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Bayes Estimation under Different Censoring Patterns on Constant-Stress PALT

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Abstract

Two-Parameter Gompertz distribution is considered here for the Bayesian inference under the Constant-Stress Partially Accelerated Life Test (CS-PALT). The first-failure Progressive (FFP) censoring pattern and its special cases have used for the analysis based on Bayes estimators of all the parameters under two different asymmetric loss functions and their special cases. A numerical study based on the real and simulated data has carried out for the analysis of the proposed method.

Keywords: constant-stress partially accelerated life test (CS-PALT), first-failure progressive (FFP) censoring pattern, LINEX loss function (LLF), Al-Bayyati loss function (AbLF), Bayes estimator, Gompertz distribution.

1. Introduction

In the life testing experiments and the consistency analysis, the cost and time limitation push to experimenters, to end the experiments before all the units on test fail. In general, it is known as censoring process. The estimate based on the censored data is less accurate than the estimate from the complete data. However, the censoring reduces the test time and expense of the life test. In literature, a number of censoring patterns available and the researcher's from time to time show their recommendations based on these censoring schemes.

Most common censoring patterns in literature are named as Type-II censoring and in this censoring, all the test units placed on a test and the test dismisses after an assumed number of units fail. The major disadvantage of Type-II censoring is, only smallest lifetimes of the test units are observed. The generalization of this censoring pattern is known as the Progressive Type-II censoring pattern. In this pattern, a well-organized scheme has used for the removal of a pre-specified number of surviving test units at each failure time during the experiments. Elimination of test units before the failure may be planned to save the time and experimental cost (See Balakrishnan and Aggarwala (2000) for more details about the Progressive censoring scheme). A huge literature is available on these censoring, a little few of them are, Ahmadia, Mir-Mostafaee, and Balakrishnan (2011), Azimi, Yaghmaei, and Azimi (2012), Prakash and Singh (2013), Prakash (2015 a), and Prakash (2016).

It is known facts that, the basic goal of Progressive censoring is to save some live test units for another test, which is mainly useful when units being tested are expensive. Johnson (1964) discussed a live test in which the experimenter should divide the test units into a number of sets and each set as an assembly of test units, and run simultaneously. The test terminated when the first failure is occurring in each group. This censoring pattern is named as first failure censoring. If an experimenter wants to remove some test units before first failures occur in these sets, this life test plan is called a First-Failure Progressive (FFP) censoring scheme (Wu and Kus (2009)). The focus of the present study is to study about the Bayes estimators under different cases of the FFP censoring based on CP-ALT. Four different censoring plans are the special cases of the FFP censoring and used here them.

Again, in some experiments where the life of the product was tested, it is not always easy to collect lifetimes on highly reliable products with a long lifetime under the normal operating conditions. The accelerated life testing (ALT) or partially accelerated life testing (PALT) are applicable and widely used in such test situation. In ALT, all test units are kept under higher stress levels, but in the PALT only few of them are running under severe condition.

Three different methods of stress loading in ALT are available. The first method is constant-stress ALT, in which stress is kept at a constant level throughout the test (Bagdonavicius and Nikulin (2002)). The stress applied to the test product is continuously increasing with time, known as progressive-stress ALT (Al-Hussaini and Abdel-Hamid (2010)). In step-stress ALT, the test condition changes at a given time or upon the occurrence of a specified number of failures (Ismail, Abdel-Ghalyb, and El-Khodary (2011), Tangi, Guani, Xu, and Xu (2012), Hyun and Lee (2015), Abdel-Hamid (2016) and Prakash (2017 a)).

In all the above cases the acceleration factor is pre-assumed, but in case when acceleration factor can't assume as a known value, the partially accelerated life test (PALT) will be a better choice for the life testing. In PALT, the test units are tested in both the conditions and also has three major stress loading named as progressive-stress, constant-stress and step-stress. In the Constant-Stress PALT (CS-PALT), the item under the test, runs either use or accelerated condition only.

2. FFP censoring under CS-PALT

The Gompertz distribution inhabits a significant place in modeling human mortality and fitting actuarial tables. The present distribution is getting some more attention nowadays in the areas of technology, medicine, biology and natural sciences also. The underlying distribution was introduced by Gompertz (1825), and contribution to the statistical method and characterization of this distribution by many authors. Few of them are Gordon (1990), Rao and Damaraju (1992) and Wu and Lee (1999).

Garg, Rao, and Redmond (1970) studied the properties of the maximum likelihood estimates, whereas Chen (1997) developed an exact confidence interval and exact joint confidence region for the parameters of the concerned distribution. Applications and more survey for the Gompertz model are given by Al-Hussaini, Al-Dayian, and Adham (2000). Wu, Hung, and Tsai (2004) uses the least square method for the estimation of the parameters, while the progressive first-failure censored data used for the estimation of the parameters by Soliman, Ahmed, Naser, and Gamal (2012).

The probability density and cumulative density function of the two-parameter Gompertz life distribution are given as

$$f(x;\theta,\sigma)=\sigma\, e^{\theta x}\, exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x}-1\right)\right)\;;\; x>0,\sigma>0,\theta>0 \tag{1}$$

and

$$F(x;\theta,\sigma)=1-exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x}-1\right)\right)\;;\; x>0,\sigma>0,\theta>0. \tag{2}$$

Here, the parameter θ is the shape parameter and parameter σ is called the scale parameter respectively. The underlying distribution is an Uni-model with positive skewness and an increasing hazard rate function. It is also noted that when $\theta\to 0$, the distribution will tend to an Exponential distribution.

