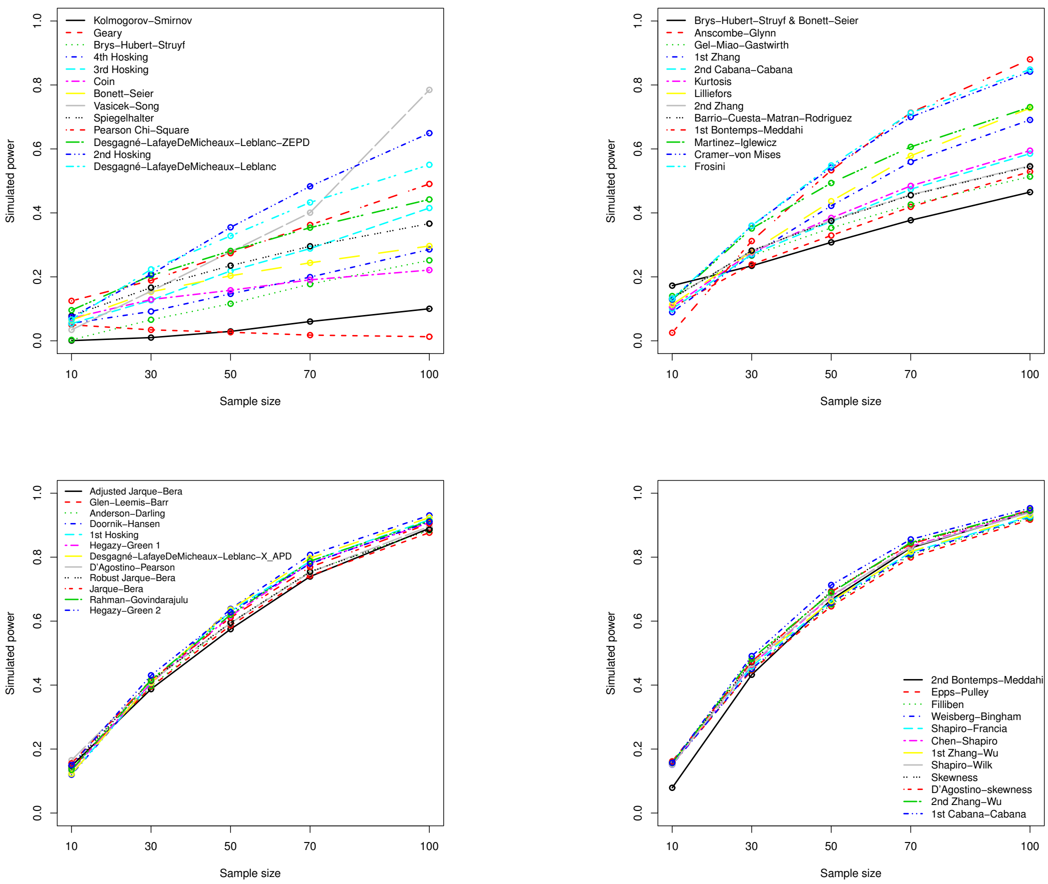

Figure 7: Simulated power curves for 50 normality tests under alternative Gumbel distribution $(0, 3)$ for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

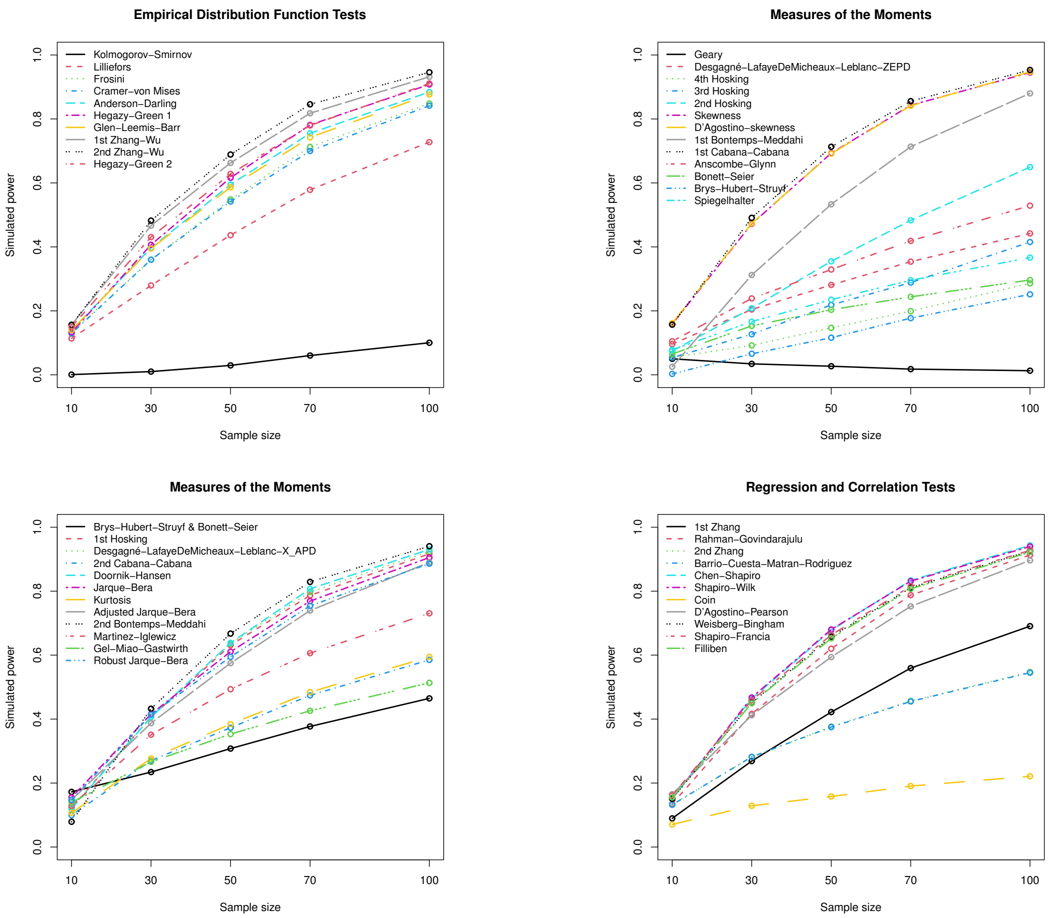

Figure 8: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Gumbel distribution $(0, 3)$ for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 5: Power comparisons of normality tests under alternative Skew-Normal distribution ($location = 0,\ scale = 3, shape = 7$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0006	0.0054	0.0246	0.0442	0.1098
2	Coin	0.0584	0.0718	0.0722	0.0860	0.0798
3	Bonett-Seier	0.0616	0.0876	0.0924	0.0956	0.1044
4	Geary	0.0696	0.0900	0.0866	0.0880	0.0838
5	4th Hosking	0.0524	0.0970	0.1792	0.2512	0.3702
6	Brys-Hubert-Struyf	0.0036	0.1074	0.2096	0.3022	0.4220
7	Spiegelhalter	0.0786	0.1096	0.1152	0.1060	0.0992
8	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.0872	0.1270	0.1418	0.1586	0.1754
9	3rd Hosking	0.0524	0.1306	0.2592	0.3820	0.5362
10	Gel–Miao–Gastwirth	0.1140	0.1428	0.1696	0.1778	0.2166
11	Desgagné-LafayeDeMicheaux-Leblanc	0.0614	0.1510	0.2218	0.2912	0.3912
12	Anscombe-Glynn	0.0920	0.1626	0.1936	0.2274	0.2700
13	2nd Cabana-Cabana	0.0896	0.1650	0.2102	0.2512	0.3110
14	Kurtosis	0.1002	0.1748	0.2204	0.2650	0.3244
15	2nd Zhang	0.1076	0.1766	0.2310	0.2728	0.3234
16	Barrio-Cuesta-Matran-Rodriguez	0.1100	0.1822	0.2296	0.2720	0.3188
17	Vasicek-Song	0.0376	0.1944	0.3884	0.5526	0.9154
18	Brys-Hubert-Struyf & Bonett-Seier	0.1480	0.2052	0.2798	0.3596	0.4530
19	1st Bontemps-Meddahi	0.0166	0.2122	0.4364	0.6586	0.8898
20	2nd Hosking	0.0644	0.2226	0.4164	0.6046	0.7736
21	Pearson Chi-Square	0.1338	0.2252	0.4066	0.5548	0.7254
22	Martinez-Iglewicz	0.1072	0.2462	0.3516	0.4412	0.5434
23	Lilliefors	0.1164	0.2854	0.4648	0.6518	0.8036
24	Adjusted Jarque–Bera	0.1146	0.2866	0.4870	0.6908	0.8950
25	Robust Jarque–Bera	0.1184	0.3108	0.4820	0.6504	0.8524
26	Jarque–Bera	0.1270	0.3272	0.5500	0.7576	0.9328
27	D'Agostino-Pearson	0.1320	0.3290	0.5260	0.7318	0.9216
28	Cramer-von Mises	0.1330	0.3850	0.6144	0.8036	0.9198
29	2nd Bontemps-Meddahi	0.0622	0.3908	0.6982	0.8828	0.9784
30	Frosini	0.1400	0.4062	0.6382	0.8196	0.9318
31	Hegazy–Green 2	0.1362	0.4124	0.6608	0.8410	0.9622
32	Doornik-Hansen	0.1152	0.4144	0.7136	0.8912	0.9812
33	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1266	0.4190	0.7014	0.8766	0.9706
34	Skewness	0.1332	0.4258	0.6824	0.8452	0.9562
35	D'Agostino-skewness	0.1350	0.4264	0.6822	0.8438	0.9558
36	Glen-Leemis-Barr	0.1432	0.4318	0.6738	0.8530	0.9516
37	1st Hosking	0.1060	0.4322	0.7264	0.8906	0.9762
38	Anderson-Darling	0.1432	0.4394	0.6860	0.8628	0.9580
39	Filliben	0.1434	0.4452	0.7138	0.8804	0.9780
40	Weisberg–Bingham	0.1426	0.4564	0.7228	0.8914	0.9790
41	Hegazy–Green 1	0.1414	0.4600	0.7264	0.8856	0.9710
42	1st Cabana-Cabana	0.1334	0.4634	0.7302	0.8856	0.9756
43	Shapiro-Francia	0.1500	0.4646	0.7298	0.8940	0.9812
44	Epps-Pulley	0.1508	0.4778	0.7284	0.8878	0.9670
45	1st Zhang	0.1228	0.4798	0.7352	0.8610	0.9398
46	1st Zhang-Wu	0.1560	0.5054	0.7764	0.9158	0.9886
47	Rahman-Govindarajulu	0.1472	0.5128	0.7878	0.9216	0.9864
48	Shapiro-Wilk	0.1518	0.5198	0.7894	0.9220	0.9898
49	Chen-Shapiro	0.1540	0.5278	0.8010	0.9304	0.9904
50	2nd Zhang-Wu	0.1530	0.5476	0.8340	0.9494	0.9942

