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Assessing Performance of the Generalized Exponential Model in the Presence of the Interval Censored Data with Covariate

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Abstract

This study aims to extend the generalized exponential model (GEM) to include covariates in the presence of interval-censored data. The maximum likelihood estimator (MLE) was obtained for the parameter of the model formulated. Afterward, a thorough simulation study was carried out to evaluate the estimator's performance based on the values of bias, standard error (SE), and root mean square error (RMSE). The result indicated that the (SE) and (RMSE) decrease with the increase in sample sizes and decrease in censoring proportions. Finally, the performance of the Wald confidence interval estimation technique for the GE model with interval-censored data covariate was assessed by a coverage probability study at several censoring proportions and different sample sizes.

Keywords: generalized exponential distribution, maximum likelihood estimator, interval-censoring, converge probability, fixed covariate.

1. Introduction

Survival time data analysis, concisely referred to as survival analysis is a collection of statistical methods for analyzing time to an event data. The focus of these methods is to describe the distribution of T on a population and relationship between T and some covariates. According to (Lee and Wang 2003) survival time, T is the time for a subject to initiate an event such that (birth, marriage), to some final event (death, divorce). T is non-negative random variable and usually continuous, unless stated otherwise. A severe analytical problem in analyzing survival time data arises when a portion or even all t_i, $i = 1, 2, ..., n$, are censored data. (Klein and Moeschberger 2006) explained that the i^{th} subject's survival time, t_i, is censored data when its exact value is unknown due to one of these four main reasons; (1) The subject lost to follow-up, (2) The subject withdraws from the study, (3) The subject does not experience the event before the study ends, and (4) The subject is not under continuous observation .
Interval-censoring arises when the failure time cannot be precisely observed, but known to exist between two subsequent examination points. However, in interval-censored data, the only information about the failure time t is that it occurred between two known time point L

and R such that $t \in [L, R]$ where R and L are the right and left endpoints, respectively. The first approach in survival analysis based on interval-censored data was presented by (Turnbull 1976), who discussed the estimation of the non-parametric survival function. Further studies on interval-censoring were explored by (Brunel, Comte *et al.* 2009), where other types of smooth estimators were proposed for interval-censored data.The survival model with doubly interval-censored data and time dependent covariate in which the lifetime is the elapsed time between two related events which means that the first event and the second event are interval-censored that discussed on Kiani and Arasan (2018).
In this study, the performance of the generalized exponential model (GEM) with interval censored data in presence of covariates was explored. A simulation study was carried out to investigate the maximum likelihood estimation (MLE) procedure for the parameters of the (GEM) with interval-censored at various censoring proportions cp and sample sizes n by computing the values of bias, standard error (SE) and root mean square error (RMSE). Further more, the performance of the Wald confidence interval was also examined by conducting coverage probability study at various cp, n, and α.

2. Methodology

2.1. The generalized exponential model with interval-censored data

(Gupta and Kundu 1999) introduced the three-parameter GE distribution and studied the theoretical properties of this family. The GE distribution has the following cumulative distribution function(CDF):

$$F(t; \alpha, \lambda, \mu) = (1 - e^{-(t-\mu)/\lambda})^{\alpha}, (t > \mu, \alpha > 0, \lambda > 0). \tag{1}$$

and probability density function (pdf),

$$f(t; \alpha, \lambda, \mu) = \frac{\alpha}{\lambda}(1 - e^{-(t-\mu)/\lambda})^{\alpha-1} e^{-(t-\mu)/\lambda}, (t > \mu, \alpha > 0, \lambda > 0). \tag{2}$$

Here, α is a shape parameter; λ is a scale parameter, and μ is a location parameter. In addition, the survival function and hazard function are given respectively by:

$$S(t; \alpha, \lambda, \mu) = 1 - F(t; \alpha, \lambda, \mu) = 1 - (1 - e^{-(t-\mu)/\lambda})^{\alpha}, (t > \mu). \tag{3}$$

$$h(t; \alpha, \lambda, \mu) = \frac{f(t; \alpha, \lambda, \mu)}{S(t; \alpha, \lambda, \mu)} = \frac{\alpha}{\lambda} \frac{(1 - e^{-(t-\mu)/\lambda})^{\alpha-1} e^{-(t-\mu)/\lambda}}{1 - (1 - e^{-(t-\mu)/\lambda})^{\alpha}}, (t > \mu). \tag{4}$$

2.2. Maximum likelihood estimation

In this subsection, the MLE was employed to obtain the parameters estimates for the GEM with interval-censored data. First, we consider the three-parameter GE model, and for the sake of simplicity, we reparametrize $\beta = 1/\lambda$. In order to incorporate the interval-censored event time and likelihood function, the definition of some indicator variables for non-censored event times, left, right and interval-censored data is necessary. The indicator variables for i^{th} observation are defined as follows:

$$\delta_{E_i} = \begin{cases} 1, & \text{if the data is uncensored at } t_i \\ 0, & \text{otherwise;} \end{cases}$$

$$\delta_{R_i} = \begin{cases} 1, & \text{if the data is right censored at } t_i \\ 0, & \text{otherwise;} \end{cases}$$

$$\delta_{L_i} = \begin{cases} 1, & \text{if the event is left censored at } t_i \\ 0, & \text{otherwise;} \end{cases}$$

$$\delta_{I_i} = \begin{cases} 1, & \text{if the event is interval-censored at } (L_i, R_i) \\ 0, & \text{otherwise.} \end{cases}$$

Sparling, Younes, Lachin, and Bautista (2006), among others, represent the likelihood construction for right, left, and interval-censored data for complete sample with $i = i, .., n$ is then:

$$L = \prod_{i=1}^{n} \left\{ f_i(t_i)^{\delta_{E_i}} F_i(t_{L_i})^{\delta_{L_i}} \left[1 - F_i(t_{R_i})\right]^{\delta_{R_i}} \left[F_i(t_{R_i}) - F_i(t_{L_i})\right]^{\delta_{I_i}} \right\},$$

or

$$L = \prod_{i=1}^{n} \left\{ f_i(t_i)^{\delta_{E_i}} F_i(t_{L_i})^{\delta_{L_i}} \left[S_i(t_{R_i})\right]^{\delta_{R_i}} \left[S_i(t_{L_i}) - S_i(t_{R_i})\right]^{\delta_{I_i}} \right\}.$$

where $L_i < t_i < R_i$,
and the log−likelihood function is given by the following:

$$l = \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln\left[f_i(t_i)\right] + \delta_{L_i} \ln\left[F_i(t_{L_i})\right] + \delta_{R_i} \ln\left[S_i(t_{R_i})\right] + \delta_{I_i} \ln\left[S_i(t_{L_i}) - S(t_{R_i})\right] \right\}. \quad (5)$$

Substituting the probability distribution and survivorship function of the model into the log−likelihood function, we obtain:

$$\begin{aligned} l &= \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln\left[\alpha\beta(1 - e^{-\beta(t_i-\mu)})^{\alpha-1} e^{-\beta(t_i-\mu)}\right] + \delta_{L_i} \ln\left[(1 - e^{-\beta(t_{L_i}-\mu)})^{\alpha}\right] \right. \\ &+ \left. \delta_{R_i} \ln\left[1 - (1 - e^{-\beta(t_{R_i}-\mu)})^{\alpha}\right] + \delta_{I_i} \ln\left[1 - (1 - e^{-\beta(tL_i-\mu)})^{\alpha} - (1 - (1 - e^{-\beta(t_{R_i}-\mu)})^{\alpha})\right] \right\} \quad (6) \end{aligned}$$

2.3. The generalized exponential model with interval-censored data and fixed covariate

If we invoporate a single fixed covariate into the model (6) by setting $\beta = e^{(b_0+b_1x_i)}$, then:

$$\begin{aligned} l &= \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln\alpha + \delta_{E_i} \ln e^{(b_0+b_1x_i)} + \delta_{E_i}(\alpha - 1)\ln(1 - e^{-e^{(b_0+b_1x_i)}(t_i-\mu)}) \right. \\ &- \delta_{E_i} e^{(b_0+b_1x_i)}(t_i - \mu) + \delta_{L_i}\alpha\ln\left[(1 - e^{-e^{(b_0+b_1x_i)}(t_{L_i}-\mu)})\right] \\ &- \delta_{R_i}\alpha\ln\left[(1 - e^{-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)})\right] + \delta_{I_i}\alpha\ln(1 - e^{-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}) \\ &- \left. \delta_{I_i}\alpha\ln(1 - e^{-exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}) \right\}. \quad (7) \end{aligned}$$