In CS-PALT, n_1 test units (selected randomly) from total of n test units, are run at the normal test condition and the remaining $n_2(=n-n_1)$ test units are at accelerated test condition. The probability density function, distribution function and failure rate are defined, when an item tested in normal test condition as

$$f_1(x_1;\theta,\sigma)=\sigma\, e^{\theta x_1}\, exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x_1}-1\right)\right), \tag{3}$$

$$F_1(x_1;\theta,\sigma)=1-exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x_1}-1\right)\right) \tag{4}$$

and

$$\rho_1(x_1)=\sigma e^{\theta x_1}\;;\; x_1>0,\sigma>0,\theta>0. \tag{5}$$

Let the failure rate function $\rho_2(x_2)$ is denoted for an item tested at accelerated condition with the acceleration factor $\lambda(>1)$, and defined for considering model given in Eq. (3) as

$$\rho_2(x_2)=\lambda\sigma e^{\theta x_2}\;;\; x_2>0,\sigma>0,\theta>0. \tag{6}$$

Based on failure rate function $\rho_2(x_2)$ given in Eq. (6), the probability density function and the distribution function under the accelerated condition are obtained as

$$f_2(x_2;\theta,\sigma)=\sigma\lambda\, e^{\theta x_2}\, exp\left(-\frac{\sigma\lambda}{\theta}\left(e^{\theta x_2}-1\right)\right), \tag{7}$$

and

$$F_2(x_2;\theta,\sigma)=1-exp\left(-\frac{\sigma\lambda}{\theta}\left(e^{\theta x_2}-1\right)\right). \tag{8}$$

Following Prakash (2017 b), the joint probability density function of order statistics based on first-failure progressively Type-II censoring scheme (FFP) under CS-PALT is defined as

$$L(\theta,\sigma,\lambda|\underline{x})\propto\left\{\prod_{i=1}^{m_1} f_1\left(x_{(1i)};\theta,\sigma\right)\left(1-F_1\left(x_{(1i)};\theta,\sigma\right)\right)^{k(R_{1i}+1)-1}\right\}$$
$$\times\left\{\prod_{i=1}^{m_2} f_2\left(x_{(2i)};\theta,\sigma,\lambda\right)\left(1-F_2\left(x_{(2i)};\theta,\sigma,\lambda\right)\right)^{k(R_{2i}+1)-1}\right\}. \tag{9}$$

In FPP censoring, total test units are divided into a number of independent groups with an equal number of test units in each group. In the present case, FFP censoring is combined with CS-PALT. Hence, the total n test units divided between two groups, each have n_1 and n_2 test units. The n_1 units are kept in a normal test condition and n_2 are at accelerated test condition. Each group further break into m groups with an equal number of test units k. Here, R_{1i} and R_{2i} are the progressive censoring scheme for normal and accelerated test group, respectively and FFP censoring runs until m_1 and m_2 failure are observed in each test condition respectively. Using Eq. (3-4) & Eq. (7-8) in Eq. (9), we get

$$L(\theta,\sigma,\lambda|\underline{x}) \propto \left\{ \prod_{i=1}^{m_1} \sigma\, e^{\theta x_{(1i)}}\, exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x_{(1i)}}-1\right)\right)\left(exp\left(-\frac{\sigma}{\theta}\left(e^{\theta x_{(1i)}}-1\right)\right)\right)^{k(R_{1i}+1)-1} \right\}$$

$$\times \left\{ \prod_{i=1}^{m_2} \lambda\sigma\, e^{\theta x_{(2i)}}\, exp\left(-\frac{\lambda\sigma}{\theta}\left(e^{\theta x_{(2i)}}-1\right)\right)\left(exp\left(-\frac{\lambda\sigma}{\theta}\left(e^{\theta x_{(2i)}}-1\right)\right)\right)^{k(R_{2i}+1)-1} \right\}$$

$$L(\theta,\sigma,\lambda|\underline{x}) \propto \sigma^{m_1+m_2}\lambda^{m_2}e^{\theta T_0}\, exp\left\{-k\frac{\sigma}{\theta}\left(T_1(\underline{x},\theta)+\lambda T_2(\underline{x},\theta)\right)\right\}; \tag{10}$$

where $T_0 = \sum_{i=1}^{m_1} x_{(1i)} + \sum_{i=1}^{m_2} x_{(2i)}$, $T_1(\underline{x},\theta) = \sum_{i=1}^{m_1}(1+R_{1i})\left(e^{\theta x_{(1i)}}-1\right)$ and $T_2(\underline{x},\theta) = \sum_{i=1}^{m_2}(1+R_{2i})\left(e^{\theta x_{(2i)}}-1\right)$.

The Maximum Likelihood (ML) estimation of the parameter under consideration are obtained by taking logarithm of Eq. (10) as

$$Log\, L(\theta,\sigma,\lambda|\underline{x}) = (m_1+m_2)\, log\, \sigma + m_2\, log\, \lambda + \theta T_0$$

$$-k\frac{\sigma}{\theta}T_1(\underline{x},\theta) - k\lambda\frac{\sigma}{\theta}T_2(\underline{x},\theta). \tag{11}$$

The first order Differentiation of Eq. (11) with respect to the parameter θ, σ and λ are given respectively as

$$T_0 = k\frac{\hat{\sigma}_{ML}}{\hat{\theta}_{ML}}\left\{\sum_{i=1}^{m_1}(1+R_{1i})\, x_{(1i)}\, e^{\hat{\theta}_{ML}x_{(1i)}} + \hat{\lambda}_{ML}\sum_{i=1}^{m_2}(1+R_{2i})\, x_{(2i)}\, e^{\hat{\theta}_{ML}x_{(2i)}}\right\}$$

$$-k\,\frac{\hat{\sigma}_{ML}}{\hat{\theta}^2_{ML}}\left(T_1(\underline{x},\theta)+\hat{\lambda}_{ML}T_2(\underline{x},\theta)\right), \tag{12}$$

$$\hat{\sigma}_{ML} = \frac{\hat{\theta}_{ML}(m_1+m_2)}{k\left\{\sum_{i=1}^{m_1}(1+R_{1i})\left(e^{\hat{\theta}_{ML}x_{(1i)}}-1\right)+\hat{\lambda}_{ML}\sum_{i=1}^{m_2}(1+R_{2i})\left(e^{\hat{\theta}_{ML}x_{(2i)}}-1\right)\right\}} \tag{13}$$

and

$$\hat{\lambda}_{ML} = \frac{\hat{\theta}_{ML}m_2}{k\hat{\sigma}_{ML}\sum_{i=1}^{m_2}(1+R_{2i})\left(e^{\hat{\theta}_{ML}x_{(2i)}}-1\right)}. \tag{14}$$

