Retropolation Models of Economical Aggregates

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Abstract: The contemporary practice used for determination of quarterly values of GDP comes out to prevailing extent from exhaustive surveys. This approach is demanding from the time, labour and financial points of view. Therefore, so-called indirect methods become increasingly important at present. Here the quarterly value of certain aggregates is estimated using a regression model of time series, in which a short-timely determined indicator enters as explaining variable. As for explained variable, there are only yearly values at our disposal in this model. The contribution deals with a specific approach to parameter estimation of such a regression model and with possibilities of its use under conditions of Czech statistics.


Keywords: National Accounting, Quarterly Values of GDP, Indirect Methods.

1 Introduction

This paper follows up with general methodology of the allocation of yearly data into seasons worked out by the authors of this contribution in French version Kozák, Hindls, Hronová (2000). The methodology of the allocation of yearly values of economical indicators contained here was gradually corrected according to experience with official data produced by Czech Statistical Office. The result of this collation of theory and practice represents the content of this paper.

2 The Allocation Problem

1. Let us start with the necessary summary of basic premises. For the years $i = 1, 2, ..., m$ and seasons $j = 1, 2, ..., n$ with $N = mn$, let us define $N$-dimensional vector of explained variable

$$
y = [y_{11}, y_{12}, ..., y_{1n}, y_{21}, y_{22}, ..., y_{mn}]$$
with the values ordered in years according to seasons. Let us suppose, it can be described using a classical linear model
\[ y = X\beta + \varepsilon, \quad E(\varepsilon) = 0_N \text{ and } E(\varepsilon\varepsilon') = \sigma^2 I_N, \]

where \( X \) denotes a non-stochastic matrix of known explaining variables with dimensions \( Np, 1 < p < N \), and with rank \( p \), \( \beta \) denotes the \( p \)-dimensional vector of unknown parameters and \( \varepsilon \) is an unobservable \( N \)-dimensional vector of mutually independent random disturbances with zero mean values and constant variances \( \sigma^2 \).

2. We shall work under conditions, when the vector \( y \) is unknown; only yearly sums are considered as known and they are ordered into \( m \)-dimensional vector \( Y = [Y_1, Y_2, \ldots, Y_m]' \)

using denotation
\[ Y_i = \sum_{j=1}^{n} y_{ij} \text{ for } i = 1, 2, \ldots, m \]

for yearly sums. There is linear relation between \( y \) and \( Y \)
\[ Qy = Y, \]

where
\[
Q = \begin{bmatrix}
1,  & 1, & \ldots, & 1, & 0, & 0, & \ldots, & 0, & 0, & \ldots, & 0, & 0, & \ldots, & 0
\end{bmatrix} \\
0,  & 0, & \ldots, & 0, & 1, & 1, & \ldots, & 1, & 0, & \ldots, & 0, & 0, & \ldots, & 0
\end{bmatrix}
\]

denotes the non-stochastic matrix with dimension \( mN \). Therefore, introducing the non-stochastic matrix with dimension \( mp \)
\[ W = QX, \]

with denotation
\[ \xi = Q\varepsilon \]

for \( m \)-dimensional vector of unobservable random disturbances and, with respect to identity
\[ QQ' = nI_m \]

using above mentioned model for \( y \), the vector \( Y \) will have form
\[ y = X\beta + \xi, \quad E(\xi) = 0_m \text{ and } E(\xi\xi') = n\sigma^2 I_m. \]

3. In the introduction of the paper quoted, we appreciate namely the fact, it contains the proposal of unique method of the allocation of yearly data into seasons
\[ u = n^{-1}Q'Y + (X - n^{-1}Q'W)B, \]

where \( N \)-dimensional vector \( u \) can be interpreted as an estimator of unknown vector \( y \), with \( B \) as the estimator of unknown vector \( \beta \). Above all, the attention deserves the specific property of the estimation proposed, because, for arbitrary vector \( B \), it holds
\[ Qu = Y, \]
and the estimation of class proposed preserves known yearly values.

4. But, the allocation method proposed generates the new problem, which was mentioned in the material quoted only as marginal one: how reasonably select the vector B representing the estimator of unknown vector β?

a) The problem would not exist in the case of known vector y. In this case, the estimator of β can be obtained with the use of common least squares method

\[
B_0 = (X'X)^{-1} X'y
\]

with matrix X of dimensions \(N_p\), which represents the estimator with optimum properties in double sense. Taking into consideration the identity

\[
E(B_0) = \beta,
\]

it is an unbiased estimator providing minimum square error

\[
R(B_0, f) = E(f' (B_0 - \beta)^2) = \sigma^2 f' (X'X)^{-1} f,
\]

where f denotes given \(p\)-dimensional non-zero vector.