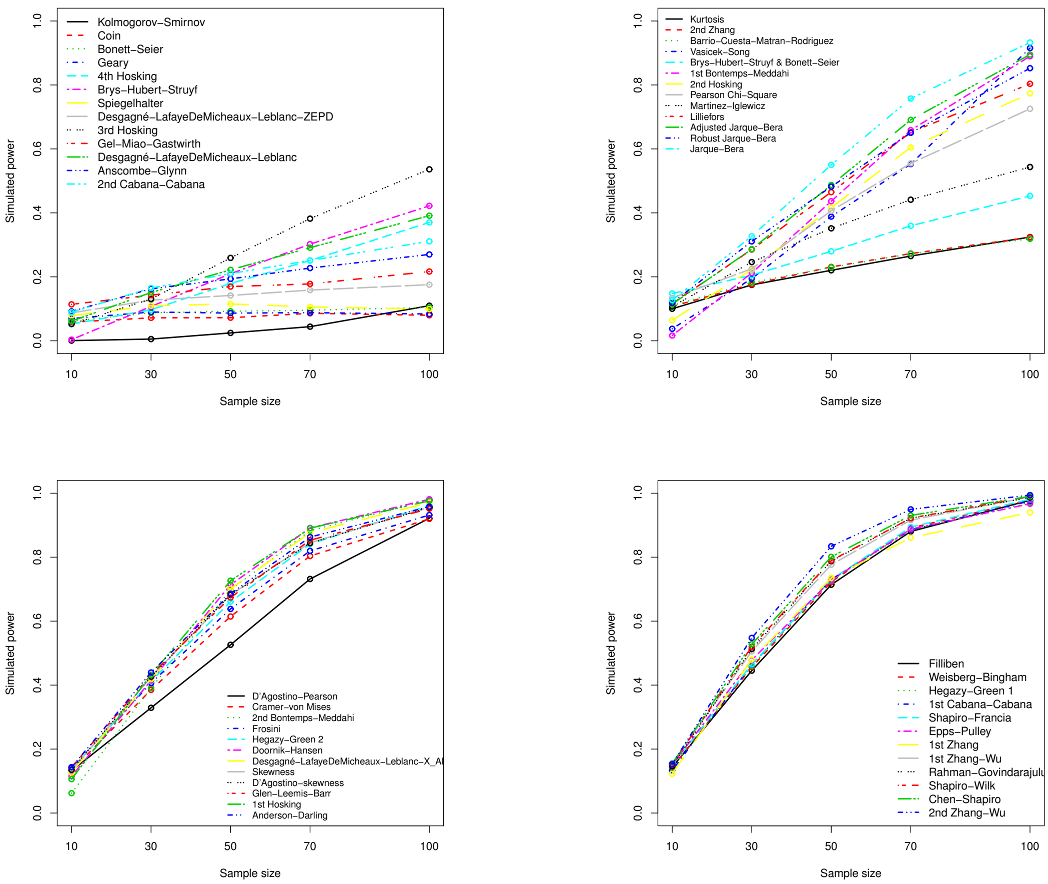

Figure 9: Simulated power curves for 50 normality tests under alternative Skew-Normal distribution ($location = 0,\ scale = 3,\ shape = 7$) for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

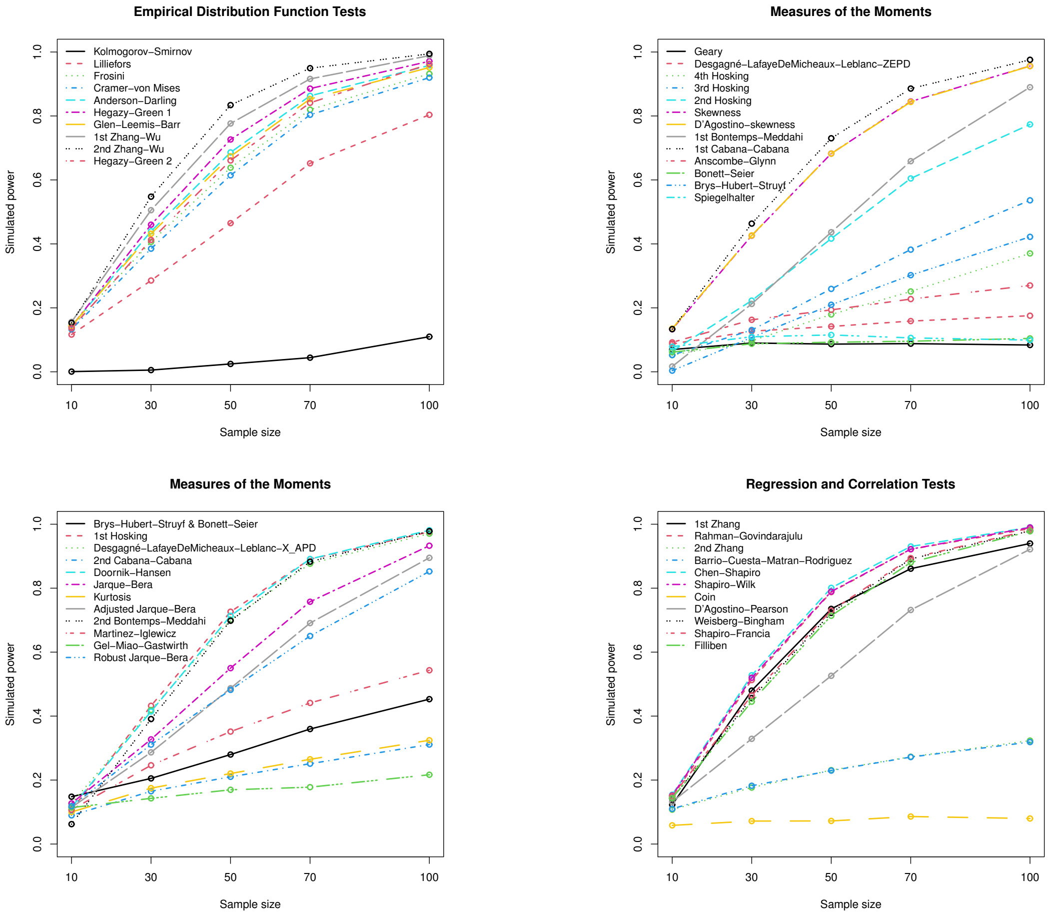

Figure 10: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Skew-Normal distribution ($location = 0,\ scale = 3,\ shape = 7$) for sample sizes $n = 10, 30, 50, 70$ and 100 ($\alpha = 0.05$)