The unknown parameters are involved in the expressions of ML estimation. A numerical technique (Newton Raphson integral method) is applied here for the numerical findings of these ML estimates.

3. Bayes estimation under asymmetric loss function

Selection of prior density in the Bayesian analysis plays an important role, however, researchers have pointed out from time to time that there is no any true way of choice of prior distribution. It depends completely upon personal belief of the researchers. If one may have satisfactory information about the parameter under study, one should use, informative prior; otherwise it is preferable to use non-informative prior.

Osman (1987) was derived a compound Gompertz model by considering one of the parameters of the Gompertz distribution as a random variable following the Gamma distribution. He studied the properties of compound Gompertz distribution and suggested its use for modeling lifetime data and analyzing the survivals in heterogeneous populations. Following, him in the present study, informative one-parameter Gamma distribution assumed here as the conjugate family of prior density for the parameters under study and defined as

$$\pi_\theta = \frac{\theta^{\alpha-1}\, e^{-\theta}}{\Gamma(\alpha)}$$

and

$$\pi_\sigma = \frac{\sigma^{\beta-1}\, e^{-\sigma}}{\Gamma(\beta)}.$$

The vague prior is selected for the acceleration factor λ here, so that the prior do not play any significant roles in the analyses. The selected vague prior for the parameter λ and the joint prior distribution are given as

$$\pi_\lambda = \frac{1}{\lambda}\ ;\ \lambda > 0$$

and

$$\pi_{(\theta,\sigma,\lambda)} \propto \theta^{\alpha-1}\, \sigma^{\beta-1}\, \lambda^{-1}\, e^{-\theta-\sigma}.$$

The resultant joint and marginal posterior densities corresponding to the parameters θ, σ and λ are obtained and given as

$$\pi^*_{(\theta,\sigma,\lambda)} = \Omega\, \sigma^{m_1+m_2+\beta-1}\, \theta^{\alpha-1}\, \lambda^{m_2-1}\, e^{\theta(T_0-1)} e^{-\sigma}$$

$$\times exp\left\{-k\frac{\sigma}{\theta}\left(T_1\left(\underline{x},\theta\right)+\lambda T_2\left(\underline{x},\theta\right)\right)\right\},$$

$$\pi^*_{(\theta)} = \Omega_\theta \frac{\theta^{m_2+\alpha-1}\, e^{\theta(T_0-1)}}{\left(T_2\left(\underline{x},\theta\right)\right)^{m_2}\ \left(1+\frac{k}{\theta}T_1\left(\underline{x},\theta\right)\right)^{m_1+\beta}}, \tag{15}$$

$$\pi^*_{(\sigma)} = \Omega_\sigma\, \sigma^{m_1+\beta-1}\, e^{-\sigma} \int_\theta \frac{\theta^{m_2+\alpha-1}\, e^{\theta(T_0-1)}}{\left(T_2\left(\underline{x},\theta\right)\right)^{m_2}}\, exp\left\{-k\frac{\sigma}{\theta}T_1\left(\underline{x},\theta\right)\right\} d\theta \tag{16}$$

and

$$\pi^*_{(\lambda)} = \Omega_\lambda \lambda^{m_2-1} \int_\theta \frac{\theta^{\alpha-1}\, e^{\theta(T_0-1)}}{\left(1+\frac{k}{\theta}\left(T_1\left(\underline{x},\theta\right)+\lambda T_2\left(\underline{x},\theta\right)\right)\right)^{m_1+m_2+\beta}} d\theta \tag{17}$$

where $\Omega_\theta = \Omega\, \frac{\Gamma(m_1+\beta)\,\Gamma(m_2)}{k^{m_2}}$, $\Omega_\sigma = \Omega\, \frac{\Gamma(m_2)}{k^{m_2}}$, $\Omega_\lambda = \Omega\, \Gamma\left(m_1+m_2+\beta\right)$ and $\Omega = \left\{\frac{\Gamma(m_2)}{k^{m_2}}\right.$ $\left.\Gamma\left(m_1+\beta\right)\ \int_\theta \frac{\theta^{m_2+\alpha-1}\, e^{\theta(T_0-1)}}{\left(T_2(\underline{x},\theta)\right)^{m_2}\ \left(1+\frac{k}{\theta}T_1(\underline{x},\theta)\right)^{m_1+\beta}} d\theta\right\}^{-1}.$

Again, the selection of loss function is also crucial in Bayesian analysis. The most common loss function is squared error loss function, which is the posterior mean of the parameter under consideration. This is symmetric in nature, but in real life experiments, there is not always possible the symmetric situation. In some situation overestimation is more serious than underestimation and vice versa. In the present article, we considered two different kinds of the asymmetric loss function. One is derived by Al-Bayyati (2002) named as Al-Bayyati loss function (AbLF) and the second one is Linex loss function (LLF) derived Varian (1975).