The 1^{st} derivative of the log likelihood function (7) with respect to the parameters, α, μ, b_0, b_1, are:

$$\begin{aligned} \frac{\partial l}{\partial \alpha} &= \sum_{i=1}^{n} n\log\left[e^{-exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\left(-1 + e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\right)\right]\delta_{L_i} \\ &+ \sum_{i=1}^{n} -n\log\left[e^{-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\left(-1 + e^{exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\right)\right]\delta_{R_i} \\ &+ \sum_{i=1}^{n} \delta_{E_i}\left(\frac{n}{\alpha} + \sum_{i=1}^{n}\log\left[1 - e^{-exp^{(b_0+b_1x_i)}(t_i-\mu)}\right]\right). \quad (8) \end{aligned}$$

$$\begin{aligned}
\frac{\partial l}{\partial \mu} &= \sum_{i=1}^{n}\Big(e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\Big(-e^{(b_0+b_1x_i)}+e^{(b_0+b_1x_i)-exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)} \\
&\quad \Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\Big)\Big)n\alpha\delta_{L_i}\Big)\Big/\Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\Big) \\
&+ \sum_{i=1}^{n}-\Big(\Big(e^{exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\Big(-e^{(b_0+b_1x_i)}+e^{(b_0+b_1x_i)-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)} \\
&\quad \Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\Big)\Big)n\alpha\delta_{R_i}\Big)\Big/\Big(-1+e^{e^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\Big)\Big) \\
&+ \sum_{i=1}^{n}\delta_{E_i}\left[ne^{(b_0+b_1x_i)}+(\alpha-1)\sum_{i=1}^{n}-\frac{e^{(b_0+b_1x_i)-exp^{(b_0+b_1x_i)}(t_i-\mu)}}{1-e^{-exp^{(b_0+b_1x_i)}(t_i-\mu)}}\right]. \qquad (9)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial l}{\partial b_j} &= \sum_{i=1}^{n}\Big(e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}n\alpha\delta_{L_i}\Big(e^{(b_0+b_1x_i)}x_{ij}(t_{L_i}-\mu)+e^{(b_0+b_1x_i)-exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)} \\
&\quad (-1+e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)})x_{ij}(t_{L_i}-\mu)\Big)\Big)\Big/\Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)}\Big) \\
&+ \sum_{i=1}^{n}-\Big(\Big(e^{exp^{(b_0+b_1x_i)(t_{R_i}-\mu)}}n\alpha\delta_{R_i}\Big(e^{(b_0+b_1x_i)}(t_{R_i}-\mu)-e^{(b_0+b_1x_i)-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)} \\
&\quad \Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\Big)x_{ij}(t_{R_i}-\mu)\Big)\Big)\Big/\Big(-1+e^{exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)}\Big)\Big) \\
&+ \sum_{i=1}^{n}\delta_{E_i}\Big(\frac{nx_{ij}}{(b_0+b_1x_i)}-\sum_{i=1}^{n}e^{(b_0+b_1x_i)}x_i(t_i-\mu)+(1-\alpha) \\
&\quad \sum_{i=1}^{n}\frac{e^{(b_0+b_1x_i)-e^{(b_0+b_1x_i)(t_i-\mu)}}x_{ij}(t_i-\mu)}{1-e^{-exp^{(b_0+b_1x_i)}(t_i-\mu)}}\Big), \qquad (10)
\end{aligned}$$

where $x_{i0}=1$, $j=0,1$.

3. Simulation study and results

The simulation study was carried out to verify the performance of parameters estimates of the GE distribution with interval-censored data and the fixed covariate. In (Olaniran and Yahya 2017), (Jamil, Abdullah, Kek, Olaniran, and Amran 2017), and (Olaniran and Abdullah 2019) similar simulation approach was used. The simulation was conducted by using R software at different combination of sample sizes n and censoring proportions. The simulation was carried out using the following true parameters as $\mu = 3$, $\alpha = 4$, $b_0 = 0.5$ and $b_1 = -0.5$.

3.1. Assessing performance of the parameter estimates

The simulation study was conducted to assess the bias, SE, and RMSE of the estimates at different cp levels and simple sizes n. The bias, SE and RMSE were computed respectively by,

$$bias = E(\hat{\theta}) - \theta, \qquad (11)$$

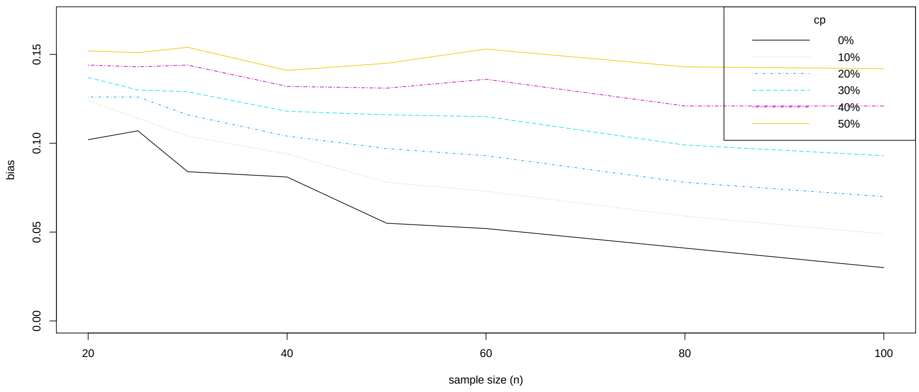
$$SE = \sqrt{E([\hat{\theta} - E(\hat{\theta})]^2)}, \qquad (12)$$

$$RMSE = \sqrt{(SE^2) + (bias^2)}, \qquad (13)$$

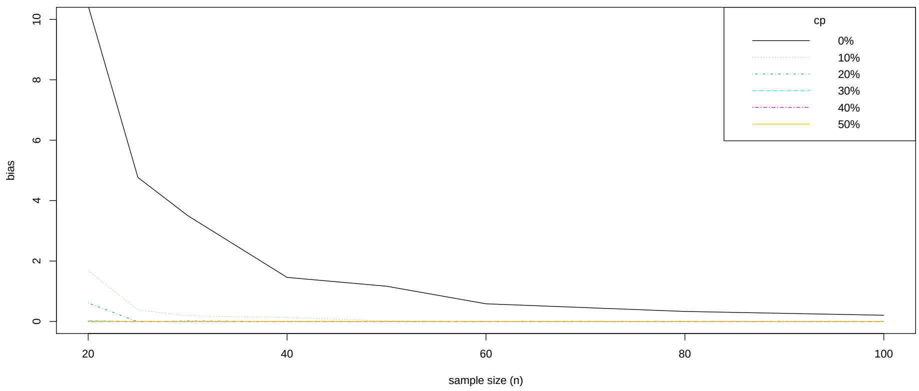
The plots of the bias the parameter estimates is presented in Figure 1. From Table 1, it revealed that the bias is very close to zero, which indicates the state of unbiasedness of the parameters which is its closeness to the real values. Also, Figure 1 revealed that the biases reduce as the sample sizes increase. This occurs across all the censoring proportions. But as the censoring proportions increase, the values of the biases increase across all samples considered.