Table 6: Power comparisons of normality tests under alternative Gamma distribution ($shape = 2,\ rate = 1$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0006	0.0232	0.0830	0.1708	0.3444
2	Geary	0.0586	0.0440	0.0356	0.0254	0.0208
3	Coin	0.0778	0.1216	0.1444	0.1584	0.1884
4	4th Hosking	0.0580	0.1290	0.2592	0.3622	0.5304
5	Brys-Hubert-Struyf	0.0040	0.1340	0.2796	0.4148	0.5792
6	Bonett-Seier	0.0774	0.1688	0.2140	0.2612	0.3238
7	3rd Hosking	0.0610	0.2000	0.3836	0.5324	0.7156
8	Spiegelhalter	0.0996	0.2012	0.2630	0.3218	0.3888
9	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.1148	0.2498	0.3358	0.4200	0.5138
10	Brys-Hubert-Struyf & Bonett-Seier	0.1488	0.2890	0.4318	0.5598	0.6962
11	Desgagné-LafayeDeMicheaux-Leblanc	0.0840	0.3060	0.4616	0.5842	0.7310
12	Anscombe-Glynn	0.1340	0.3092	0.4218	0.5258	0.6470
13	Gel–Miao–Gastwirth	0.1742	0.3358	0.4408	0.5344	0.6242
14	2nd Cabana-Cabana	0.1254	0.3408	0.4754	0.5876	0.7014
15	Kurtosis	0.1344	0.3512	0.4842	0.5934	0.7086
16	2nd Hosking	0.0816	0.3534	0.6158	0.7862	0.9210
17	2nd Zhang	0.1608	0.3556	0.4704	0.5536	0.6560
18	Barrio-Cuesta-Matran-Rodriguez	0.1608	0.3576	0.4676	0.5552	0.6550
19	Vasicek-Song	0.0594	0.3996	0.6988	0.8720	0.9976
20	Pearson Chi-Square	0.1944	0.4094	0.6488	0.8184	0.9474
21	1st Bontemps-Meddahi	0.0436	0.4414	0.7386	0.9106	0.9902
22	Lilliefors	0.1708	0.4604	0.6958	0.8470	0.9542
23	Martinez-Iglewicz	0.1806	0.4810	0.6628	0.7892	0.8870
24	Adjusted Jarque–Bera	0.1810	0.5244	0.7774	0.9212	0.9912
25	Robust Jarque–Bera	0.1894	0.5388	0.7610	0.9112	0.9830
26	D'Agostino-Pearson	0.2092	0.5566	0.7936	0.9288	0.9934
27	Jarque–Bera	0.2042	0.5754	0.8222	0.9482	0.9954
28	Cramer-von Mises	0.1958	0.5986	0.8366	0.9444	0.9886
29	Frosini	0.2014	0.6168	0.8524	0.9506	0.9920
30	2nd Bontemps-Meddahi	0.1110	0.6354	0.9034	0.9816	0.9992
31	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1842	0.6498	0.8996	0.9768	0.9974
32	Glen-Leemis-Barr	0.2102	0.6534	0.8810	0.9688	0.9966
33	Anderson-Darling	0.2096	0.6566	0.8904	0.9748	0.9968
34	Hegazy–Green 2	0.2140	0.6584	0.8944	0.9774	0.9982
35	Doornik-Hansen	0.1726	0.6598	0.9154	0.9852	0.9998
36	1st Hosking	0.1736	0.6674	0.9134	0.9836	0.9982
37	D'Agostino-skewness	0.2188	0.6738	0.8918	0.9734	0.9964
38	Skewness	0.2178	0.6738	0.8918	0.9738	0.9966
39	Hegazy–Green 1	0.2090	0.6896	0.9128	0.9804	0.9974
40	Filliben	0.2242	0.6942	0.9170	0.9858	0.9992
41	Epps-Pulley	0.2278	0.7010	0.9064	0.9784	0.9970
42	1st Cabana-Cabana	0.2190	0.7072	0.9216	0.9836	0.9998
43	Weisberg–Bingham	0.2206	0.7072	0.9236	0.9884	0.9990
44	Shapiro-Francia	0.2308	0.7106	0.9270	0.9896	0.9992
45	1st Zhang	0.1650	0.7198	0.9578	0.9960	1.0000
46	Rahman-Govindarajulu	0.2132	0.7364	0.9494	0.9952	1.0000
47	1st Zhang-Wu	0.2306	0.7474	0.9494	0.9928	0.9998
48	Shapiro-Wilk	0.2254	0.7522	0.9498	0.9934	0.9998
49	Chen-Shapiro	0.2272	0.7578	0.9554	0.9938	1.0000
50	2nd Zhang-Wu	0.2376	0.7918	0.9720	0.9976	1.0000

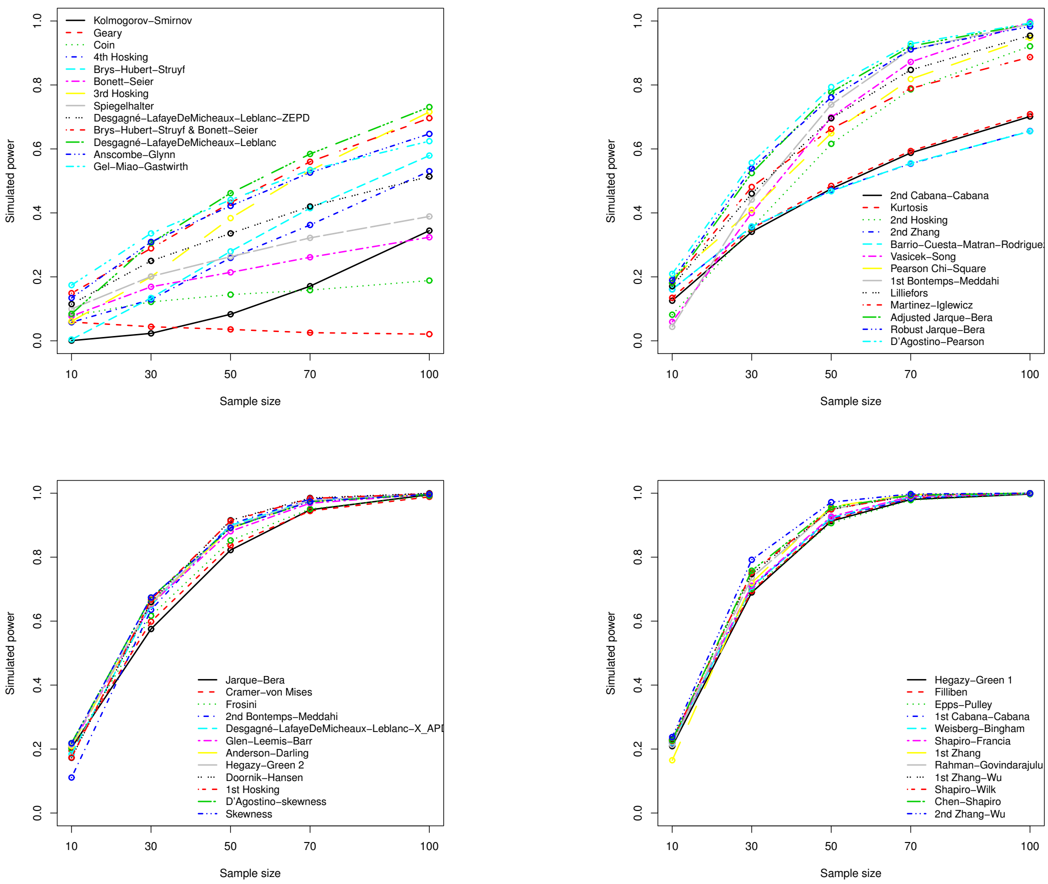

Figure 11: Simulated power curves for 50 normality tests under alternative Gamma distribution ($shape = 2,\ rate = 1$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