Properties of Bayes estimation of the life parameters of the Rayleigh distribution under Progressive censoring were recently studied by Prakash (2015 b). AbLF is the result of a slight modification in squared error loss function and is defined for any estimate $\hat{\theta}$ corresponding to the parameter θ as

$$L_{Al}\left(\hat{\theta},\theta\right)=\theta^{a}\left(\hat{\theta}-\theta\right)^{2}\ ;\ a\in R^{+}.$$

The Bayes estimator $\hat{\theta}_{Al}$ under AbLF corresponding to the any parameter θ is defined as

$$\hat{\theta}_{Al}=\frac{\int_{\theta}\theta^{a+1}\,\pi^{*}_{(\theta)}\,d\theta}{\int_{\theta}\theta^{a}\,\pi^{*}_{(\theta)}\,d\theta}. \tag{18}$$

For the particular, $a=0$ the Bayes estimator under AbLF is simply the posterior mean (Bayes estimator under squared error loss function) and for $a=-2$, the Bayes estimator under invariant squared error loss function (Prakash (2011)).

The Bayes estimators corresponding to the parameters θ,σ and λ under the AbLF given in Eq. (18) are obtained as

$$\hat{\theta}_{Al}=\frac{\int_{\theta}\frac{\theta^{m_2+\alpha+a}\,e^{\theta(T_0-1)}}{(T_2(\underline{x},\theta))^{m_2}\left(1+\frac{k}{\theta}T_1(\underline{x},\theta)\right)^{m_1+\beta}}d\theta}{\int_{\theta}\frac{\theta^{m_2+\alpha+a-1}\,e^{\theta(T_0-1)}}{(T_2(\underline{x},\theta))^{m_2}\left(1+\frac{k}{\theta}T_1(\underline{x},\theta)\right)^{m_1+\beta}}d\theta}, \tag{19}$$

$$\hat{\sigma}_{Al}=\Gamma\left(m_1+\beta+a\right)\frac{\int_{\theta}\frac{\theta^{m_2+\alpha-1}\,e^{\theta(T_0-1)}}{(T_2(\underline{x},\theta))^{m_2}\left(1+\frac{k}{\theta}T_1(\underline{x},\theta)\right)^{m_1+\beta+a+1}}d\theta}{\int_{\theta}\frac{\theta^{m_2+\alpha-1}\,e^{\theta(T_0-1)}}{(T_2(\underline{x},\theta))^{m_2}\left(1+\frac{k}{\theta}T_1(\underline{x},\theta)\right)^{m_1+\beta+a}}d\theta} \tag{20}$$

and

$$\hat{\lambda}_{Al}=\frac{\int_{\lambda}\lambda^{m_2+a}\int_{\theta}\frac{\theta^{\alpha-1}\,e^{\theta(T_0-1)}}{\left(1+\frac{k}{\theta}(T_1(\underline{x},\theta)+\lambda T_2(\underline{x},\theta))\right)^{m_1+m_2+\beta}}d\theta\,d\lambda}{\int_{\lambda}\lambda^{m_2+a-1}\int_{\theta}\frac{\theta^{\alpha-1}\,e^{\theta(T_0-1)}}{\left(1+\frac{k}{\theta}(T_1(\underline{x},\theta)+\lambda T_2(\underline{x},\theta))\right)^{m_1+m_2+\beta}}d\theta\,d\lambda} \tag{21}$$

Varian (1975) discussed about the convex loss function, which is appropriate in the situations where overestimation is more serious than underestimation and vice versa. The sign of shape parameter $'c'$ reflects the direction of asymmetry whereas the magnitude reflects the degree of asymmetry (see Prakash (2011) for more details). The Linex loss function (LLF) is defined for any parameter θ as

$$L\left(\hat{\theta},\theta\right)=e^{c\left(\hat{\theta}-\theta\right)}-c\left(\hat{\theta}-\theta\right)-1\ ;\ c\neq 0. \tag{22}$$

The Bayes estimators corresponding to the parameters θ,σ and λ under LLF are obtained as

$$\hat{\theta}_{L}=-\frac{1}{c}\,log\int_{\theta}e^{-c\theta}\,\pi^{*}_{(\theta)}\,d\theta$$

$$\Rightarrow\hat{\theta}_{L}=-\frac{1}{c}\,log\left\{\Omega_{\theta}\int_{\theta}\frac{\theta^{m_2+\alpha-1}\,e^{\theta(T_0-c-1)}}{(T_2\left(\underline{x},\theta\right))^{m_2}\ \left(1+\frac{k}{\theta}T_1\left(\underline{x},\theta\right)\right)^{m_1+\beta}}d\,\theta\right\}, \tag{23}$$

$$\hat{\sigma}_{L}=-\frac{1}{c}\,log\left\{\Omega_{\sigma}\Gamma\left(m_1+\beta\right)\int_{\theta}\frac{\theta^{m_2+\alpha-1}\,e^{\theta(T_0-1)}}{(T_2\left(\underline{x},\theta\right))^{m_2}}\left(\frac{k}{\theta}T_1\left(\underline{x},\theta\right)+c+1\right)^{-m_1-\beta}d\theta\right\} \tag{24}$$

and

$$\hat{\lambda}_L = -\frac{1}{c}\, log \left\{ \Omega_\lambda \int_\lambda \lambda^{m_2-1}\, e^{-c\lambda} \int_\theta \frac{\theta^{\alpha-1}\, e^{\theta(T_0-1)}}{\left(1+\frac{k}{\theta}\left(T_1\left(\underline{x},\theta\right)+\lambda T_2\left(\underline{x},\theta\right)\right)\right)^{m_1+m_2+\beta}} d\theta\, d\lambda \right\}. \quad (25)$$

The expressions of Bayes estimators and their corresponding Bayes risks for the unknown parameters under both loss functions are not obtained in nice closed form. A numerical technique with the simulation was applied here for obtaining the numerical findings.