In our situation, we know the vector y, because we „think out“ the methodology of the allocation (actual values serve as an criterion of estimators quality) and, therefore, we can try to obtain β vector estimation with respect to mentioned optimum properties.

The solution found will be only a starting point for comparisons from the point of view of the allocation itself, and, namely, for the possible specification of „losses“, which must be admitted during parameters estimation process.

b) In the paper quoted, only the estimator based on the model of yearly sums has been briefly mentioned

\[
B_1 = (W'W)^{-1} W'Y.
\]

With respect to the structure of the vector of yearly sums, this estimator is unbiased. In comparison with an ideal estimator \(B_0\) (not realizable with respect to the fact the vector y is unknown) this is a case of estimator reduced from \(N\) items contained in rows of X matrix to \(m = N/n\) known rows contained in the W matrix and, thus, intentionally ignoring information contained in rows of X, i.e. not taking into consideration known values of explaining variables in the seasons of individual years.

3 Simulation

1. Under the described circumstances, a feasible way to improve statistical properties of the estimation of the vector β is a simulation of elements of unknown vector y. Let us mention some possibilities in this direction under assumption of two starting points.

a) Let us mention, the knowledge of yearly sums of explained variable \(Y_i\), \(i = 1, 2, ..., m\) is supposed. Therefore, we are able to determine average values of the explained variable corresponding to one season as \(y_{0,j} = n^{-1} Y_i\) constant for \(j = 1, 2, ..., n\) in the year \(i = 1, 2, ..., m\), which can be in the first approximation considered as trend component of time series of explained variable under investigation.

b) Further, let us suppose, the matrix of explaining variables X with dimensions \(2N\) is known. After transposition, it will contain ones in the first row and the values of single explaining variable in the second row
Therefore, the vector of parameters $\mathbf{\beta}$ will contain two scalars: free parameter $\beta_0$ and the slope $\beta_1$.

2. As for the defined sequence of yearly values of explained variable, one can reproach for the fact, it does not contain stochastic component in comparison with real case. This shortage can be easily rectified by simulation. For given variance comparable e.g. with the variance $S^2$ of the sequence $y_{0,ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, we can simulate the sequence of random quantities and to create a new sequence

$$y_{1,ij} = y_{0,ij} + \psi_{ij}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n,$$

where $\psi_{ij}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ are simulated values of stochastic component arisen by „generating“ of random sample from e.g. normal distribution with zero mean value and the variance $S^2$.

3. In this connection, the possibility is offered for the estimation of parameters of the vector $\mathbf{\beta}$ containing scalars $\beta_0$ and $\beta_1$ to use the model

$$y_{1,ij} = \beta_0 + \beta_1 x_{ij} + \eta_{ij}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n,$$

where besides the quantities introduced for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, the additional two sequences play the role: the sequence $a x_{ij}$ containing the estimations of seasonally adjusted time series of used explaining variable $x_{ij}$ and the sequence of random disturbances $\eta_{ij}$. Thus, the estimations within given model can be obtained with the use of common least squares, when the estimation $b_1$ of an unknown parameter $\beta_1$ is calculated as

$$b_1 = \frac{\text{cov}(y_{1,1} a x)}{\text{var}(a x)}.$$

4. As acceptable can be considered simulation reflecting the starting idea, the simulated explained variable considered „rather“ similar in comparison with explaining variable is comparable even in the nature of seasonal component. Under validity of such a hypothesis, then, according to our opinion, it is possible to „graft“ the seasonal component contained in the sequence of explaining variable on simulated explained variable and to specify it for our purposes. This idea can be put into concrete terms e.g. by this way.

Let us suppose, during the years $i = 1, 2, \ldots, m$ and seasons $j = 1, 2, \ldots, n$, the explaining variable $x_{ij}$ can be described using a multiplicative model, i.e. by a set of seasonal indices $I_{ij}$ systematically changing their values during the seasons $j = 1, 2, \ldots, n$ and constantly repeating during the years with the condition

$$n^{-1} \sum_{j=1}^{n} I_{ij} = 1,$$

i.e. under additional assumption, the average of seasonal indices within given year equals one. If we can justify the assumption, the action of seasonal influences is equally intensive on all variables, we can simulate explained variable in the form
\[ y_{3,ij} = y_{2,ij} + \eta_{ij}, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n, \]

where
\[ y_{2,ij} = I_{ij}y_{0,ij}, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n, \]

represents the estimation of „mixed” model of regular components (both trend and seasonal one) of time series under investigation. Thus, using such a model, we suppose, for \( i = 1, 2, ..., m \) and \( j = 1, 2, ..., n \) the deterministic component of time series of explained variable is equal to the constant \( I_{ij}y_{0,ij} \) and the corresponding stochastic component is equal to simulated value \( \eta_{ij} \) in above mentioned sense of word. In this connection, it is useful to mention two facts.