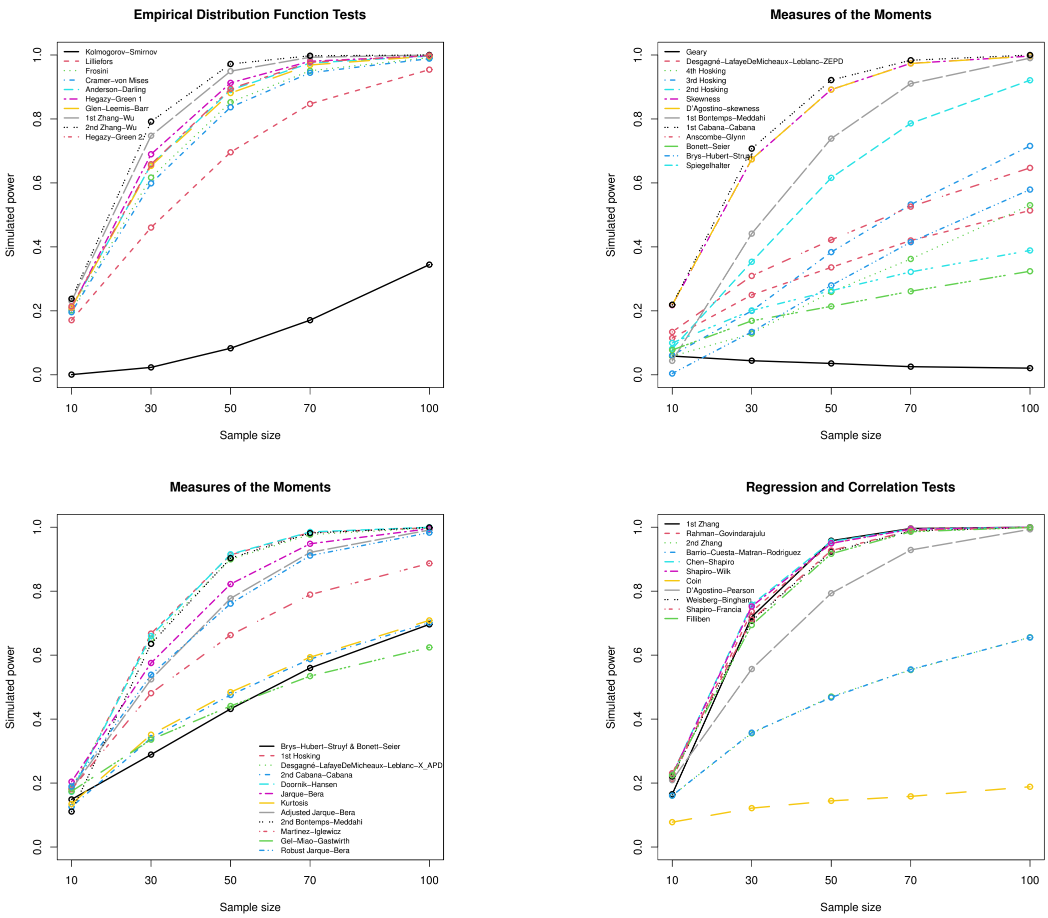

Figure 12: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Gamma distribution ($shape = 2,\ rate = 1$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 7: Power comparisons of normality tests under alternative Weibull distribution ($shape = 1.5,\ scale = 1$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0000	0.0076	0.0300	0.0638	0.1328
2	Geary	0.0664	0.0706	0.0756	0.0666	0.0570
3	Coin	0.0656	0.0792	0.1002	0.0998	0.0974
4	4th Hosking	0.0502	0.0976	0.1878	0.2544	0.3682
5	Brys-Hubert-Struyf	0.0024	0.1010	0.2142	0.3048	0.4328
6	Bonett-Seier	0.0668	0.1046	0.1310	0.1318	0.1450
7	3rd Hosking	0.0552	0.1400	0.2794	0.3836	0.5414
8	Spiegelhalter	0.0912	0.1402	0.1670	0.1642	0.1744
9	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.1040	0.1526	0.1996	0.2146	0.2714
10	Desgagné-LafayeDeMicheaux-Leblanc	0.0708	0.1862	0.3030	0.3964	0.5154
11	Gel–Miao–Gastwirth	0.1322	0.1926	0.2506	0.2818	0.3304
12	Anscombe-Glynn	0.1124	0.2036	0.2624	0.3176	0.4040
13	Brys-Hubert-Struyf & Bonett-Seier	0.1516	0.2094	0.3200	0.3880	0.4970
14	2nd Cabana-Cabana	0.1092	0.2162	0.2930	0.3650	0.4546
15	Kurtosis	0.1214	0.2242	0.3082	0.3818	0.4654
16	2nd Zhang	0.1264	0.2306	0.3098	0.3698	0.4478
17	Barrio-Cuesta-Matran-Rodriguez	0.1278	0.2322	0.3094	0.3666	0.4454
18	2nd Hosking	0.0668	0.2352	0.4606	0.6226	0.8076
19	Pearson Chi-Square	0.1518	0.2668	0.4938	0.6534	0.8534
20	Vasicek-Song	0.0434	0.2740	0.5554	0.7476	0.9890
21	1st Bontemps-Meddahi	0.0274	0.2894	0.5544	0.7634	0.9480
22	Martinez-Iglewicz	0.1292	0.3138	0.4508	0.5644	0.6904
23	Lilliefors	0.1268	0.3224	0.5362	0.7002	0.8564
24	Adjusted Jarque–Bera	0.1386	0.3662	0.6014	0.7866	0.9520
25	Robust Jarque–Bera	0.1434	0.3820	0.5910	0.7614	0.9232
26	D'Agostino-Pearson	0.1556	0.4076	0.6362	0.8134	0.9634
27	Jarque–Bera	0.1516	0.4228	0.6624	0.8364	0.9704
28	Cramer-von Mises	0.1544	0.4304	0.6860	0.8444	0.9548
29	Frosini	0.1646	0.4532	0.7136	0.8608	0.9616
30	2nd Bontemps-Meddahi	0.0840	0.4698	0.7874	0.9314	0.9934
31	Hegazy–Green 2	0.1630	0.4886	0.7706	0.9186	0.9898
32	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1504	0.4900	0.7842	0.9218	0.9892
33	Glen-Leemis-Barr	0.1656	0.4918	0.7558	0.9016	0.9780
34	Doornik-Hansen	0.1354	0.4958	0.8050	0.9406	0.9944
35	Anderson-Darling	0.1668	0.4972	0.7700	0.9106	0.9838
36	1st Hosking	0.1276	0.5062	0.8104	0.9358	0.9916
37	D'Agostino-skewness	0.1608	0.5122	0.7640	0.9106	0.9830
38	Skewness	0.1612	0.5150	0.7630	0.9106	0.9826
39	Filliben	0.1714	0.5294	0.8146	0.9424	0.9954
40	Hegazy–Green 1	0.1692	0.5304	0.7978	0.9290	0.9896
41	Epps-Pulley	0.1772	0.5380	0.7956	0.9234	0.9866
42	Weisberg–Bingham	0.1698	0.5398	0.8240	0.9492	0.9968
43	1st Cabana-Cabana	0.1636	0.5472	0.8096	0.9316	0.9922
44	Shapiro-Francia	0.1780	0.5482	0.8300	0.9528	0.9970
45	1st Zhang-Wu	0.1830	0.5990	0.8784	0.9736	0.9988
46	Rahman-Govindarajulu	0.1750	0.6024	0.8838	0.9768	0.9994
47	Shapiro-Wilk	0.1812	0.6080	0.8762	0.9718	0.9982
48	Chen-Shapiro	0.1856	0.6124	0.8858	0.9758	0.9988
49	1st Zhang	0.1476	0.6312	0.9322	0.9914	0.9996
50	2nd Zhang-Wu	0.1834	0.6678	0.9276	0.9900	0.9996

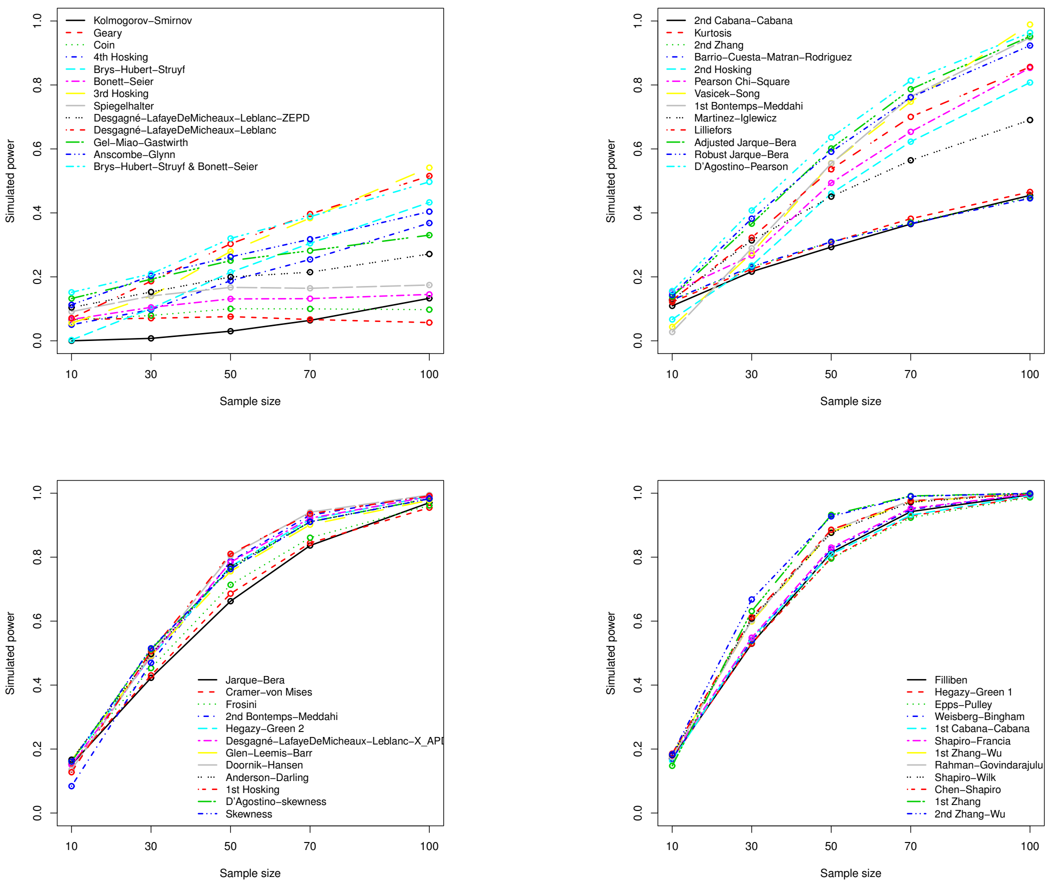

Figure 13: Simulated power curves for 50 normality tests under alternative Weibull distribution ($shape = 1.5,\ scale = 1$) for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