4. Numerical illustration

Based on pre-assumed values of $\alpha(=0.50, 1.00, 2.50)$ and $\beta(=0.50, 1.00, 2.50)$ the values of θ and σ are generated from the prior distribution π_θ and π_σ respectively. Using these generated values, a set of $10,000$ random sample generated, each of size $n = 30$ by using following relation

$$x_{(i)} = \frac{1}{\theta}\, log \left\{ 1 - \frac{\theta}{\sigma}\, log\, \left(1 - U_{(i)}\right) \right\}.$$

Here, $U_{(i)}$ are independently distributed $U(0,1)$. Monte Carlo simulation technique was applied here for generating FFP censored samples for each simulation (See details for algorithms described in Balakrishnan and Sandhu (1995)). For different special cases of FFP censoring scheme along with different values of k are given in Table 1. Here the values of m_1 and m_2 are assumed equal only for simplicity in calculation.

The calculated ML estimate of the parameters under consideration are given in Table 2 under all possible special cases of the FFP censoring. It is observed from the tables that, as hyper-parameter increases the magnitude of the ML estimate first increase and then decreases. Increasing trend also have seen when the sample size getting larger. The magnitude of the ML estimate is observed maximum for the FFP censoring whereas the minimum is noted for the Type-II censoring for all the considered values. However, the magnitudes of ML estimates are smaller.

Table 1: Special cases of FFP censoring scheme

Case	k	m_1	m_2	$R_i; 1, 2, ...,$	Different Censoring Plans
1	5	05	05	1 2 0 2 1	First-Failure Progressive Type-II Censoring (FFP)
2	5	05	05	0 0 0 0 0	Progressive Type-II Censoring (PC)
3	1	05	05	1 2 0 2 1	First-Failure Censoring (FFC)
4	1	05	05	0 0 0 0 25	Type-II Censoring (T-II)
5	1	05	05	0 0 0 0 0	Complete Sample (CS)
1	5	10	10	1 0 0 5 0 0 1 4 2 1	First-Failure Progressive Type-II Censoring (FFP)
2	5	10	10	0 0 0 0 0 0 0 0 0 0	Progressive Type-II Censoring (PC)
3	1	10	10	1 0 0 5 0 0 1 4 2 1	First-Failure Censoring (FFC)
4	1	10	10	0 0 0 0 0 0 0 0 0 20	Type-II Censoring (T-II)
5	1	10	10	0 0 0 0 0 0 0 0 0 0	Complete Sample (CS)

Table 2: ML estimate under FFP censoring scheme

	m_1	m_2	(α,β)	FFP	PC	FFC	T-II	CS
$\hat{\theta}_{ML}$	05	05	0.50, 0.50	1.4113	1.3714	1.2853	1.1204	1.3153
			1.00, 1.00	1.4323	1.3845	1.3231	1.1303	1.3207
			2.50, 2.50	1.4227	1.3759	1.3101	1.1118	1.3193
	10	10	0.50, 0.50	1.4387	1.3783	1.3112	1.1542	1.3416
			1.00, 1.00	1.4589	1.4116	1.3494	1.1743	1.3437
			2.50, 2.50	1.4473	1.3829	1.3363	1.1455	1.3356
$\hat{\sigma}_{ML}$	05	05	0.50, 0.50	1.2710	1.2498	1.1966	1.1169	1.1738
			1.00, 1.00	1.2930	1.2636	1.2009	1.1259	1.1987
			2.50, 2.50	1.2813	1.2488	1.1791	1.0991	1.1675
	10	10	0.50, 0.50	1.3018	1.2651	1.2301	1.1476	1.2171
			1.00, 1.00	1.3142	1.2712	1.2348	1.1659	1.2396
			2.50, 2.50	1.3036	1.2552	1.2229	1.1397	1.2127
$\hat{\lambda}_{ML}$	05	05	0.50, 0.50	1.1627	1.1298	1.0589	0.9230	1.0836
			1.00, 1.00	1.1800	1.1406	1.0900	0.9312	1.0880
			2.50, 2.50	1.1721	1.1335	1.0793	0.9159	1.0869
	10	10	0.50, 0.50	1.1852	1.1355	1.0802	0.9509	1.1053
			1.00, 1.00	1.2019	1.1629	1.1117	0.9675	1.1070
			2.50, 2.50	1.1923	1.1393	1.1009	0.9437	1.1003
Result Based on Real Data								
$\hat{\theta}_{ML}$	05	05	1.00, 1.00	1.4007	1.3449	1.2649	1.1162	1.2725
	10	10	1.00, 1.00	1.4178	1.3515	1.3306	1.1692	1.3050
$\hat{\sigma}_{ML}$	05	05	1.00, 1.00	1.2471	1.2262	1.1635	1.0947	1.1410
	10	10	1.00, 1.00	1.2676	1.2313	1.1767	1.1451	1.1838
$\hat{\lambda}_{ML}$	05	05	0.50, 0.50	1.1672	1.1179	1.0574	0.9139	1.0754
	10	10	0.50, 0.50	1.1691	1.1502	1.1091	0.9352	1.0844

Table 3-5 shows the Bayes risks for the Bayes estimators corresponding to the parameters θ, σ and λ respectively under AbLF. Four possible cases of AbLF are assumed here for the analysis. The values of the shape parameter $'a'$ of AbLF are assumed here as $a(= 1, -1, -2, 0)$, $a = -2$, shows the Bayes risk under ISELF and for $a = 0$ it is the Posterior risk. The numerical findings shows, the maximum risks under the complete sample case, whereas the minimum risk under FFP censoring. It is also observed that, for higher hyper-parametric values, the Bayes risks are smaller for Type-II censoring when compared with FFC. Otherwise Bayes risks increase as the censoring patterns changed. Other properties are seen similar as discussed above.