Table 1: Summary table for bias of the parameters for various n and cp

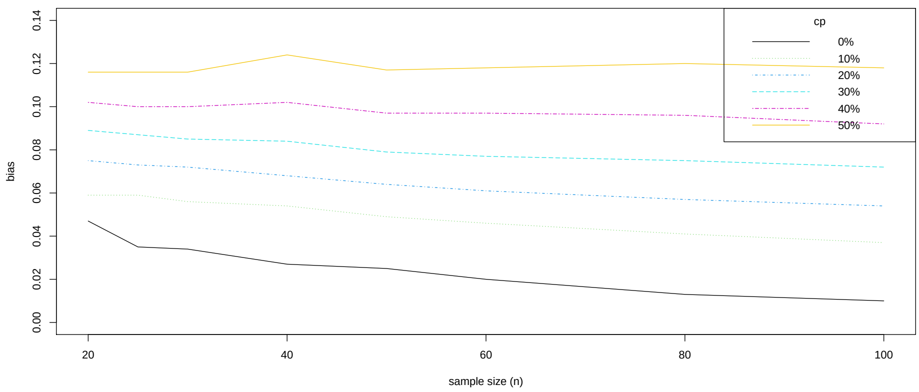
		censoring proportion					
Estmiates	Sample size (n)	0%	10%	20%	30%	40%	50%
	20	0.102	0.124	0.126	0.137	0.144	0.152
	25	0.107	0.114	0.126	0.130	0.143	0.151
	30	0.084	0.104	0.116	0.129	0.144	0.154
$\hat{\mu}$	40	0.081	0.094	0.104	0.118	0.132	0.141
	50	0.055	0.078	0.097	0.116	0.131	0.145
	60	0.052	0.073	0.093	0.115	0.136	0.153
	80	0.041	0.059	0.078	0.099	0.121	0.143
	100	0.030	0.049	0.070	0.093	0.121	0.142
	20	10.447	1.686	0.612	0.023	-0.002	0.000
	25	4.767	0.383	-0.010	0.000	0.000	0.000
	30	3.506	0.187	0.017	0.000	0.000	0.000
	40	1.458	0.132	0.000	0.000	0.000	0.000
$\hat{\alpha}$	50	1.166	0.019	0.002	0.000	0.000	0.000
	60	0.584	0.002	0.000	0.000	0.000	0.000
	80	0.331	-0.001	0.000	0.000	0.000	0.000
	100	0.205	0.000	0.000	0.000	0.000	0.000
	20	0.047	0.059	0.075	0.089	0.102	0.116
	25	0.035	0.059	0.073	0.087	0.100	0.116
	30	0.034	0.056	0.072	0.085	0.100	0.116
	40	0.027	0.054	0.068	0.084	0.102	0.124
$\hat{b}_0$	50	0.025	0.049	0.064	0.079	0.097	0.117
	60	0.020	0.046	0.061	0.077	0.097	0.118
	80	0.013	0.041	0.057	0.075	0.096	0.120
	100	0.010	0.037	0.054	0.072	0.092	0.118
	20	-0.058	0.015	0.034	0.038	0.044	0.047
	25	-0.056	0.019	0.031	0.036	0.039	0.041
	30	-0.046	0.021	0.029	0.032	0.034	0.039
	40	-0.040	0.019	0.024	0.028	0.031	0.036
$\hat{b}_1$	50	-0.027	0.022	0.026	0.029	0.031	0.035
	60	-0.022	0.020	0.023	0.023	0.027	0.030
	80	-0.019	0.017	0.020	0.022	0.025	0.028
	100	-0.016	0.016	0.017	0.019	0.019	0.023



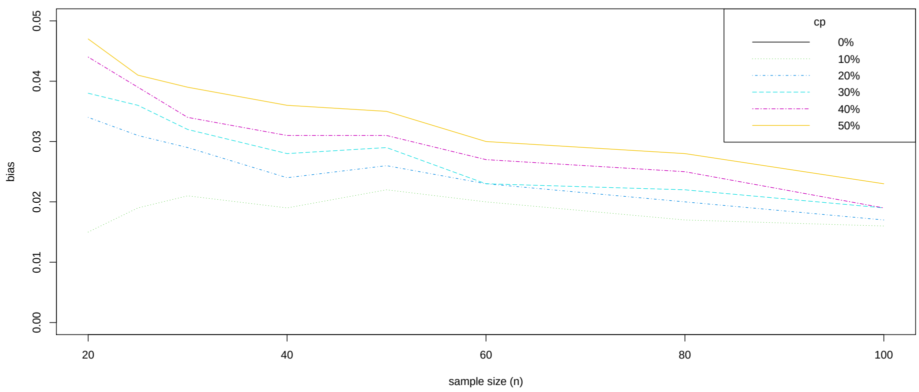
(a) Bias for μ



(b) Bias for α



(c) Bias for b_0



(d) Bias for b_1

Figure 1: Plots of Bias for parameters for various sample sizes

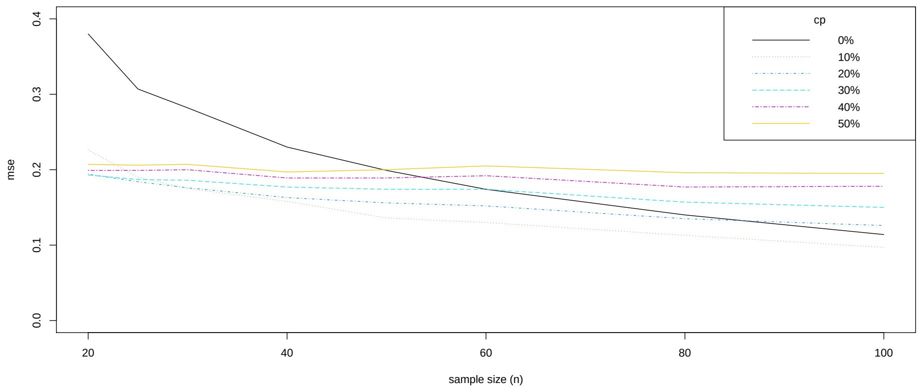
For Table 2 and 3, the standard errors and the mean square errors of the estimates are shown. It revealed that as the sample sizes increase, the SE and RMSE reduces. Figure 1 shows the plots from the RMSE for the parameter estimates of the model. This occurs across all the censoring proportions. These further shows that the estimates are efficient and consistent and therefore confirming the suitability of the estimation method. Also, it revealed that for each parameter, the RMSE increases as censoring proportions increases. This is to say that the corresponding values of RMSE for the parameter μ increases as the censoring proportion increases. Table 2 showed that as the percentages of cp increases, the values of the standard error will continue to increase. More accurate inference can be achieved by increasing the size of the sampling with low censored data proportions. All parameters of this model show good performance by having a relatively low standard error when the sample size increases and cp decreases. Figure 2 further revealed this.

Table 2: Summary table for standard error (SE) of the parameters for various n and cp

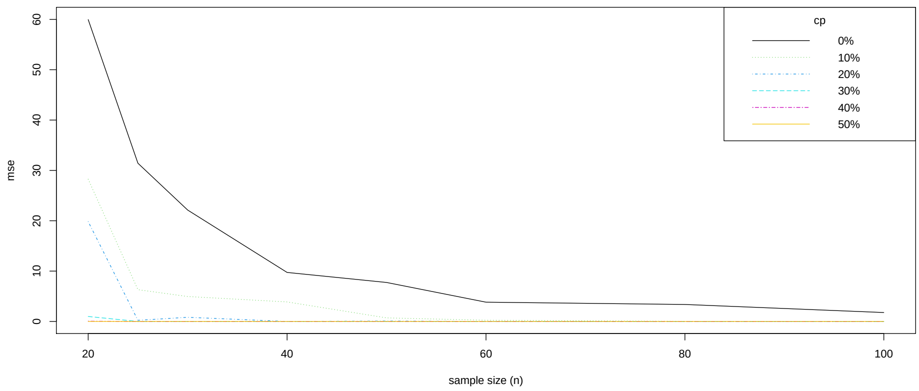
		censoring proportion					
Estmiates	Sample size (n)	0%	10%	20%	30%	40%	50%
	20	0.366	0.188	0.147	0.137	0.137	0.140
	25	0.288	0.152	0.134	0.135	0.138	0.140
	30	0.269	0.141	0.132	0.134	0.139	0.139
$\hat{\mu}$	40	0.215	0.127	0.126	0.132	0.136	0.138
	50	0.191	0.112	0.122	0.130	0.136	0.138
	60	0.166	0.107	0.120	0.130	0.136	0.136
	80	0.134	0.096	0.110	0.122	0.130	0.134
	100	0.110	0.084	0.104	0.117	0.131	0.134
	20	59.045	28.205	19.834	0.987	0.053	0.000
	25	31.065	6.300	0.229	0.000	0.000	0.000
	30	21.851	4.949	0.825	0.000	0.000	0.000
	40	9.625	3.875	0.000	0.000	0.000	0.000
$\hat{\alpha}$	50	7.655	0.708	0.082	0.000	0.000	0.000
	60	3.783	0.244	0.000	0.000	0.000	0.000
	80	3.356	0.030	0.000	0.000	0.000	0.000
	100	1.759	0.007	0.000	0.000	0.000	0.000
	20	0.284	0.149	0.106	0.101	0.107	0.114
	25	0.252	0.117	0.090	0.092	0.100	0.110
	30	0.227	0.090	0.081	0.087	0.095	0.105
	40	0.185	0.070	0.071	0.081	0.090	0.101
$\hat{b_0}$	50	0.162	0.062	0.065	0.074	0.084	0.096
	60	0.146	0.053	0.062	0.072	0.084	0.093
	80	0.123	0.045	0.055	0.064	0.074	0.085
	100	0.110	0.042	0.052	0.061	0.071	0.083
	20	0.199	0.112	0.083	0.076	0.083	0.086
	25	0.175	0.085	0.067	0.070	0.074	0.078
	30	0.151	0.073	0.062	0.066	0.069	0.075
	40	0.128	0.055	0.050	0.058	0.063	0.070
$\hat{b_1}$	50	0.116	0.049	0.052	0.058	0.062	0.069
	60	0.100	0.041	0.045	0.048	0.055	0.061
	80	0.084	0.034	0.038	0.043	0.050	0.055
	100	0.075	0.030	0.033	0.038	0.040	0.048