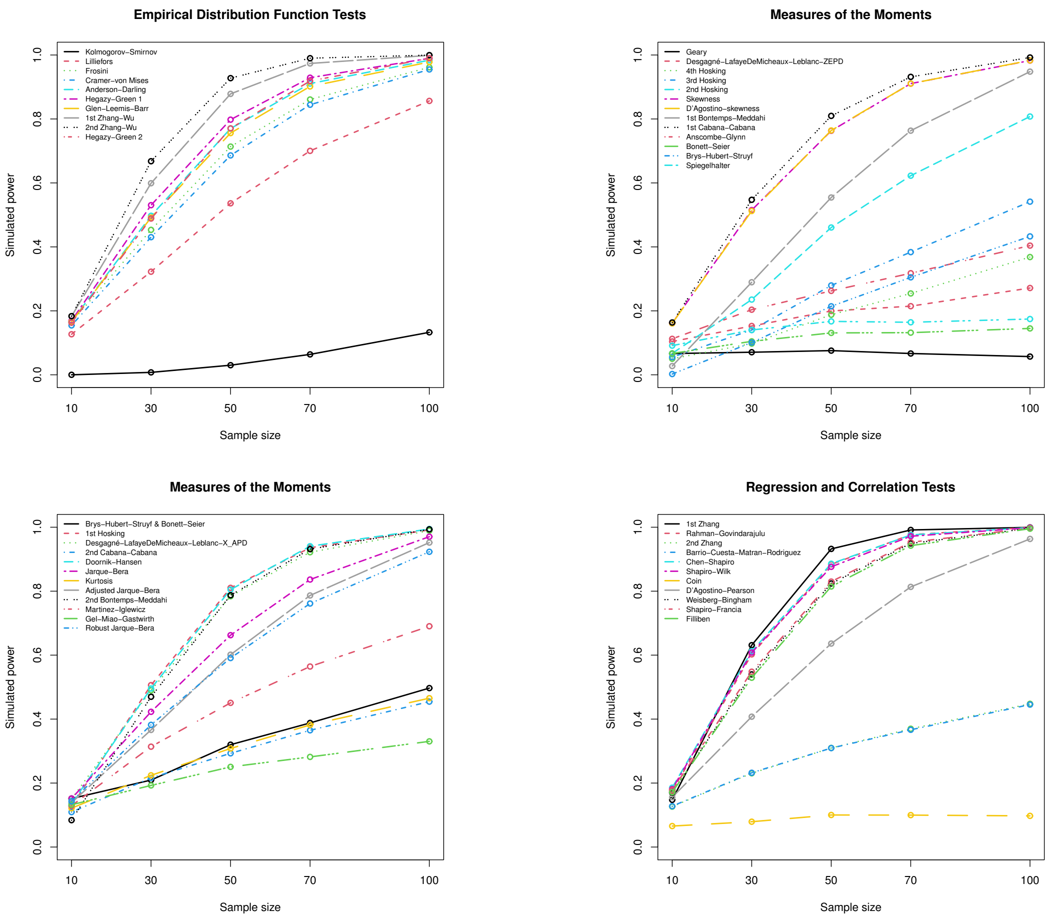

Figure 14: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Weibull distribution ($shape = 1.5,\ scale = 1$) for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 8: Power comparisons of normality tests under alternative Beta(2,5) distribution for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Kolmogorov-Smirnov	0.0000	0.0006	0.0044	0.0110	0.0164
2	Gel–Miao–Gastwirth	0.0694	0.0484	0.0466	0.0328	0.0332
3	Barrio-Cuesta-Matran-Rodriguez	0.0646	0.0612	0.0612	0.0606	0.0568
4	2nd Zhang	0.0632	0.0616	0.0592	0.0606	0.0560
5	4th Hosking	0.0546	0.0624	0.1050	0.1324	0.1874
6	1st Bontemps-Meddahi	0.0040	0.0626	0.1414	0.2450	0.4826
7	Coin	0.0454	0.0634	0.0940	0.1352	0.1876
8	2nd Cabana-Cabana	0.0670	0.0636	0.0680	0.0542	0.0562
9	Brys-Hubert-Struyf	0.0016	0.0656	0.1066	0.1590	0.2038
10	Bonett-Seier	0.0486	0.0660	0.0846	0.0984	0.1226
11	Desgagné-LafayeDeMicheaux-Leblanc	0.0496	0.0680	0.0852	0.0964	0.1228
12	Kurtosis	0.0718	0.0706	0.0800	0.0690	0.0712
13	3rd Hosking	0.0530	0.0750	0.1394	0.1834	0.2754
14	Martinez-Iglewicz	0.0640	0.0794	0.1036	0.1046	0.1194
15	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.0632	0.0824	0.1016	0.1090	0.1166
16	Spiegelhalter	0.0640	0.0872	0.0860	0.0692	0.0494
17	Anscombe-Glynn	0.0534	0.0904	0.1110	0.1044	0.1072
18	Vasicek-Song	0.0260	0.0968	0.1962	0.3032	0.7692
19	Adjusted Jarque–Bera	0.0624	0.1016	0.1768	0.2844	0.5020
20	2nd Hosking	0.0550	0.1124	0.2158	0.2956	0.4508
21	Geary	0.0726	0.1130	0.1440	0.1660	0.1942
22	Pearson Chi-Square	0.1004	0.1152	0.1878	0.2592	0.3940
23	Robust Jarque–Bera	0.0684	0.1222	0.1902	0.2548	0.4302
24	Jarque–Bera	0.0712	0.1268	0.2248	0.3496	0.5900
25	D'Agostino-Pearson	0.0732	0.1428	0.2456	0.3658	0.6046
26	Lilliefors	0.0772	0.1512	0.2672	0.3540	0.5030
27	2nd Bontemps-Meddahi	0.0272	0.1640	0.3930	0.5944	0.8136
28	Hegazy–Green 2	0.0814	0.1648	0.3120	0.4714	0.7072
29	Brys-Hubert-Struyf & Bonett-Seier	0.1382	0.1684	0.1982	0.2442	0.2956
30	Doornik-Hansen	0.0622	0.1744	0.3936	0.6002	0.8276
31	Skewness	0.0750	0.1870	0.3568	0.5050	0.7088
32	D'Agostino-skewness	0.0748	0.1898	0.3574	0.5052	0.7082
33	Cramer-von Mises	0.0820	0.1936	0.3414	0.4700	0.6460
34	Filliben	0.0852	0.1948	0.3838	0.5646	0.7904
35	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.0754	0.2038	0.4042	0.5868	0.7896
36	Frosini	0.0888	0.2072	0.3676	0.4978	0.6786
37	Weisberg–Bingham	0.0852	0.2074	0.4006	0.5908	0.8080
38	1st Hosking	0.0592	0.2124	0.4324	0.6188	0.8124
39	Shapiro-Francia	0.0900	0.2128	0.4058	0.5980	0.8124
40	1st Cabana-Cabana	0.0780	0.2144	0.4084	0.5824	0.7896
41	Glen-Leemis-Barr	0.0854	0.2180	0.3886	0.5454	0.7358
42	Anderson-Darling	0.0850	0.2210	0.3972	0.5570	0.7480
43	Hegazy–Green 1	0.0910	0.2364	0.4272	0.5950	0.7864
44	Epps-Pulley	0.0924	0.2500	0.4522	0.6178	0.7970
45	1st Zhang-Wu	0.0892	0.2598	0.5000	0.7078	0.8970
46	Shapiro-Wilk	0.0866	0.2680	0.5006	0.7020	0.8876
47	Chen-Shapiro	0.0886	0.2764	0.5290	0.7352	0.9100
48	2nd Zhang-Wu	0.0910	0.2952	0.5936	0.8130	0.9640
49	Rahman-Govindarajulu	0.0996	0.3008	0.5630	0.7582	0.9276
50	1st Zhang	0.0860	0.3262	0.6288	0.8396	0.9682