a) For \( i \)-th year, \( i = 1, 2, ..., m \), let us consider the sums
\[
\sum_{j=1}^{n} y_{2,ij} = \sum_{j=1}^{n} I_{ij}y_{0,ij}.
\]

With respect to above given definition of the quantities \( y_{0,ij} \) for \( i = 1, 2, ..., m, \) \( j = 1, 2, ..., n \), we can write
\[
\sum_{j=1}^{n} y_{2,ij} = n^{-1}Y_{i} \sum_{j=1}^{n} I_{ij}.
\]

But, using the relation
\[
\sum_{j=1}^{n} I_{ij} = n,
\]

we obtain the identities
\[
\sum_{j=1}^{n} y_{2,ij} = Y_{i}, \quad i = 1, 2, ..., m,
\]

which can be interpreted by this manner: the quantities \( y_{1,ij} \) can be considered within given year \( i, \) \( i = 1, 2, ..., m, \) as the rough allocation of the values of unknown values of explained variable \( y_{ij} \) in seasons \( j = 1, 2, ..., n. \)

b) If the hypotheses considered within this article and concerning with simulated values of explained variable are justified, we can use the linear model
\[ y_{3,ij} = \beta_{0} + \beta_{1}x_{ij} + \omega_{ij}, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n \]

and to obtain the estimation \( b_{2} \) of the parameter \( \beta_{1} \) using least squares method and the formula
\[ b_{2} = \frac{\text{cov}(y_{1}, x)}{\text{var}(x)}. \]
4 Illustration Using the Building Industry Data

The above given procedure of the allocation of yearly values into seasons is illustrated using official data from Czech Statistical Office with the goal to manifest, even in conditions of economical transformation (i.e. in conditions of relative instability of short time series) is possible to use this approach. After consultations with experts from the area of branch statistics, we decided to perform a simulation using the data from the building industry, where did not occur substantial methodological changes of the content of indicators and the way of their survey during the last period. As the explained variable \( y \) the volume of building works has been selected (in CZK-Czech Crowns-milion in fixed prices 1994), as the explaining variable \( x \) natural indicator – the production of cement in thousands of tons. The reason for the selection of natural indicator as explaining variable is its easy accessibility, the rate of its survey and the relative stability in time. These properties concern not only the indicator of cement production, but of all natural indicators investigated by Czech Statistical Office, which, due to these attributes, manifest themselves as suitable explaining variables in contemporary state of matters.

As for the modelling itself, four different possibilities have been used and we describe them now in detail.

1. Let us start with somewhat not realistic model assuming the knowledge of explained variable and based on linear regression function

\[
y_{ij} = \alpha + \beta x_{ij} + e_{ij} \quad \text{for} \quad i = 1, 2, \ldots, m \quad \text{and} \quad j = 1, 2, \ldots, n.
\]

As for practical applicability of this model, it is suitable to take into consideration the first row in Table 1, where coefficient denotes the adjusted coefficient of determination, significance expresses, whether the estimations of both parameters are sufficiently statistically significant (the answer „yes“) or not (the answer „no“), „estimation“ denotes the estimation of \( \beta \) parameter obtained by common least squares method and „lower bound“ and „upper bound“ denote lower and upper bound of 95% two-sided confidence interval of given estimation.

Table 1: Characteristics of individual models

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Variables</th>
<th>Coefficient</th>
<th>Significance</th>
<th>Estimation</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Figure No.</th>
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<td>( Y, x )</td>
<td>0.357 9</td>
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<td>12.422</td>
<td>6.222</td>
<td>18.223</td>
<td>1</td>
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<td>2</td>
<td>( y_1, x )</td>
<td>0.000 0</td>
<td>no</td>
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<td>–6.375</td>
<td>6.472</td>
<td>2</td>
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<td>( y_2, x )</td>
<td>0.613 9</td>
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<td>( y_3, x )</td>
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<td>no</td>
<td>21.939</td>
<td>15.544</td>
<td>28.333</td>
<td>4</td>
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</table>

As mentioned above, the model given has no practical meaning for the allocation itself with a view to the fact, the \( y \) variable will be unknown. The data in the first row of table 1 indicate, the measure of determination of a given model is not too high. But the positive value of estimation indicates good interpretability of direct dependence between both variables.
The situation can be better understood with the use of Figure 1 showing the time course of both variables, with the scale of $y$ variable „building works in mil. CZK in fixed prices“ and the values of $x$ variable (the cement production in thousands of tons) are created by multiplying of $y$ values by certain constant. At the abscissa, the time variable is depicted (the order of quarters during the years 1990–1998).