Table 3: Summary table for root mean square error (RMSE) of the parameters for various n and cp

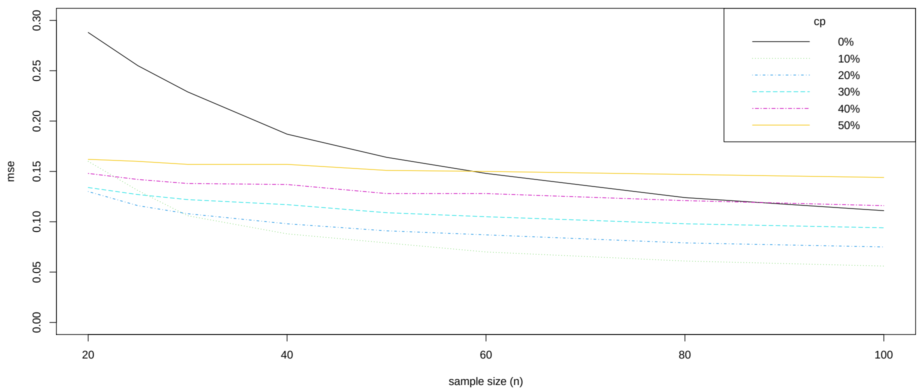
Estmiates	Sample size (n)	censoring proportion					
		0%	10%	20%	30%	40%	50%
$\hat{\mu}$	20	0.380	0.226	0.194	0.193	0.199	0.207
	25	0.307	0.190	0.184	0.187	0.199	0.206
	30	0.282	0.175	0.176	0.186	0.200	0.207
	40	0.230	0.158	0.163	0.177	0.189	0.197
	50	0.199	0.136	0.156	0.174	0.189	0.200
	60	0.174	0.130	0.152	0.174	0.192	0.205
	80	0.140	0.113	0.135	0.157	0.177	0.196
	100	0.114	0.097	0.126	0.150	0.178	0.195
$\hat{\alpha}$	20	59.963	28.255	19.834	0.987	0.053	0.000
	25	31.429	6.312	0.229	0.000	0.000	0.000
	30	22.131	4.952	0.825	0.000	0.000	0.000
	40	9.735	3.877	0.000	0.000	0.000	0.000
	50	7.743	0.708	0.082	0.000	0.000	0.000
	60	3.828	0.244	0.000	0.000	0.000	0.000
	80	3.372	0.030	0.000	0.000	0.000	0.000
	100	1.771	0.007	0.000	0.000	0.000	0.000
$\hat{b_0}$	20	0.288	0.160	0.130	0.134	0.148	0.162
	25	0.255	0.131	0.116	0.127	0.142	0.160
	30	0.229	0.106	0.108	0.122	0.138	0.157
	40	0.187	0.088	0.098	0.117	0.137	0.157
	50	0.164	0.079	0.091	0.109	0.128	0.151
	60	0.148	0.070	0.087	0.105	0.128	0.150
	80	0.124	0.061	0.079	0.098	0.121	0.147
	100	0.111	0.056	0.075	0.094	0.116	0.144
$\hat{b_1}$	20	0.208	0.113	0.089	0.085	0.094	0.098
	25	0.184	0.087	0.074	0.079	0.084	0.088
	30	0.158	0.076	0.068	0.073	0.077	0.085
	40	0.134	0.059	0.056	0.065	0.070	0.078
	50	0.119	0.054	0.058	0.064	0.069	0.078
	60	0.103	0.046	0.050	0.054	0.061	0.068
	80	0.086	0.038	0.043	0.048	0.055	0.061
	100	0.077	0.034	0.037	0.042	0.045	0.053



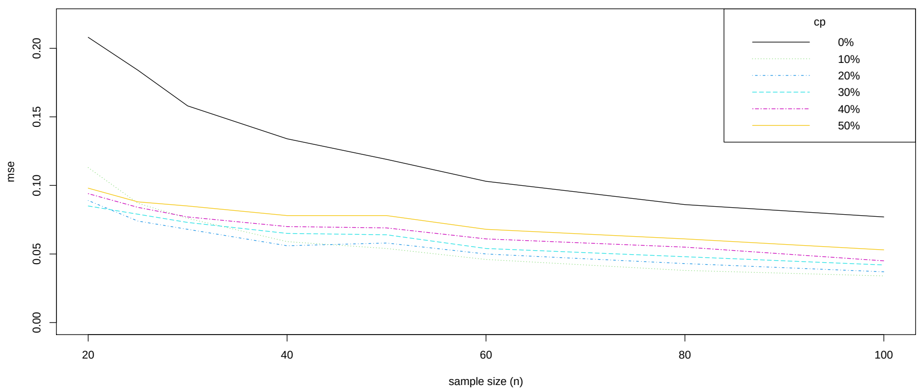
(a) RMSE for μ



(b) RMSE for α



(c) RMSE for b_0



(d) RMSE for b_1

Figure 2: Plots of RMSE for parameters for various sample sizes

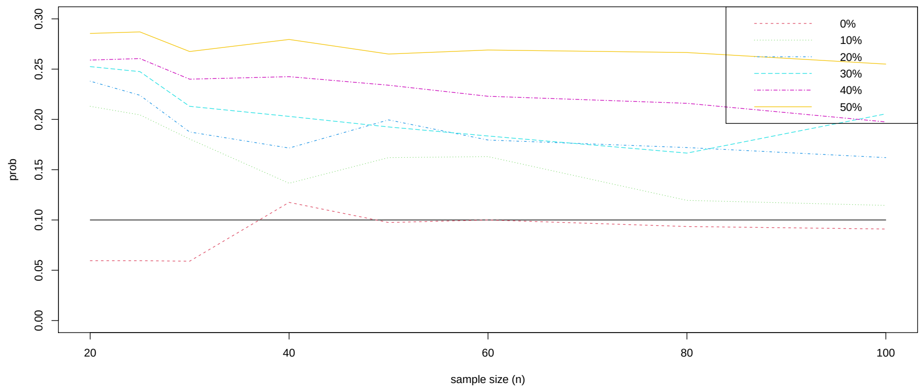
3.2. Assessing performance of the coverage probability

The coverage probability is the proportion of time a confidence interval includes the true population parameter value. Here, we study the coverage probability for the Wald confidence interval using a coverage probability study. In this study, N= 2000 replications of size $n = 20, 25, 30, 40, 50, 60, 80$ and 100 were simulated. The nominal Type I error α was set to $\alpha = 0.05$ and $\alpha = 0.10$. The approach of (Manoharan, Arasan, Midi, and Adam 2018) was adopted to calculate the error probabilities from the left (lep) and right (rep) and the corresponding total error probability (tep) which is the sum of lep and rep. Upon computing tep, the Wald confidence is termed anticonservative if $tep > \alpha + 2.58s.e.(\hat{\alpha})$, conservative (C) if $tep < \alpha + 2.58s.e.(\hat{\alpha})$ with $s.e.(\hat{\alpha}) = \sqrt{\alpha(1-\alpha)/N}$. For the asymmetric (AS), the larger error probability will be 1.5 times the smaller one. The Wald confidence interval is optimal if AC, C and AS are close to zero and lep, rep and tep are close 0.025(0.05) and 0.05(0.10) respectively. Tables 4 - 5 show the results of estimated error probabilities for all parameter using the Wald over different sample sizes and censoring proportions at 10% and 5% significance levels. The results show that the lep and rep are asymmetrical under most settings i.e, at various sample sizes and censoring proportions, the lep or rep are 1.5 times larger than the other for all parameters. However, the tep values are closer to the nominal 0.05 and 0.1 at all settings for parameter b_0 and b_1. This was further revealed in Figure 3 and Figure 4 which show the plots of the tep for 10% and 5% significance level respectively.