The Gompertz distribution is often applied to describe the distribution of adult lifespans by actuaries and demographers. Computer scientists have also started to model the failure rates of computer codes by said model. Biology and gerontology also considered the Gompertz distribution for the analysis of survival. More recently, In Marketing Science, it has been used as an individual-level simulation for customer lifetime value modeling. In all the said applications, overestimation is more serious than underestimation. Hence, the positive value of shape parameter $c(> 0)$ is considered here for LLF and it shows the loss function is quite asymmetric about 0 with overestimation being more costly than underestimation.

The Bayes risks corresponding to the Bayes estimators under LLF are given in Tables 6-8 respectively for $c = 0.50, 1.00$. The minimum Bayes risks are obtained for FFP censoring whereas the maximum for complete sample case and the risk increases as the censoring patterns changed. The Bayes risks are increased when the shape parameter of LLF $'c'$ increases. Other properties are seen similar as discussed above. Hence, for minimum Bayes risk one may prefer the FFP censoring over other censoring patterns.

5. Real data illustration

Real life data of the cancer survival times in years (Bekker, Roux, and Mostert (2000)) has been examined to illustrate the real-world applicability of the results. All the results have been obtained for analysis purpose for all selected parametric values as assumed in previous section, but presents only the results for $\alpha = \beta = 1.00$ in concerned tables. All the properties have been seen similar as discussed above. One remarkable point is that, the risk magnitude is smaller as compared to simulated data, however the difference in risk magnitude is nominal.

Table 3: Bayes risk of $\hat{\theta}_{Al}$ under AbLF

m_1	m_2	a	(α, β)	FFP	PC	FFC	T-II	CS
05	05	1	0.50, 0.50	0.9415	1.0601	1.0953	1.1425	1.1761
			1.00, 1.00	0.9499	1.0919	1.1099	1.1634	1.1837
			2.50, 2.50	0.9343	1.0809	1.1563	1.1087	1.1756
		-1	0.50, 0.50	1.0189	1.1033	1.1313	1.1839	1.2213
			1.00, 1.00	1.0282	1.1364	1.1387	1.1961	1.2409
			2.50, 2.50	1.0208	1.1264	1.1881	1.1351	1.2319
		-2 (ISELF)	0.50, 0.50	1.1116	1.2603	1.3001	1.3529	1.4131
			1.00, 1.00	1.1259	1.3079	1.3155	1.3861	1.4332
			2.50, 2.50	1.1074	1.2841	1.3574	1.3049	1.4216
		0 (SELF)	0.50, 0.50	1.1651	1.3066	1.3677	1.4261	1.4876
			1.00, 1.00	1.1754	1.3259	1.3834	1.4307	1.5094
			2.50, 2.50	1.1561	1.3123	1.4301	1.3719	1.4940
10	10	1	0.50, 0.50	0.9699	1.1020	1.1274	1.1482	1.1809
			1.00, 1.00	0.9869	1.1292	1.1340	1.1663	1.2026
			2.50, 2.50	0.9626	1.1224	1.1581	1.1230	1.1962
		-1	0.50, 0.50	1.0405	1.1276	1.1559	1.1903	1.2469
			1.00, 1.00	1.0594	1.1579	1.1633	1.2216	1.2658
			2.50, 2.50	1.0424	1.1504	1.1946	1.1511	1.2549
		-2 (ISELF)	0.50, 0.50	1.1296	1.2861	1.3063	1.3760	1.4158
			1.00, 1.00	1.1397	1.3241	1.3284	1.3890	1.4367
			2.50, 2.50	1.1241	1.2904	1.3805	1.3111	1.4271
		0 (SELF)	0.50, 0.50	1.2002	1.3635	1.3951	1.4333	1.4961
			1.00, 1.00	1.2211	1.3973	1.4032	1.4679	1.5171
			2.50, 2.50	1.1912	1.3889	1.4380	1.3896	1.5050
Result Based on Real Data								
05	05	1	1.00, 1.00	0.9155	1.0651	1.0864	1.1299	1.1595
		-1	1.00, 1.00	0.9930	1.1091	1.1149	1.1622	1.2161
		-2 (ISELF)	1.00, 1.00	1.0897	1.2789	1.2899	1.3503	1.4065
		0 (SELF)	1.00, 1.00	1.1387	1.2967	1.3572	1.3945	1.4819
10	10	1	1.00, 1.00	0.9521	1.1020	1.1102	1.1327	1.1782
		-1	1.00, 1.00	1.0239	1.1304	1.1393	1.1875	1.2407
		-2 (ISELF)	1.00, 1.00	1.1034	1.2950	1.3027	1.3532	1.4099
		0 (SELF)	1.00, 1.00	1.1840	1.3674	1.3768	1.4313	1.4895