![Figure 1](image)

2. From the point of view of practical application of the allocation problem, the additional model

$$y_{1ij} = \alpha + \beta a_{ij} + e_{ij} \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n,$$

based on the search for the relation between known yearly averages of explained variable with added simulated random disturbances (i.e. variable $y_1$ differing from variable $y_0$ by simulated values generated from normal distribution with zero mean value and standard deviation 3000) on the one hand and the variable $a_x$ representing seasonally adjusted values of $x$ variable could be the most simple multiplicative method. The second row in Table 1 manifests the „futileness“ of such an attempt from all three points of view considered – the measure of determination, significance and bad interpretability of the estimation.

State of matters can be seen from Figure 2, where time variable is again depicted at the abscissa, the axis of ordinates denotes the variable $y_1$ (expressed in mil. CZK in fixed prices) and the second variable $a_x$ (again in thousands of tons) denotes seasonally adjusted time series of explaining variable scaled in the same way as at the Figure 1. The picture indicates that seasonal adjustment of explaining variable cannot be considered as good „step“ during simplifying of the relation between both variables.
3. The preceding attempt indicates, it is necessary to employ either the model
\[ y_{2,ij} = \alpha + \beta x_{ij} + e_{2,ij} \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n, \]
or the model
\[ y_{3,ij} = \alpha + \beta x_{ij} + e_{3,ij} \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n, \]
i.e. the models similar to theoretical model 1 with the difference, its left side differs in comparison with „ideal“ model in simulated values of explained variable. Table 1
shows in the third and fourth row good interpretability of results from all three judged points of view. The state of matters is illustrated in Figures 3 and 4 having similar character like in Figure 1 with the only difference, namely, at the ordinate axis is given the variable \( y_2 \) in Figure 3 and the variable \( y_3 \) in Figure 4 and the second variable \( x \) is scaled in the same manner like in preceding two Figures 1 and 2.

![Figure 4](image)

![Figure 5](image)
Now, it is suitable to close the explanation by own considerations based on three types of allocation of the indicator of building works.

a) The first one is theoretical allocation represented by time series of explained variable, i.e. of empirical values of indicator, which naturally meets the condition needed on prescribed yearly sums. This allocation can serve only as a basis for the comparison for additional practically usable allocation, of which we use only two.

b) The example of practically usable allocation is simple allocation represented by the values of indicator of dependent variable adjusted according to the above mentioned pattern of $y_2$, i.e. of rough estimation of theoretical values of this time series constructed for $i$-th year and $j$-th quarter from the multiple of known yearly average and the value of seasonal index.

c) The additional allocation was obtained according to original allocation formula suggested by authors with the use off the estimation of parameters of the fourth model. The calculation was performed using the formulae

$$y_{4,ij} = y_{0,ij} + b(x_{ij} - x_{0,ij}) \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n ,$$

$$x_{0,ij} = \frac{1}{n} \sum_{j=1}^{n} x_{ij} \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n ,$$

where for given $i$ and $j = 1, 2, \ldots, n$ denotes yearly average of explaining variable and $b = 21.9$, according to the last row of Table 1 denotes the estimation of $\beta$ contained in above formulated model 4. The values of the allocations mentioned are given in Table 2 for given years and quarters. They are depicted in Figure 5.

### Table 2: Recapitulation

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</tbody>
</table>
In this connection, a natural question arises, which of mentioned practical allocations is "better" in the sense, it is closer to the theoretical sequence $y$. For $h$-th allocation, one can use the measure defined as the square root from average square error of the values allocated, i.e. the square root of the characteristics

$$ASE(y_h) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (y_{h,ij} - y_j)^2}{mn}.$$

Using this formula the data from Table 2 and calculating square roots, we get for $h = 2$ and 4 the values of square roots of average square errors as

$$\sqrt{ASE(y_2)} = 5419.17 \quad \text{and} \quad \sqrt{ASE(y_4)} = 4365.30,$$

of which follows:

a) because the square root of average square error can be considered as the average square deviation of the value allocated from ideal allocation, we can say, this value is, regardless of the sign, equal to about 5.4 mil. CZK in fixed prices in case of the second allocation and about 4.4 mil. CZK in fixed prices in case of the improved fourth allocation.

b) therefore, the improved allocation $y_4$ is roughly by 20% better in comparison with simple allocation $y_2$ (measured by the square root of average square error).

### Basic References


### Other References


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