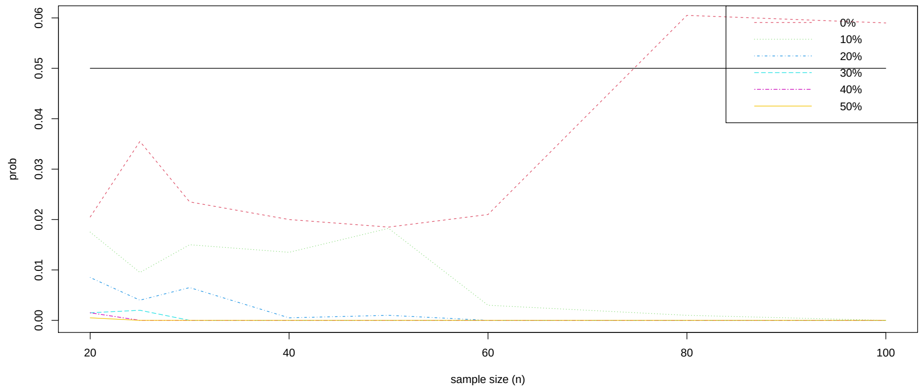
Tables 6 - 7 show the results for total number of AC, C, and AS intervals for all parameters using Wald intervals over varying sample sizes and censored proportions and 0.05 and 0.10 level of significance. The results show that the Wald intervals are anti-conservative for parameters μ and α at most settings but asymmetric for b_0 and b_1.

Table 4: Estimated error probabilities for all parameter using the Wald over different sample sizes and censoring proportions at 10% significance level

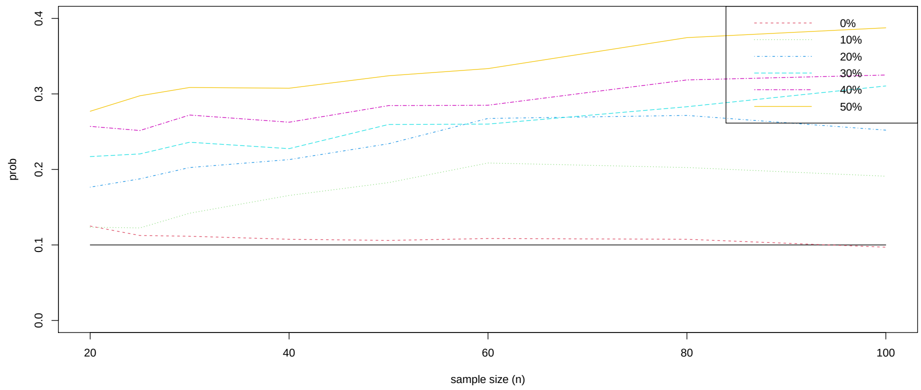
Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
0	20	0.0135	0.0460	0.0595	0.0205	0.0000	0.0205	0.0695	0.0555	0.1250	0.0190	0.0950	0.1140
	25	0.0115	0.0480	0.0595	0.0355	0.0000	0.0355	0.0660	0.0465	0.1125	0.0230	0.0935	0.1165
	30	0.0110	0.0480	0.0590	0.0235	0.0000	0.0235	0.0640	0.0475	0.1115	0.0325	0.0940	0.1265
	40	0.0815	0.0360	0.1175	0.0200	0.0000	0.0200	0.0585	0.0490	0.1075	0.0305	0.0825	0.1130
	50	0.0630	0.0345	0.0975	0.0185	0.0000	0.0185	0.0605	0.0455	0.1060	0.0305	0.0820	0.1125
	60	0.0660	0.0340	0.1000	0.0210	0.0000	0.0210	0.0665	0.0420	0.1085	0.0310	0.0855	0.1165
	80	0.0600	0.0335	0.0935	0.0605	0.0000	0.0605	0.0555	0.0520	0.1075	0.0335	0.0865	0.1200
	100	0.0660	0.0250	0.0910	0.0590	0.0000	0.0590	0.0515	0.0455	0.0970	0.0335	0.0800	0.1135
0.1	20	0.1940	0.0190	0.2130	0.0175	0.0000	0.0175	0.0995	0.0240	0.1235	0.0705	0.0490	0.1195
	25	0.1935	0.0110	0.2045	0.0095	0.0000	0.0095	0.1020	0.0205	0.1225	0.0810	0.0280	0.1090
	30	0.1760	0.0045	0.1805	0.0095	0.0055	0.0150	0.1230	0.0190	0.1420	0.0775	0.0275	0.1050
	40	0.1330	0.0035	0.1365	0.0070	0.0065	0.0135	0.1565	0.0090	0.1655	0.1205	0.0105	0.1310
	50	0.1605	0.0015	0.1620	0.0040	0.0075	0.1825	0.1785	0.0040	0.1825	0.1060	0.0055	0.1115
	60	0.1630	0.0000	0.1630	0.0015	0.0015	0.0030	0.2075	0.0010	0.2085	0.1100	0.0015	0.1115
	80	0.1195	0.0000	0.1195	0.0000	0.0010	0.0010	0.2020	0.0005	0.2025	0.1120	0.0005	0.1125
	100	0.1145	0.0000	0.1145	0.0000	0.0000	0.0000	0.1910	0.0000	0.1910	0.1210	0.0000	0.1210
0.2	20	0.2355	0.0025	0.2380	0.0050	0.0035	0.0085	0.1705	0.0060	0.1765	0.1110	0.0090	0.1200
	25	0.2235	0.0005	0.2240	0.0020	0.0020	0.0040	0.1855	0.0020	0.1875	0.1045	0.0040	0.1085
	30	0.1875	0.0000	0.1875	0.0020	0.0045	0.0065	0.2010	0.0015	0.2025	0.1100	0.0010	0.1110
	40	0.1715	0.0000	0.1715	0.0000	0.0005	0.0005	0.2125	0.0005	0.2130	0.1295	0.0005	0.1300
	50	0.1995	0.0000	0.1995	0.0005	0.0005	0.0010	0.2340	0.0000	0.2340	0.1190	0.0000	0.1190
	60	0.1795	0.0000	0.1795	0.0000	0.0000	0.0000	0.2675	0.0000	0.2675	0.1235	0.0000	0.1235
	80	0.1720	0.0000	0.1720	0.0000	0.0000	0.0000	0.2715	0.0000	0.2715	0.1130	0.0000	0.1130
	100	0.1620	0.0000	0.1620	0.0000	0.0000	0.0000	0.2520	0.0000	0.2520	0.1240	0.0000	0.1240
0.3	20	0.2515	0.0010	0.2525	0.0010	0.0005	0.0015	0.2165	0.0005	0.2170	0.1215	0.0015	0.1230
	25	0.2475	0.0000	0.2475	0.0000	0.0020	0.0020	0.2200	0.0005	0.2205	0.1130	0.0000	0.1130
	30	0.2130	0.0000	0.2130	0.0000	0.0000	0.0000	0.2360	0.0000	0.2360	0.1200	0.0000	0.1200
	40	0.2030	0.0000	0.2030	0.0000	0.0000	0.0000	0.2275	0.0000	0.2275	0.1345	0.0000	0.1345
	50	0.1925	0.0000	0.1925	0.0000	0.0000	0.0000	0.2595	0.0000	0.2595	0.1315	0.0000	0.1315
	60	0.1835	0.0000	0.1835	0.0000	0.0000	0.0000	0.2600	0.0000	0.2600	0.1275	0.0000	0.1275
	80	0.1665	0.0000	0.1665	0.0000	0.0000	0.0000	0.2830	0.0000	0.2830	0.1200	0.0000	0.1200
	100	0.2055	0.0000	0.2055	0.0000	0.0000	0.0000	0.3105	0.0000	0.3105	0.1195	0.0000	0.1195
0.4	20	0.2580	0.0010	0.2590	0.0015	0.0000	0.0015	0.2570	0.0000	0.2570	0.1245	0.0005	0.1250
	25	0.2605	0.0000	0.2605	0.0000	0.0000	0.0000	0.2515	0.0000	0.2515	0.1235	0.0000	0.1235
	30	0.2400	0.0000	0.2400	0.0000	0.0000	0.0000	0.2720	0.0000	0.2720	0.1265	0.0000	0.1265
	40	0.2425	0.0000	0.2425	0.0000	0.0000	0.0000	0.2625	0.0000	0.2625	0.1170	0.0000	0.1170
	50	0.2340	0.0000	0.2340	0.0000	0.0000	0.0000	0.2845	0.0000	0.2845	0.1175	0.1175	0.1175
	60	0.2230	0.0000	0.2230	0.0000	0.0000	0.0000	0.2850	0.0000	0.2850	0.1385	0.0000	0.1385
	80	0.2160	0.0000	0.2160	0.0000	0.0000	0.0000	0.3185	0.0000	0.3185	0.1140	0.0000	0.1140
	100	0.1975	0.0000	0.1975	0.0000	0.0000	0.0000	0.3250	0.0000	0.3250	0.1195	0.0000	0.1195
0.5	20	0.2855	0.0000	0.2855	0.0005	0.0000	0.0005	0.2770	0.0000	0.2770	0.1390	0.0005	0.1395
	25	0.2870	0.0000	0.2870	0.0000	0.0000	0.0000	0.2975	0.0000	0.2975	0.1305	0.0000	0.1305
	30	0.2675	0.0000	0.2675	0.0000	0.0000	0.0000	0.3085	0.0000	0.3085	0.1325	0.0000	0.1325
	40	0.2795	0.0000	0.2795	0.0000	0.0000	0.0000	0.3075	0.0000	0.3075	0.1210	0.0000	0.1210
	50	0.2650	0.0000	0.2650	0.0000	0.0000	0.0000	0.3240	0.0000	0.3240	0.1220	0.0000	0.1220
	60	0.2690	0.0000	0.2690	0.0000	0.0000	0.0000	0.3335	0.0000	0.3335	0.1435	0.0000	0.1435
	80	0.2665	0.0000	0.2665	0.0000	0.0000	0.0000	0.3745	0.0000	0.3745	0.1205	0.0000	0.1205
	100	0.2550	0.0000	0.2550	0.0000	0.0000	0.0000	0.3875	0.0000	0.3875	0.1175	0.0000	0.1175