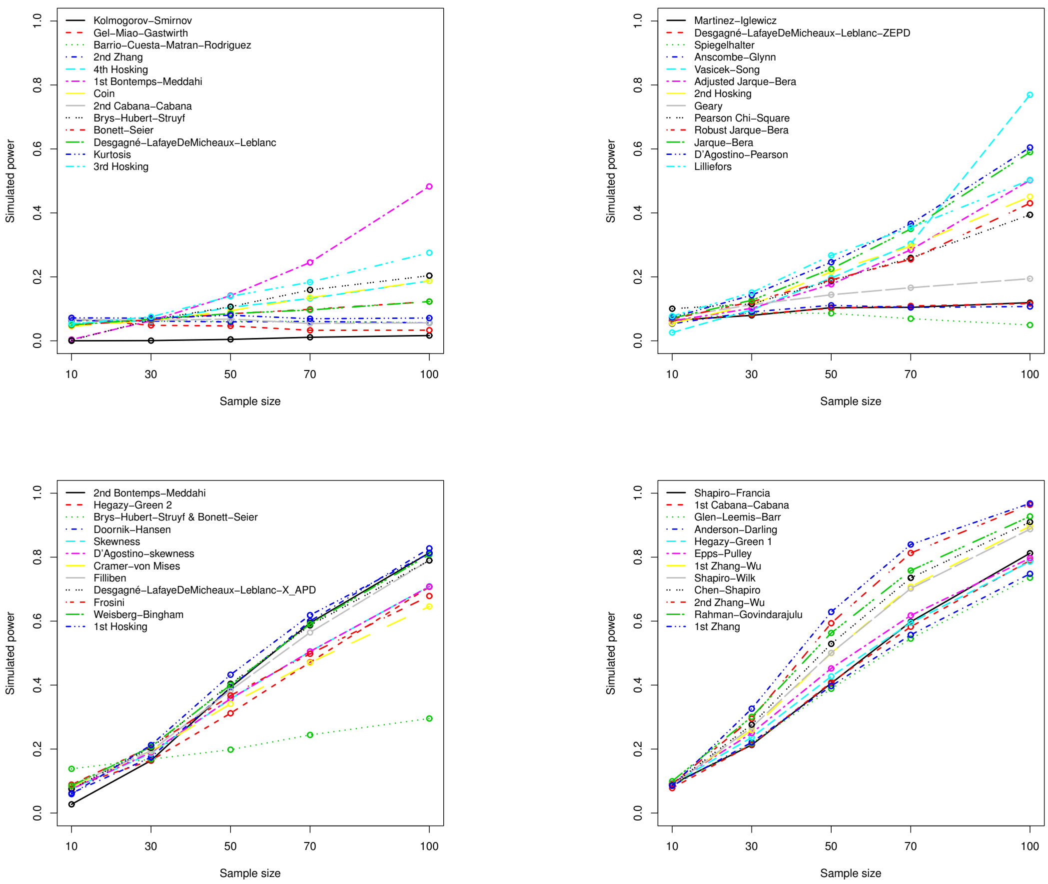

Figure 15: Simulated power curves for 50 normality tests under alternative Beta(2,5) distribution for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

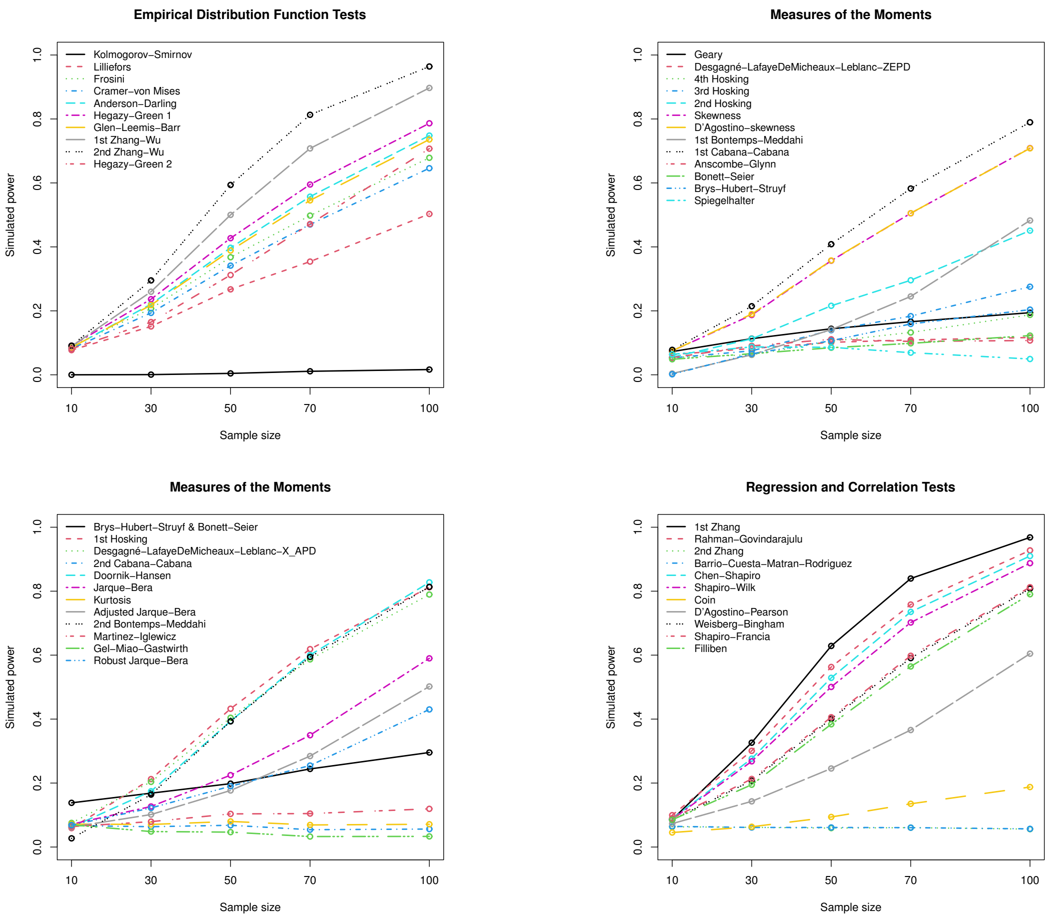

Figure 16: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Beta(2,5) distribution for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

Table 9: Power comparisons of normality tests under the alternative Uniform distribution $(0, 1)$ for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

		Sample Size (n)				
	Tests for Normality (in order of increasing power)	10	30	50	70	100
1	Robust Jarque–Bera	0.0172	0.0014	0.0004	0.0000	0.0436
2	Kolmogorov-Smirnov	0.0006	0.0232	0.0830	0.1708	0.3444
3	Geary	0.0586	0.0440	0.0356	0.0254	0.0208
4	Coin	0.0778	0.1216	0.1444	0.1584	0.1884
5	4th Hosking	0.0580	0.1290	0.2592	0.3622	0.5304
6	Brys-Hubert-Struyf	0.0040	0.1340	0.2796	0.4148	0.5792
7	Bonett-Seier	0.0774	0.1688	0.2140	0.2612	0.3238
8	3rd Hosking	0.0610	0.2000	0.3836	0.5324	0.7156
9	Spiegelhalter	0.0996	0.2012	0.2630	0.3218	0.3888
10	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	0.1148	0.2498	0.3358	0.4200	0.5138
11	Brys-Hubert-Struyf & Bonett-Seier	0.1488	0.2890	0.4318	0.5598	0.6962
12	Desgagné-LafayeDeMicheaux-Leblanc	0.0840	0.3060	0.4616	0.5842	0.7310
13	Anscombe-Glynn	0.1340	0.3092	0.4218	0.5258	0.6470
14	Gel–Miao–Gastwirth	0.1742	0.3358	0.4408	0.5344	0.6242
15	2nd Cabana-Cabana	0.1254	0.3408	0.4754	0.5876	0.7014
16	Kurtosis	0.1344	0.3512	0.4842	0.5934	0.7086
17	2nd Hosking	0.0816	0.3534	0.6158	0.7862	0.9210
18	2nd Zhang	0.1608	0.3556	0.4704	0.5536	0.6560
19	Barrio-Cuesta-Matran-Rodriguez	0.1608	0.3576	0.4676	0.5552	0.6550
20	Vasicek-Song	0.0594	0.3996	0.6988	0.8720	0.9976
21	Pearson Chi-Square	0.1944	0.4094	0.6488	0.8184	0.9474
22	1st Bontemps-Meddahi	0.0436	0.4414	0.7386	0.9106	0.9902
23	Lilliefors	0.1708	0.4604	0.6958	0.8470	0.9542
24	Martinez-Iglewicz	0.1806	0.4810	0.6628	0.7892	0.8870
25	Adjusted Jarque–Bera	0.1810	0.5244	0.7774	0.9212	0.9912
26	D'Agostino-Pearson	0.2092	0.5566	0.7936	0.9288	0.9934
27	Jarque–Bera	0.2042	0.5754	0.8222	0.9482	0.9954
28	Cramer-von Mises	0.1958	0.5986	0.8366	0.9444	0.9886
29	Frosini	0.2014	0.6168	0.8524	0.9506	0.9920
30	2nd Bontemps-Meddahi	0.1110	0.6354	0.9034	0.9816	0.9992
31	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	0.1842	0.6498	0.8996	0.9768	0.9974
32	Glen-Leemis-Barr	0.2102	0.6534	0.8810	0.9688	0.9966
33	Anderson-Darling	0.2096	0.6566	0.8904	0.9748	0.9968
34	Hegazy–Green 2	0.2140	0.6584	0.8944	0.9774	0.9982
35	Doornik-Hansen	0.1726	0.6598	0.9154	0.9852	0.9998
36	1st Hosking	0.1736	0.6674	0.9134	0.9836	0.9982
37	D'Agostino-skewness	0.2188	0.6738	0.8918	0.9734	0.9964
38	Skewness	0.2178	0.6738	0.8918	0.9738	0.9966
39	Hegazy–Green 1	0.2090	0.6896	0.9128	0.9804	0.9974
40	Filliben	0.2242	0.6942	0.9170	0.9858	0.9992
41	Epps-Pulley	0.2278	0.7010	0.9064	0.9784	0.9970
42	1st Cabana-Cabana	0.2190	0.7072	0.9216	0.9836	0.9998
43	Weisberg–Bingham	0.2206	0.7072	0.9236	0.9884	0.9990
44	Shapiro-Francia	0.2308	0.7106	0.9270	0.9896	0.9992
45	1st Zhang	0.1650	0.7198	0.9578	0.9960	1.0000
46	Rahman-Govindarajulu	0.2132	0.7364	0.9494	0.9952	1.0000
47	1st Zhang-Wu	0.2306	0.7474	0.9494	0.9928	0.9998
48	Shapiro-Wilk	0.2254	0.7522	0.9498	0.9934	0.9998
49	Chen-Shapiro	0.2272	0.7578	0.9554	0.9938	1.0000
50	2nd Zhang-Wu	0.2376	0.7918	0.9720	0.9976	1.0000