Table 4: Bayes risk of $\hat{\sigma}_{Al}$ under AbLF

m_1	m_2	a	(α, β)	FFP	PC	FFC	T-II	CS
05	05	1	0.50, 0.50	0.8352	0.9596	1.0445	1.0746	1.0922
			1.00, 1.00	0.8426	0.9882	1.0545	1.0905	1.0963
			2.50, 2.50	0.8287	0.9783	1.0832	1.0480	1.0952
		-1	0.50, 0.50	0.9316	1.0702	1.1153	1.1625	1.1960
			1.00, 1.00	0.9398	1.1012	1.1199	1.1736	1.2137
			2.50, 2.50	0.9243	1.0901	1.1663	1.1187	1.2056
		-2 (ISELF)	0.50, 0.50	1.0018	1.1076	1.1193	1.1962	1.2319
			1.00, 1.00	1.0106	1.1409	1.1631	1.2180	1.2607
			2.50, 2.50	0.9941	1.1396	1.1514	1.2003	1.2521
		0 (SELF)	0.50, 0.50	1.0459	1.1998	1.2177	1.2501	1.3074
			1.00, 1.00	1.0551	1.2351	1.2528	1.2624	1.3137
			2.50, 2.50	1.0378	1.2029	1.2544	1.2315	1.3008
10	10	1	0.50, 0.50	0.8706	0.9891	1.0497	1.0853	1.0952
			1.00, 1.00	0.8858	1.0179	1.0749	1.1005	1.1137
			2.50, 2.50	0.8642	1.0081	1.0918	1.0532	1.1076
		-1	0.50, 0.50	0.9699	1.1019	1.1274	1.1683	1.2090
			1.00, 1.00	0.9868	1.1293	1.1340	1.1963	1.2260
			2.50, 2.50	0.9627	1.1225	1.1721	1.1230	1.2163
		-2 (ISELF)	0.50, 0.50	1.0320	1.1795	1.1924	1.2324	1.2864
			1.00, 1.00	1.0499	1.2015	1.2065	1.2622	1.3044
			2.50, 2.50	1.0243	1.1943	1.2364	1.1948	1.2941
		0 (SELF)	0.50, 0.50	1.0774	1.2123	1.2240	1.2866	1.3430
			1.00, 1.00	1.0961	1.2543	1.2596	1.3177	1.3618
			2.50, 2.50	1.0693	1.2468	1.2908	1.2474	1.3510
Result Based on Real Data								
05	05	1	1.00, 1.00	0.7940	0.9445	1.0125	1.0380	1.0531
		-1	1.00, 1.00	0.8885	1.0544	1.0760	1.1187	1.1672
		-2 (ISELF)	1.00, 1.00	0.9573	1.0929	1.1180	1.1619	1.2129
		0 (SELF)	1.00, 1.00	1.0005	1.1845	1.2052	1.2150	1.2644
10	10	1	1.00, 1.00	0.8360	0.9734	1.0323	1.0477	1.0701
		-1	1.00, 1.00	0.9342	1.0817	1.0897	1.1408	1.1792
		-2 (ISELF)	1.00, 1.00	0.9955	1.1518	1.1602	1.2048	1.2554
		0 (SELF)	1.00, 1.00	1.0404	1.2032	1.2118	1.2588	1.3112

Table 5: Bayes risk of $\hat{\lambda}_{Al}$ under AbLF

m_1	m_2	a	(α, β)	FFP	PC	FFC	T-II	CS
05	05	1	0.50, 0.50	0.7051	0.8104	0.8126	0.8348	0.9039
			1.00, 1.00	0.7214	0.8245	0.8466	0.8632	0.9183
			2.50, 2.50	0.7096	0.8104	0.8577	0.8312	0.8904
		-1	0.50, 0.50	0.7350	0.8291	0.9023	0.9364	0.9588
			1.00, 1.00	0.7518	0.9052	0.9284	0.9655	0.9835
			2.50, 2.50	0.7490	0.8913	0.9395	0.9126	0.9639
		-2 (ISELF)	0.50, 0.50	0.8307	0.9144	0.9678	1.0148	1.0308
			1.00, 1.00	0.8418	0.9227	1.0017	1.0207	1.0559
			2.50, 2.50	0.8324	0.9168	1.0082	0.9899	1.0455
		0 (SELF)	0.50, 0.50	0.9377	1.0757	1.0917	1.1208	1.1721
			1.00, 1.00	0.9459	1.1073	1.1232	1.1318	1.1778
			2.50, 2.50	0.9304	1.0784	1.1246	1.1041	1.1662
10	10	1	0.50, 0.50	0.7366	0.8189	0.8369	0.8897	0.9283
			1.00, 1.00	0.7495	0.8276	0.8613	0.9101	0.9411
			2.50, 2.50	0.7312	0.8125	0.8826	0.8529	0.9337
		-1	0.50, 0.50	0.7684	0.8784	0.9107	0.9434	0.9652
			1.00, 1.00	0.8022	0.9094	0.9334	0.9665	1.0091
			2.50, 2.50	0.7924	0.8939	0.9466	0.9243	1.0011
		-2 (ISELF)	0.50, 0.50	0.8466	0.9244	0.9739	1.0542	1.0795
			1.00, 1.00	0.8811	1.0082	1.0225	1.0792	1.0946
			2.50, 2.50	0.8595	1.0022	1.0176	1.0026	1.0759
		0 (SELF)	0.50, 0.50	0.9659	1.0869	1.0974	1.1535	1.2041
			1.00, 1.00	0.9827	1.1245	1.1293	1.1814	1.2209
			2.50, 2.50	0.9587	1.1178	1.1573	1.1183	1.2112
Result Based on Real Data								
05	05	1	1.00, 1.00	0.6794	0.7891	0.8142	0.8209	0.8842
		-1	1.00, 1.00	0.7091	0.8679	0.8941	0.9208	0.9479
		-2 (ISELF)	1.00, 1.00	0.7970	0.8850	0.9657	0.9747	1.0186
		0 (SELF)	1.00, 1.00	0.8987	1.0653	1.0743	1.0832	1.1376
10	10	1	1.00, 1.00	0.7069	0.7922	0.8286	0.8667	0.9065
		-1	1.00, 1.00	0.7583	0.8720	0.8990	0.9218	0.9729
		-2 (ISELF)	1.00, 1.00	0.8354	0.9685	0.9860	1.0318	1.0564
		0 (SELF)	1.00, 1.00	0.9346	1.0821	1.0903	1.1316	1.1797