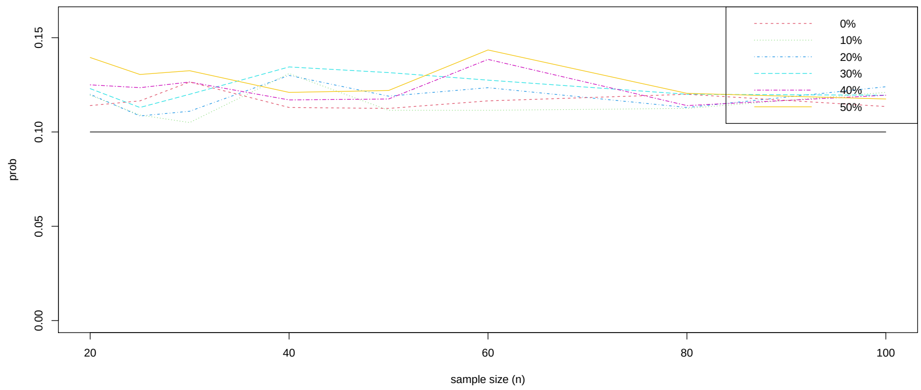
(a) tep for μ



(b) tep for α



(c) tep for b_0

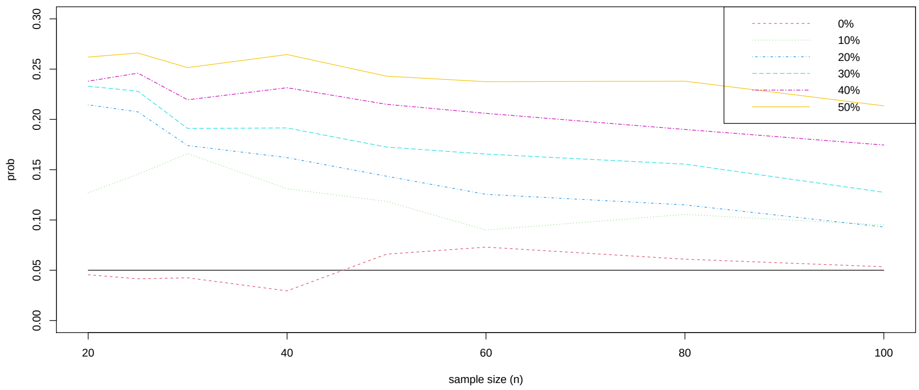


(d) tep for b_1

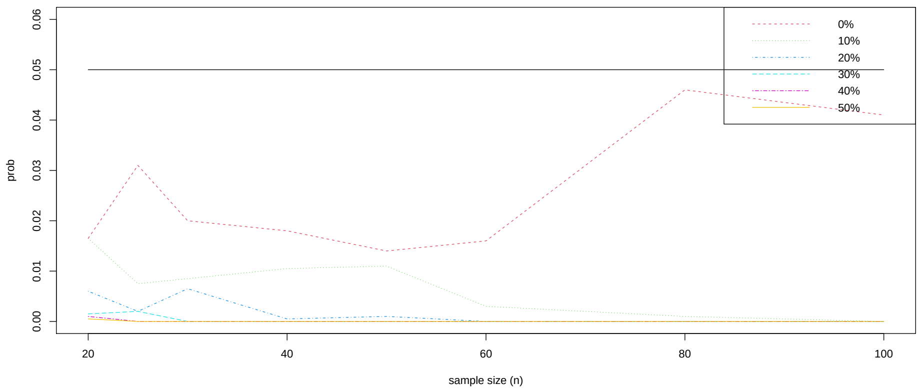
Figure 3: Plots of tep for parameters for various sample sizes for Wald Interval 10%

Table 5: Estimated error probabilities for all parameter using the Wald at different sample sizes and censoring proportions at 5% significance level