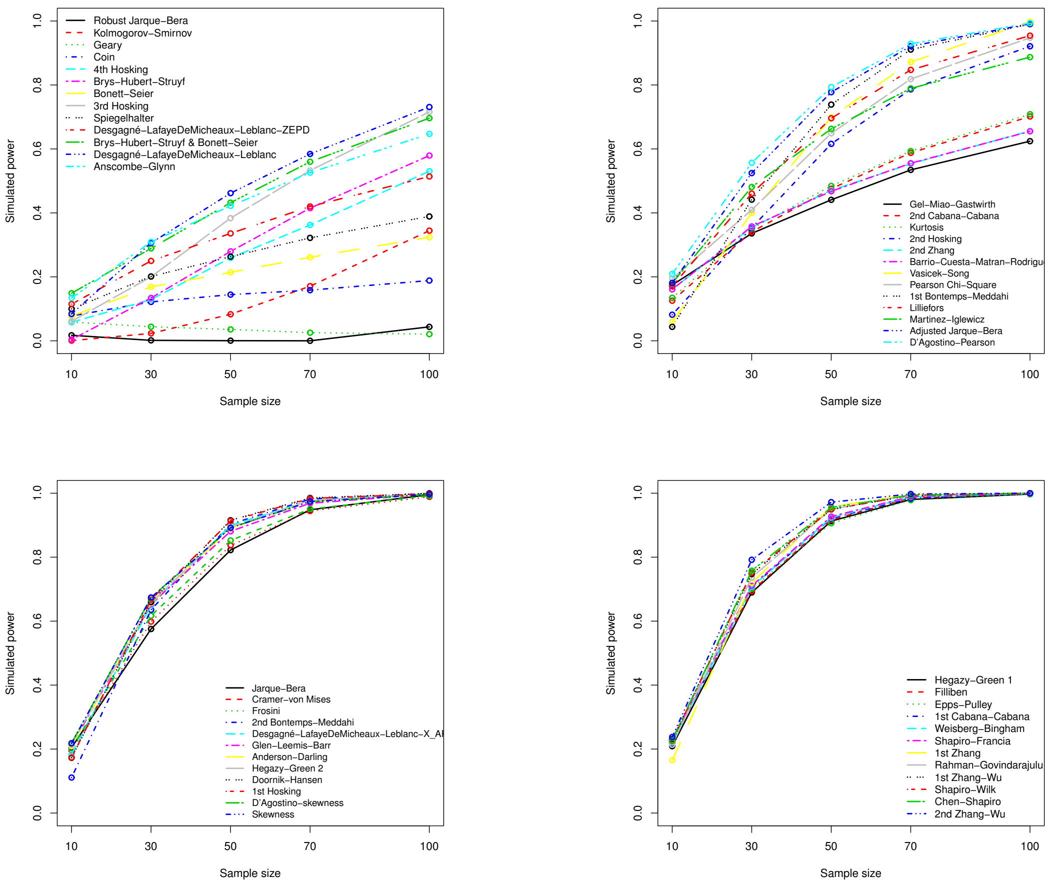

Figure 17: Simulated power curves for 50 normality tests under alternative Uniform distribution $(0, 1)$ for sample sizes n = 10, 30, 50, 70 and 100 ($\alpha = 0.05$)

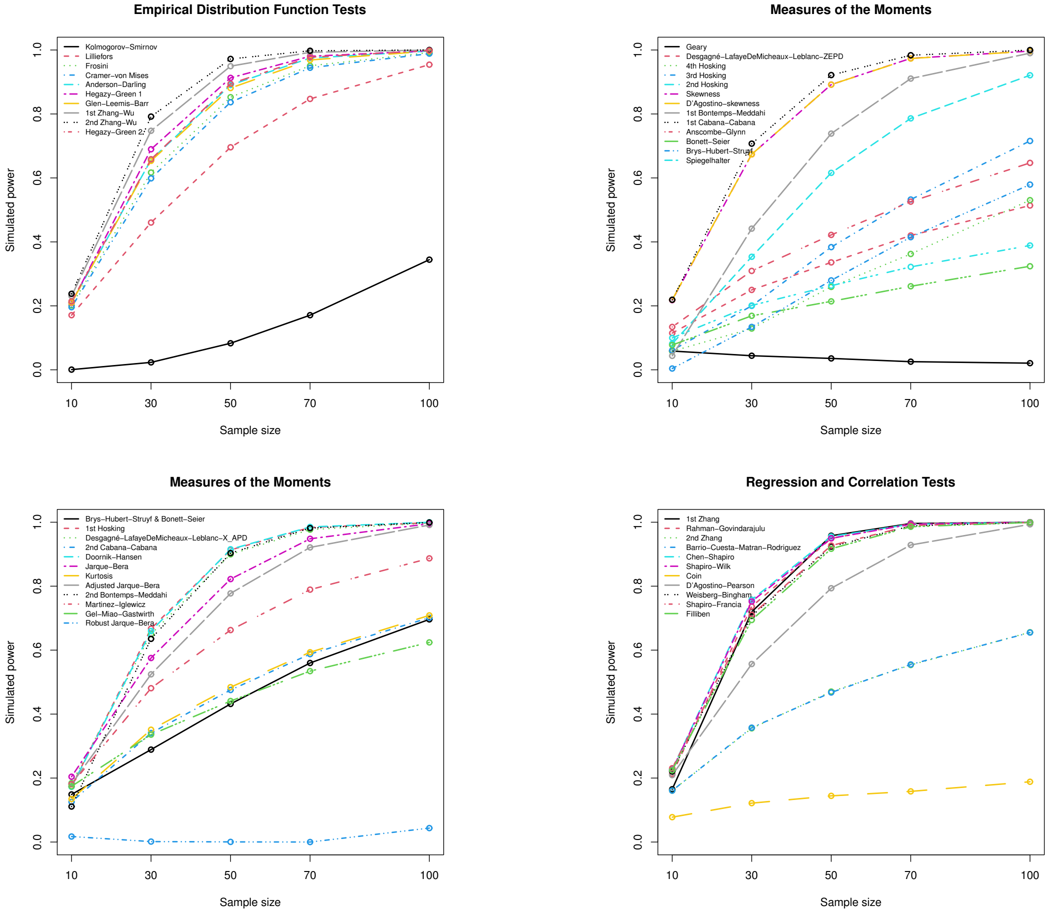

Figure 18: Simulated power curves for normality tests based on Empirical distribution function, Measures of the moments, and Regression and correlation tests under alternative Uniform distribution $(0, 1)$ for sample sizes $n = 10$, 30, 50, 70 and 100 ($\alpha = 0.05$)

5. Results and recommendations

Table 10 contains the ranking from the first to the tenth of normality tests that have the most power obtained from Table 1 - 9 for the four groups of the alternative distributions, respectively.

Table 11 contains the ranking from the first to the tenth of normality tests that have the least power obtained from Table 1 - 9 for the four groups of the alternative distributions, respectively.

For Group I: Symmetric distributions with support on $(-\infty, \infty)$, as shown in Table 1 - 3 and Table 10 - 11 and Figure 1 - 5, we can see that the tests Geary and Kolmogorov-Smirnov have the least power and the tests Robust Jarque–Bera and Gel–Miao–Gastwirth have the most power.

For Group II: Asymmetric distributions with support on $(-\infty, \infty)$, as shown in Table 4 - 5 and Table 10 - 11 and Figure 7 - 9, we can see that the test Kolmogorov-Smirnov has the least power and the test 2nd Zhang-Wu has the most power.

For Group III: Distributions with support on $(0, \infty)$, as shown in Table 6 - 7 and Table 10 - 11 and Figure 11 - 13, we can see that the tests Kolmogorov-Smirnov and Geary have the least power and the test 2nd Zhang-Wu has the most power.