Table 6: Bayes risk of $\hat{\theta}_L$ under LLF

m_1	m_2	c	(α, β)	FFP	PC	FFC	T-II	CS
05	05	0.50	0.50, 0.50	0.8675	0.9188	0.9574	1.0103	1.0354
			1.00, 1.00	0.8853	0.9376	0.9715	1.0231	1.0427
			2.50, 2.50	0.8608	0.9276	0.9703	1.0163	1.0349
		1.00	0.50, 0.50	0.8989	0.9234	0.9908	1.0348	1.0561
			1.00, 1.00	0.9107	0.9541	1.0004	1.0702	1.0732
			2.50, 2.50	0.9002	0.9435	0.9933	1.0477	1.0556
10	10	0.50	0.50, 0.50	0.9139	0.9471	0.9707	1.0201	1.0506
			1.00, 1.00	0.9298	0.9724	0.9869	1.0569	1.0808
			2.50, 2.50	0.9171	0.9661	0.9706	1.0393	1.0548
		1.00	0.50, 0.50	0.9363	1.0176	1.0410	1.0602	1.0903
			1.00, 1.00	0.9528	1.0426	1.0471	1.0768	1.1104
			2.50, 2.50	0.9293	1.0364	1.0369	1.0693	1.1045
Result Based on Real Data								
05	05	0.50	0.50, 0.50	0.8359	0.8866	0.9195	0.9696	0.9886
		1.00	0.50, 0.50	0.8791	0.9204	0.9344	1.0024	1.0255
10	10	0.50	0.50, 0.50	0.8121	0.8769	0.9183	0.9630	0.9810
		1.00	0.50, 0.50	0.8667	0.9143	0.9186	0.9853	1.0003

Table 7: Bayes risk of $\hat{\sigma}_L$ under LLF

m_1	m_2	c	(α, β)	FFP	PC	FFC	T-II	CS
05	05	0.50	0.50, 0.50	0.8364	0.8764	0.9141	0.9457	1.0202
			1.00, 1.00	0.8537	0.9048	0.9478	0.9982	1.0473
			2.50, 2.50	0.8498	0.9015	0.9267	0.9816	1.0297
		1.00	0.50, 0.50	0.8771	0.9009	0.9567	1.0196	1.0304
			1.00, 1.00	0.8985	0.9409	0.9776	1.0241	1.0479
			2.50, 2.50	0.8783	0.9305	0.9691	1.0122	1.0299
10	10	0.50	0.50, 0.50	0.8916	0.9214	0.9471	0.9953	1.0325
			1.00, 1.00	0.9172	0.9487	0.9629	1.0312	1.0545
			2.50, 2.50	0.9048	0.9326	0.9437	1.0114	1.0301
		1.00	0.50, 0.50	0.9235	0.9728	1.0157	1.0284	1.0538
			1.00, 1.00	0.9296	1.0102	1.0216	1.0506	1.0834
			2.50, 2.50	0.9067	1.0012	1.0117	1.0433	1.0776
Result Based on Real Data								
05	05	0.50	0.50, 0.50	0.8052	0.8548	0.8965	0.9454	0.9930
		1.00	0.50, 0.50	0.8668	0.8974	0.9112	0.9774	1.0082
10	10	0.50	0.50, 0.50	0.8014	0.8516	0.8760	0.9293	0.9760
		1.00	0.50, 0.50	0.8548	0.8818	0.8925	0.9582	0.9764

Table 8: Bayes risk of $\hat{\lambda}_L$ under LLF

m_1	m_2	c	(α, β)	FFP	PC	FFC	T-II	CS
05	05	0.50	0.50, 0.50	0.9197	0.9632	0.9842	1.0286	1.0996
			1.00, 1.00	0.9385	0.9941	1.0209	1.0857	1.1191
			2.50, 2.50	0.9343	0.9905	1.0079	1.0676	1.1012
		1.00	0.50, 0.50	0.9439	0.9899	1.0406	1.0909	1.1207
			1.00, 1.00	0.9673	1.0134	1.0733	1.1139	1.1397
			2.50, 2.50	0.9553	1.0021	1.0154	1.0809	1.1102
10	10	0.50	0.50, 0.50	0.9497	1.0122	1.0401	1.0725	1.1230
			1.00, 1.00	0.9776	1.0419	1.0673	1.0996	1.1469
			2.50, 2.50	0.9641	1.0243	1.0264	1.0901	1.1204
		1.00	0.50, 0.50	1.0044	1.0381	1.1047	1.1085	1.1462
			1.00, 1.00	1.0101	1.0987	1.1211	1.1427	1.1784
			2.50, 2.50	0.9762	1.0890	1.1004	1.1347	1.1721
Result Based on Real Data								
05	05	0.50	0.50, 0.50	0.8875	0.9414	0.9674	1.0303	1.0627
		1.00	0.50, 0.50	0.9254	0.9878	1.0124	1.0438	1.0897
10	10	0.50	0.50, 0.50	0.8834	0.9379	0.9548	1.0127	1.0453
		1.00	0.50, 0.50	0.9123	0.9707	0.9728	1.0346	1.0640

6. Conclusion

In the life testing experiments, the cost and time limitation pushed to experimenters, to end the experiments before all the units on test fail. The First-Failure Progressive (FFP) censoring scheme is the combination of first-failure censoring and Progressive Type-II censoring and gives an opportunity to remove some sets of test units before observing the first failures in these sets.

Two-parameter Gompertz distribution is considered here for the Bayesian inference under FFP censoring with Constant-Stress Partially Accelerated Life Test. The properties of the Bayes estimators of all the parameters under two different asymmetric loss functions and special cases of FFP censoring have been study based on real and simulated data.

The numerical findings shows, the highest risks for the complete sample case, whereas the least risk has been noted under FFP censoring. It is also observed that, for higher hyper parameter values, the Bayes risks are smaller for Type-II censoring when compared with FFC. Hence, one may prefer the FFP censoring over others, for the selected parametric values and small censored sample size.

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