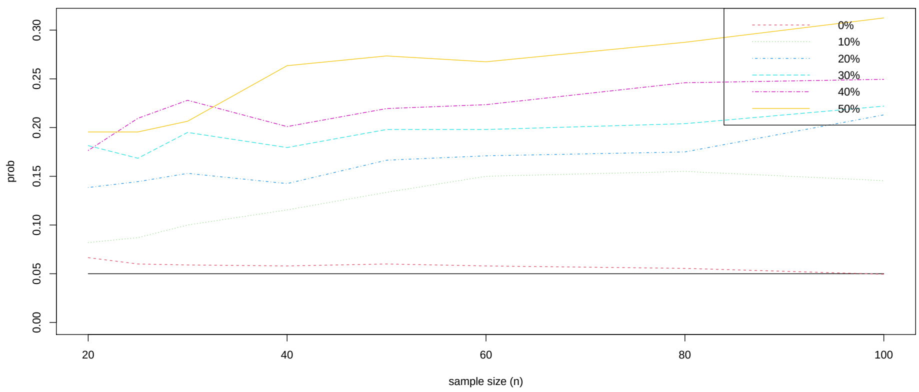
Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
	20	0.0055	0.0400	0.0455	0.0165	0.0000	0.0165	0.0340	0.0325	0.0665	0.0065	0.0560	0.0625
	25	0.0030	0.0385	0.0415	0.0310	0.0000	0.0310	0.0315	0.0285	0.0600	0.0055	0.0585	0.0640
	30	0.0040	0.0385	0.0425	0.0200	0.0000	0.0200	0.0290	0.0300	0.0590	0.0130	0.0550	0.0680
0	40	0.0035	0.0260	0.0295	0.0180	0.0000	0.0180	0.0280	0.0300	0.0580	0.0180	0.0550	0.0730
	50	0.0390	0.0270	0.0660	0.0140	0.0000	0.0140	0.0310	0.0290	0.0600	0.0180	0.0450	0.0630
	60	0.0500	0.0230	0.0730	0.0160	0.0000	0.0160	0.0335	0.0245	0.0580	0.0150	0.0440	0.0590
	80	0.0400	0.0210	0.0610	0.0460	0.0000	0.0460	0.0285	0.0270	0.0555	0.0160	0.0485	0.0645
	100	0.0380	0.0155	0.0535	0.0410	0.0000	0.0410	0.0255	0.0240	0.0495	0.0150	0.0430	0.0580
	20	0.1100	0.0170	0.1270	0.0165	0.0000	0.0165	0.0625	0.0195	0.0820	0.0345	0.0400	0.0745
	25	0.1360	0.0095	0.1455	0.0075	0.0000	0.0075	0.0725	0.0145	0.0870	0.0685	0.0245	0.0930
	30	0.1620	0.0040	0.1660	0.0085	0.0000	0.0085	0.0850	0.0150	0.1000	0.0575	0.0235	0.0810
0.1	40	0.1275	0.0035	0.1310	0.0070	0.0035	0.0105	0.1080	0.0075	0.1155	0.0755	0.0090	0.0845
	50	0.1170	0.0015	0.1185	0.0040	0.0070	0.0110	0.1300	0.0035	0.1335	0.0895	0.0050	0.0945
	60	0.0900	0.0000	0.0900	0.0015	0.0015	0.0030	0.1490	0.0010	0.1500	0.0935	0.0015	0.0950
	80	0.1055	0.0000	0.1055	0.0000	0.0010	0.0010	0.1545	0.0005	0.1550	0.0920	0.0005	0.0925
	100	0.0950	0.0000	0.0950	0.0000	0.0000	0.0000	0.1455	0.0000	0.1455	0.0930	0.0000	0.0930
	20	0.2120	0.0025	0.2145	0.0045	0.0015	0.0060	0.1325	0.0060	0.1385	0.0920	0.0075	0.0995
	25	0.2070	0.0005	0.2075	0.0020	0.0000	0.0020	0.1430	0.0015	0.1445	0.0860	0.0035	0.0895
	30	0.1740	0.0000	0.1740	0.0020	0.0045	0.0065	0.1515	0.0015	0.1530	0.0860	0.0010	0.0870
0.2	40	0.1620	0.0000	0.1620	0.0000	0.0005	0.0005	0.1420	0.0005	0.1425	0.0805	0.0005	0.0810
	50	0.1435	0.0000	0.1435	0.0005	0.0005	0.0010	0.1665	0.0000	0.1665	0.1025	0.0000	0.1025
	60	0.1255	0.0000	0.1255	0.0000	0.0000	0.0000	0.1710	0.0000	0.1710	0.1060	0.0000	0.1060
	80	0.1150	0.0000	0.1150	0.0000	0.0000	0.0000	0.1750	0.0000	0.1750	0.0905	0.0000	0.0905
	100	0.0930	0.0000	0.0930	0.0000	0.0000	0.0000	0.2130	0.0000	0.2130	0.1010	0.0000	0.1010
	20	0.2320	0.0010	0.2330	0.0010	0.0005	0.0015	0.1810	0.0005	0.1815	0.1040	0.0015	0.1055
	25	0.2280	0.0000	0.2280	0.0000	0.0020	0.0020	0.1680	0.0005	0.1685	0.0940	0.0000	0.0940
	30	0.1910	0.0000	0.1910	0.0000	0.0000	0.0000	0.1950	0.0000	0.1950	0.0995	0.0000	0.0995
	40	0.1915	0.0000	0.1915	0.0000	0.0000	0.0000	0.1795	0.0000	0.1795	0.0905	0.0000	0.0905
0.3	50	0.1725	0.0000	0.1725	0.0000	0.0000	0.0000	0.1980	0.0000	0.1980	0.0950	0.0000	0.0950
	60	0.1655	0.0000	0.1655	0.0000	0.0000	0.0000	0.1980	0.0000	0.1980	0.1140	0.0000	0.1140
	80	0.1555	0.0000	0.1555	0.0000	0.0000	0.0000	0.2040	0.0000	0.2040	0.0990	0.0000	0.0990
	100	0.1275	0.0000	0.1275	0.0000	0.0000	0.0000	0.2220	0.0000	0.2220	0.0980	0.0000	0.0980
	20	0.2370	0.0010	0.2380	0.0010	0.0000	0.0010	0.1765	0.0000	0.1765	0.1070	0.0005	0.1075
	25	0.2460	0.0000	0.2460	0.0000	0.0000	0.0000	0.2095	0.0000	0.2095	0.1030	0.0000	0.1030
	30	0.2195	0.0000	0.2195	0.0000	0.0000	0.0000	0.2280	0.0000	0.2280	0.1035	0.0000	0.1035
0.4	40	0.2315	0.0000	0.2315	0.0000	0.0000	0.0000	0.2010	0.0000	0.2010	0.0955	0.0000	0.0955
	50	0.2150	0.0000	0.2150	0.0000	0.0000	0.0000	0.2195	0.0000	0.2195	0.0950	0.0000	0.0950
	60	0.2060	0.0000	0.2060	0.0000	0.0000	0.0000	0.2235	0.0000	0.2235	0.0905	0.0000	0.0905
	80	0.1900	0.0000	0.1900	0.0000	0.0000	0.0000	0.2460	0.0000	0.2460	0.0990	0.0000	0.0990
	100	0.1745	0.0000	0.1745	0.0000	0.0000	0.0000	0.2495	0.0000	0.2495	0.1030	0.0000	0.1030
	20	0.2620	0.0000	0.2620	0.0005	0.0000	0.0005	0.1955	0.0000	0.1955	0.1200	0.0005	0.1205
	25	0.2660	0.0000	0.2660	0.0000	0.0000	0.0000	0.1955	0.0000	0.1955	0.1110	0.0000	0.1110
	30	0.2515	0.0000	0.2515	0.0000	0.0000	0.0000	0.2065	0.0000	0.2065	0.1125	0.0000	0.1125
0.5	40	0.2645	0.0000	0.2645	0.0000	0.0000	0.0000	0.2635	0.0000	0.2635	0.1045	0.0000	0.1045
	50	0.2430	0.0000	0.2430	0.0000	0.0000	0.0000	0.2735	0.0000	0.2735	0.0980	0.0000	0.0980
	60	0.2375	0.0000	0.2375	0.0000	0.0000	0.0000	0.2675	0.0000	0.2675	0.0985	0.0000	0.0985
	80	0.2380	0.0000	0.2380	0.0000	0.0000	0.0000	0.2875	0.0000	0.2875	0.0840	0.0000	0.0840
	100	0.2135	0.0000	0.2135	0.0000	0.0000	0.0000	0.3125	0.0000	0.3125	0.1020	0.0000	0.1020