For Group IV: Distributions with support on $(0, 1)$, as shown in Table 8 - 9 and Table 10 - 11 and Figures 15 - 17, we can see that the test Kolmogorov-Smirnov has the least power and the tests 1st Zhang and the test 2nd Zhang-Wu have the most power.

In terms of the selected normality tests based on the empirical distribution function, for the case of the symmetric distributions, the test Hegazy–Green-2 has the most power and the tests Kolmogorov-Smirnov and Lilliefors have the least power (see Figure 2, 4, and 6). For the asymmetric distributions, the tests 1st Zhang-Wu and 2nd Zhang-Wu have the most power and the tests Kolmogorov-Smirnov and Lilliefors have the least power (see Figure 8 and 10). For distributions with support on $(0, \infty)$ and distributions with support on $(0, 1)$, the tests 1st Zhang-Wu and 2nd Zhang-Wu have the most power and the tests Kolmogorov-Smirnov and Lilliefors have the least power (see Figure 12, 14, 16 and 18).

In terms of the selected normality tests based on measures of the moments, the tests Robust Jarque–Bera and Gel–Miao–Gastwirth generally have the most power and the tests Geary and Brys-Hubert-Struyf have the least power for the symmetric distributions (see Figure 2, 4, and 6). For the case of asymmetric distributions, the test 1st Cabana-Cabana has the most power and the test Geary has the least power (see Figure 8 and 10). For distributions with support on $(0, \infty)$ the test 1st Cabana-Cabana and Skewness have the most power and the tests Geary and 4th Hosking have the least power (see Figure 12 and 14). For distributions with support on $(0, 1)$ the test 1st Cabana-Cabana has the most power and the test 4th Hosking has the least power (see Figure 16 and 18) .

In terms of regression and correlation tests, for the case of the symmetric distributions, the tests Filliben and Shapiro–Francia have the most power and the tests 1st Zhang Q and Rahman-Govindarajulu have the least power (see Figure 2, 4, and 6). For the asymmetric distributions, the tests Shapiro-Wilk and Chen-Shapiro have the most power and the test Coin has the least power (see Figure 8 and 10). For distributions with support on $(0, \infty)$ and distributions with support on $(0, 1)$, the tests Shapiro-Wilk and Chen-Shapiro have the most power and the tests Coin and 2nd Zhang Q have the least power (see Figure 12, 14, 16 and 18).

Kolmogorov-Smirnov test is one of the most famous tests of normality among practitioners, mostly because it is available in any statistical software. However, it has the least power against all alternatives, this agrees with the conclusions of Fortiana and Grané (2003), Grané (2012), and Grané and Tchirina (2013).

Geary's test has the least power, this agrees with the conclusion of D'Agostino and Rosman (1974).

General recommendations based on the analysis of the power of the selected tests indicate the best choices for normality testing are the tests Robust Jarque–Bera and Gel–Miao–Gastwirth for symmetric distributions, the tests 1st Cabana-Cabana and 2nd Zhang-Wu for asymmetric distributions, and the test 2nd Zhang-Wu for distributions with support on $(0, \infty)$ and distributions with support on $(0, 1)$.

Table 10: Ranking from the first to the tenth of normality tests that have the most power for the four groups of the alternative distributions

	Group I Symmetric distributions			Group II Asymmetric distributions		Group III Distributions on $(0,\infty)$		Group IV Distributions on $(0,1)$	
Rank	Student's t-distribution df=3	Laplace (0,1) distribution	Logistic (0,4) distribution	Gumbel (0,3) distribution	Skew-Normal (location = 0, scale = 3, shape = 7) distribution	Gamma(shape = 2, rate = 1) distribution	Weibull distribution (shape=1.5, scale = 1)	Beta(2,5) distribution	Uniform (0,1) distribution
1	Robust Jarque–Bera	Gel–Miao–Gastwirth	Robust Jarque–Bera	1st Cabana-Cabana	2nd Zhang-Wu	2nd Zhang-Wu	2nd Zhang-Wu	1st Zhang	2nd Zhang-Wu
2	Gel–Miao–Gastwirth	Robust Jarque–Bera	Gel–Miao–Gastwirth	2nd Zhang-Wu	Chen-Shapiro	Chen-Shapiro	1st Zhang	Rahman-Govindarajulu	Chen-Shapiro
3	Martinez-Iglewicz	Martinez-Iglewicz	Brys-Hubert-Struyf & Bonett-Seier	D'Agostino-skewness	Shapiro-Wilk	Shapiro-Wilk	Chen-Shapiro	2nd Zhang-Wu	Shapiro-Wilk
4	Hegazy-Green 2	Hegazy-Green 2	Martinez-Iglewicz	Skewness	Rahman-Govindarajulu	1st Zhang-Wu	Shapiro-Wilk	Chen-Shapiro	1st Zhang-Wu
5	2nd Bontemps-Meddahi	Brys-Hubert-Struyf & Bonett-Seier	Adjusted Jarque–Bera	Shapiro-Wilk	1st Zhang-Wu	Rahman-Govindarajulu	Rahman-Govindarajulu	Shapiro-Wilk	Rahman-Govindarajulu
6	Adjusted Jarque–Bera	1st Hosking	Jarque–Bera	1st Zhang-Wu	1st Zhang	1st Zhang	1st Zhang-Wu	1st Zhang-Wu	1st Zhang
7	Kurtosis	Spiegelhalter	Kurtosis	Chen-Shapiro	Epps-Pulley	Shapiro-Francia	Shapiro-Francia	Epps-Pulley	Shapiro-Francia
8	Filliben	2nd Bontemps-Meddahi	Hegazy-Green 2	Shapiro-Francia	Shapiro-Francia	Weisberg–Bingham	1st Cabana-Cabana	Hegazy-Green 1	Weisberg–Bingham
9	Shapiro-Francia	Filliben	D'Agostino-Pearson	Weisberg–Bingham	1st Cabana-Cabana	1st Cabana-Cabana	Weisberg–Bingham	Anderson-Darling	1st Cabana-Cabana
10	Jarque–Bera	Desgagné-LafayeDeMicheaux-Leblanc-X_APD	2nd Bontemps-Meddahi	Filliben	Hegazy-Green 1	Epps-Pulley	Epps-Pulley	Glen-Leemis-Barr	Epps-Pulley

Table 11: Ranking from the first to the tenth of normality tests that have the least power for the four groups of the alternative distributions

	Group I Symmetric distributions			Group II Asymmetric distributions		Group III Distributions on $(0, \infty)$		Group IV Distributions on $(0, 1)$	
Rank	Student's t-distribution df=3	Laplace (0,1) distribution	Logistic (0,4) distribution	Gumbel (0,3) distribution	Skew-Normal (location = 0, scale = 3, shape = 7) distribution	Gamma(shape = 2, rate = 1) distibution	Weibull distribution (shape=1.5, scale = 1)	Beta(2,5) distribution	Uniform (0,1) distribution
1	Geary	Geary	Kolmogorov-Smirnov	Kolmogorov-Smirnov	Kolmogorov-Smirnov	Kolmogorov-Smirnov	Kolmogorov-Smirnov	Kolmogorov-Smirnov	Robust Jarque–Bera
2	Brys-Hubert-Struyf	Kolmogorov-Smirnov	Geary	Geary	Coin	Geary	Geary	Gel–Miao–Gastwirth	Kolmogorov-Smirnov
3	Kolmogorov-Smirnov	Brys-Hubert-Struyf	Vasicek-Song	Brys-Hubert-Struyf	Bonett-Seier	Coin	Coin	Barrio-Cuesta-Matran-Rodriguez	Geary
4	4th Hosking	Vasicek-Song	Brys-Hubert-Struyf	4th Hosking	Geary	4th Hosking	4th Hosking	2nd Zhang	Coin
5	3rd Hosking	4th Hosking	4th Hosking	3rd Hosking	4th Hosking	Brys-Hubert-Struyf	Brys-Hubert-Struyf	4th Hosking	4th Hosking
6	Vasicek-Song	Rahman-Govindarajulu	3rd Hosking	Coin	Brys-Hubert-Struyf	Bonett-Seier	Bonett-Seier	1st Bontemps-Meddahi	Brys-Hubert-Struyf
7	2nd Hosking	Pearson Chi-Square	Pearson Chi-Square	Bonett-Seier	Spiegelhalter	3rd Hosking	3rd Hosking	Coin	Bonett-Seier
8	Pearson Chi-Square	3rd Hosking	Rahman-Govindarajulu	Vasicek-Song	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	Spiegelhalter	Spiegelhalter	2nd Cabana-Cabana	3rd Hosking
9	1st Zhang	1st Zhang	2nd Hosking	Spiegelhalter	3rd Hosking	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD	Brys-Hubert-Struyf	Spiegelhalter
10	Rahman-Govindarajulu	Barrio-Cuesta-Matran-Rodriguez	Lilliefors	Pearson Chi-Square	Gel–Miao–Gastwirth	Brys-Hubert-Struyf & Bonett-Seier	Desgagné-LafayeDeMicheaux-Leblanc	Bonett-Seier	Desgagné-LafayeDeMicheaux-Leblanc-ZEPD

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