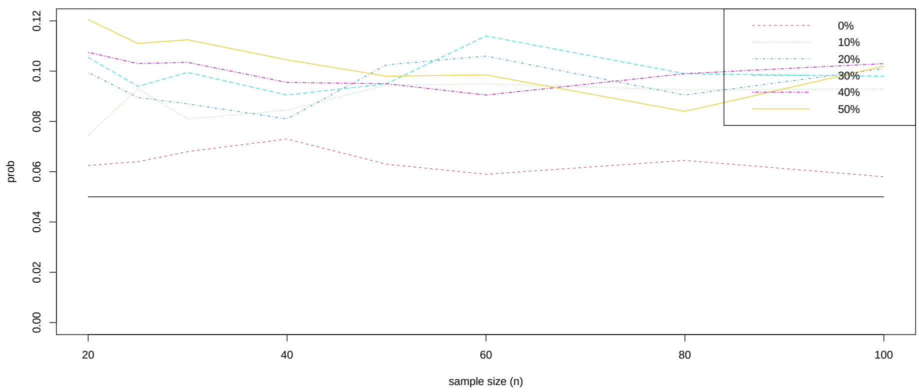
(a) tep for μ



(b) tep for α



(c) tep for b_0



(d) tep for b_1

Figure 4: Plots of tep for parameters for various sample sizes for Wald Interval 5%

Table 6: Total number of AC, C, and AS intervals for all parameters using Wald intervals at various sample sizes, censored proportions and 10% significance level

α	0.10												
Parameter		$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$		
cp	n	AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
0	20	0	10	10	0	10	10	3	0	2	3	0	10
	25	0	10	10	0	10	10	3	0	0	4	0	10
	30	0	10	10	0	10	10	0	0	0	6	0	10
	40	4	0	9	0	10	10	0	0	4	7	0	10
	50	1	0	10	0	10	10	0	0	2	1	0	10
	60	0	0	10	0	10	10	0	0	1	4	0	10
	80	0	1	10	0	10	10	0	0	0	1	0	10
	100	0	3	8	0	10	10	0	0	1	1	0	10
0.1	20	10	0	10	0	10	10	3	0	10	4	0	7
	25	10	0	10	0	10	10	9	0	10	0	0	10
	30	10	0	10	0	10	9	10	0	10	0	0	10
	40	10	0	10	0	10	3	10	0	10	10	0	10
	50	10	0	10	0	10	7	10	0	10	7	0	10
	60	10	0	10	0	10	7	10	0	10	6	0	10
	80	10	0	10	0	10	7	10	0	10	5	0	10
	100	5	0	10	0	10	4	10	0	10	9	0	10
0.2	20	10	0	10	0	10	9	10	0	10	5	0	10
	25	10	0	10	0	10	7	10	0	10	1	0	10
	30	10	0	10	0	10	9	10	0	10	5	0	10
	40	10	0	10	0	10	8	10	0	10	10	0	10
	50	10	0	10	0	10	3	10	0	10	9	0	10
	60	10	0	10	0	10	1	10	0	10	8	0	10
	80	10	0	10	0	10	0	10	0	10	8	0	10
	100	10	0	10	0	10	0	10	0	10	9	0	10
0.3	20	10	0	10	0	10	9	10	0	10	10	0	10
	25	10	0	10	0	10	8	10	0	10	7	0	10
	30	10	0	10	0	10	3	10	0	10	6	0	10
	40	10	0	10	0	10	0	10	0	10	10	0	10
	50	10	0	10	0	10	0	10	0	10	10	0	10
	60	10	0	10	0	10	0	10	0	10	9	0	10
	80	10	0	10	0	10	0	10	0	10	5	0	10
	100	10	0	10	0	10	0	10	0	10	7	0	10
0.4	20	10	0	10	0	10	2	10	0	10	10	0	10
	25	10	0	10	0	10	0	10	0	10	10	0	10
	30	10	0	10	0	10	0	10	0	10	10	0	10
	40	10	0	10	0	10	0	10	0	10	3	0	10
	50	10	0	10	0	10	0	10	0	10	8	0	10
	60	10	0	10	0	10	0	10	0	10	10	0	10
	80	10	0	10	0	10	0	10	0	10	7	0	10
	100	10	0	10	0	10	0	10	0	10	6	0	10
0.5	20	10	0	10	0	10	1	10	0	10	10	0	10
	25	10	0	10	0	10	0	10	0	10	10	0	10
	30	10	0	10	0	10	0	10	0	10	10	0	10
	40	10	0	10	0	10	0	10	0	10	10	0	10
	50	10	0	10	0	10	0	10	0	10	6	0	10
	60	10	0	10	0	10	0	10	0	10	7	0	10
	80	10	0	10	0	10	0	10	0	10	10	0	10
	100	10	0	10	0	10	0	10	0	10	8	0	10

Table 7: Total number of AC, C, and AS intervals for all parameters using Wald intervals at various sample sizes, censored proportions and 5% significance level

α		0.05											
Parameter			$\hat{\mu}$			$\hat{\alpha}$			$\hat{b_0}$			$\hat{b_1}$	
cp	n	AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
0	20	0	0	10	0	10	10	6	0	2	8	0	10
	25	0	2	10	0	10	10	5	0	0	9	0	10
	30	0	7	10	0	10	10	3	0	0	10	0	10
	40	0	10	10	0	10	10	1	0	1	10	0	10
	50	7	0	8	0	10	10	1	0	1	7	0	10
	60	10	0	10	0	10	10	1	0	1	6	0	10
	80	1	0	10	0	6	10	0	0	0	4	0	10
	100	0	0	6	0	4	10	0	0	3	1	0	10
0.1	20	10	0	10	0	10	10	10	0	9	10	0	1
	25	10	0	10	0	10	10	10	0	10	10	0	10
	30	10	0	10	0	10	10	10	0	10	10	0	10
	40	10	0	10	0	10	9	10	0	10	10	0	10
	50	10	0	10	0	10	7	10	0	10	10	0	10
	60	10	0	10	0	10	7	10	0	10	10	0	10
	80	10	0	10	0	10	7	10	0	10	10	0	10
	100	10	0	10	0	10	4	10	0	10	10	0	10
0.2	20	10	0	10	0	10	10	10	0	10	10	0	10
	25	10	0	10	0	10	9	10	0	10	10	0	10
	30	10	0	10	0	10	9	10	0	10	10	0	10
	40	10	0	10	0	10	8	10	0	10	10	0	10
	50	10	0	10	0	10	3	10	0	10	10	0	10
	60	10	0	10	0	10	1	10	0	10	10	0	10
	80	10	0	10	0	10	0	10	0	10	10	0	10
	100	10	0	10	0	10	0	10	0	10	10	0	10
0.3	20	10	0	10	0	10	9	10	0	10	10	0	10
	25	10	0	10	0	10	8	10	0	10	10	0	10
	30	10	0	10	0	10	3	10	0	10	10	0	10
	40	10	0	10	0	10	0	10	0	10	10	0	10
	50	10	0	10	0	10	0	10	0	10	10	0	10
	60	10	0	10	0	10	0	10	0	10	10	0	10
	80	10	0	10	0	10	0	10	0	10	10	0	10
	100	10	0	10	0	10	0	10	0	10	10	0	10
0.4	20	10	0	10	0	10	2	10	0	10	10	0	10
	25	10	0	10	0	10	0	10	0	10	10	0	10
	30	10	0	10	0	10	0	10	0	10	10	0	10
	40	10	0	10	0	10	0	10	0	10	10	0	10
	50	10	0	10	0	10	0	10	0	10	10	0	10
	60	10	0	10	0	10	0	10	0	10	10	0	10
	80	10	0	10	0	10	0	10	0	10	10	0	10
	100	10	0	10	0	10	0	10	0	10	10	0	10
0.5	20	10	0	10	0	10	1	10	0	10	10	0	10
	25	10	0	10	0	10	0	10	0	10	10	0	10
	30	10	0	10	0	10	0	10	0	10	10	0	10
	40	10	0	10	0	10	0	10	0	10	10	0	10
	50	10	0	10	0	10	0	10	0	10	10	0	10
	60	10	0	10	0	10	0	10	0	10	10	0	10
	80	10	0	10	0	10	0	10	0	10	10	0	10
	100	10	0	10	0	10	0	10	0	10	10	0	10

4. Conclusion

The extension of the GEM with a fixed covariate and interval-censored data shows that the bias, SE, and RMSE performed for the parameter estimates with higher censoring proportions, and smaller sample sizes. Accordingly, the estimates perform better when the sample size is larger, and censoring proportion is lower. The coverage probability for the Wald confidence interval have a notably high error probability for small sample size and lower error probabilities for large sample size. Also, the estimated error probabilities are lower when the censoring proportion of the data is low.

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